

On the CCN (de)activation nonlinearities

Sylwester Arabas and Shin-ichiro Shima

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 - ❖ 2017–2018: AETHON, Athens (H2020 “Innovation Associate”)

Shin-ichiro Shima

≡ Google Scholar



Shin-ichiro Shima

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Nonlinear Processes
in Geophysics



On the CCN (de)activation nonlinearities

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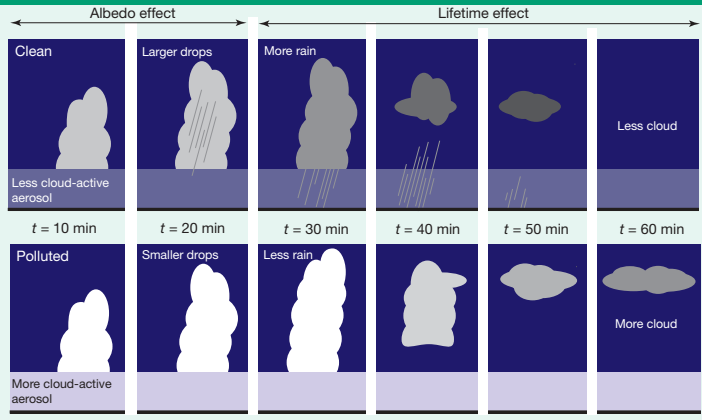
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one-slide aerosol-cloud (micro-macro) interaction primer

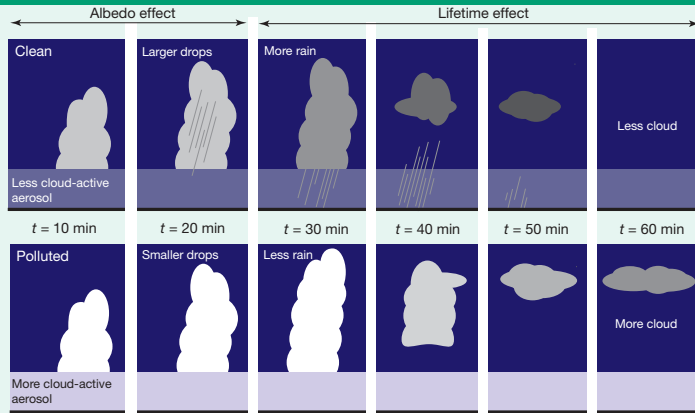
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Stevens and Feingold, 2009 (Nature)



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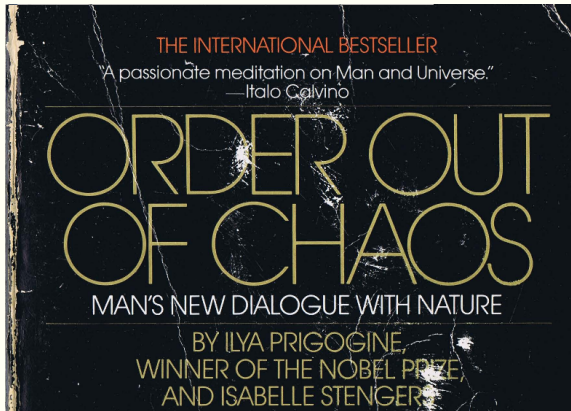


Stevens and Boucher, 2012 (Nature)

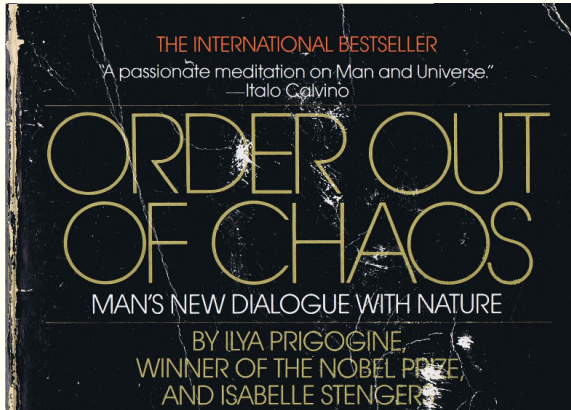
"there is something captivating about the idea that fine particulate matter, suspended almost invisibly in the atmosphere, holds the key to some of the greatest mysteries of climate science"

... others captivated by micro-macro interactions

... others captivated by micro-macro interactions



... others captivated by micro-macro interactions



Prigogine and Stengers 1984

"Much of this book has centered around the relation between the microscopic and the macroscopic. One of the most important problems in evolutionary theory is the eventual feedback between macroscopic structures and microscopic events: macroscopic structures emerging from microscopic events would in turn lead to a modification of the microscopic mechanisms."

regime-transition (bifurcation) example from P&S 1984

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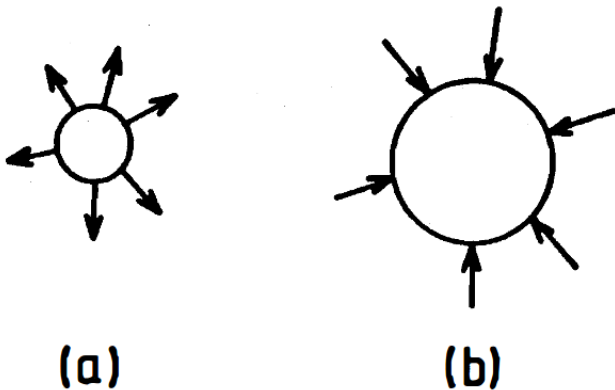


Figure 19. Nucleation of a liquid droplet in a supersaturated vapor. (a) droplet smaller than the critical size; (b) droplet larger than the critical size. The existence of the threshold has been experimentally verified for dissipative structures.

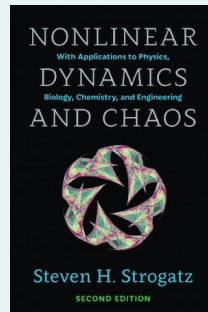
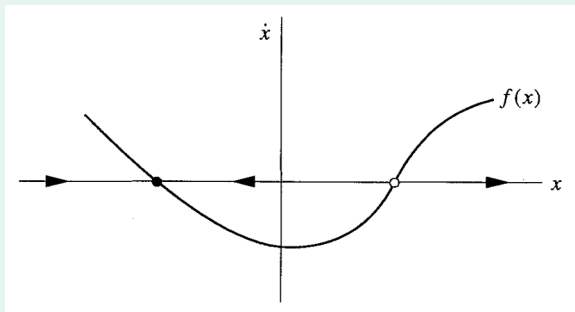
two-slide bifurcation analysis primer (1/2)

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Strogatz 2014 (sect. 2.2): fixed points and stability

graphical (qualitative) analysis
of a non-linear one-dimensional dynamical system:

$$\dot{x} = f(x)$$



two-slide bifurcation analysis primer (2/2)

Strogatz 2014 (sect. 3.1): saddle-node bifurcation

prototypical example of saddle-node bifurcation:

$$\dot{x} = r + x^2$$

r : parameter (distinct regimes if positive, negative or zero)

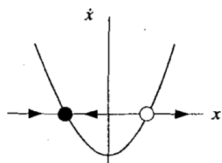
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Strogatz 2014 (sect. 3.1): saddle-node bifurcation

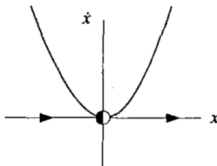
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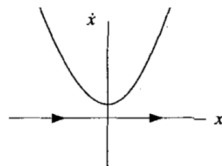
r : parameter (distinct regimes if positive, negative or zero)



(a) $r < 0$



(b) $r = 0$



(c) $r > 0$

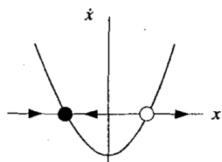
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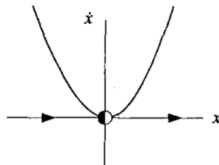
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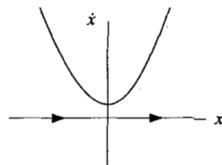
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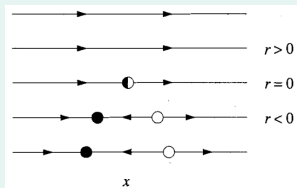
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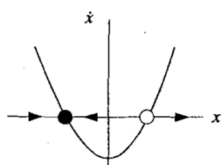
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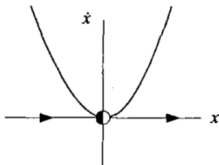
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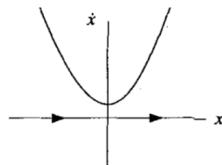
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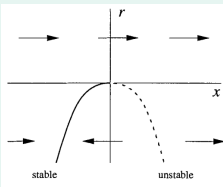
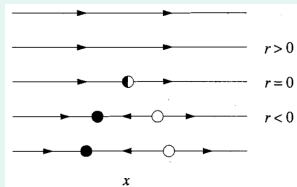
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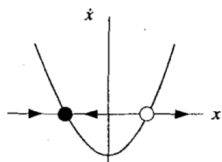
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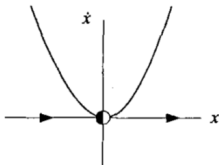
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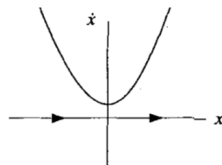
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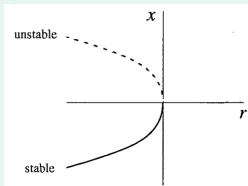
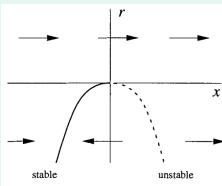
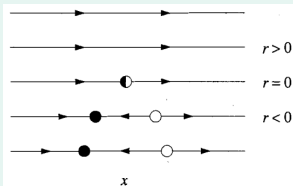
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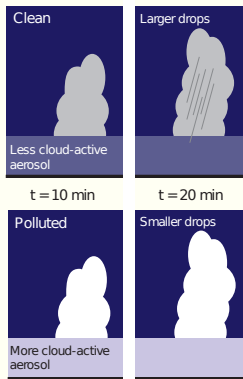
(b) $r = 0$



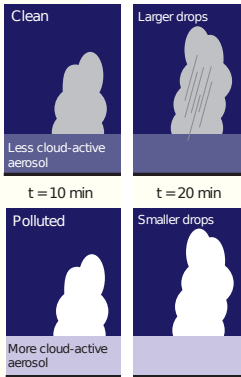
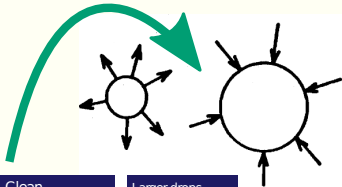
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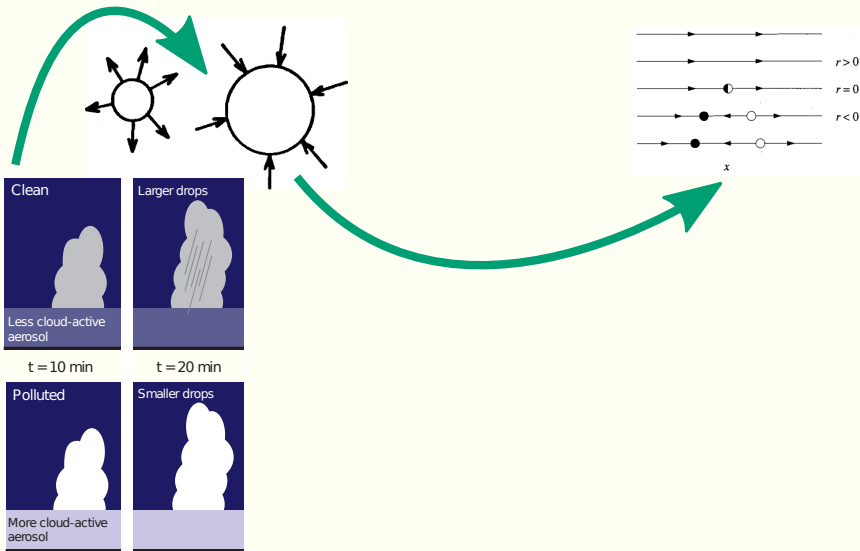
connecting the dots ...



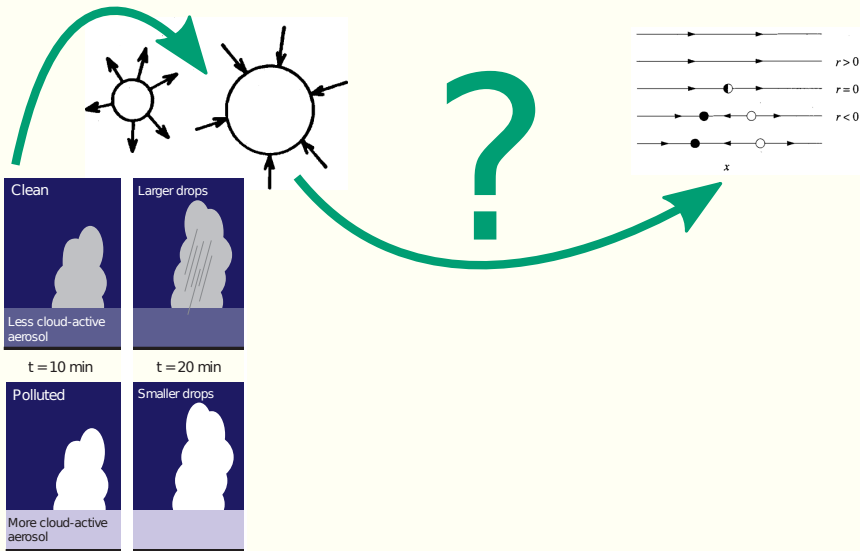
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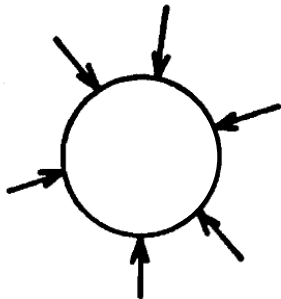
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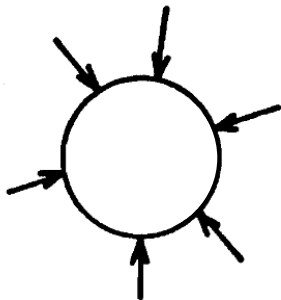


droplet growth laws in a nutshell: mass and heat diffusion



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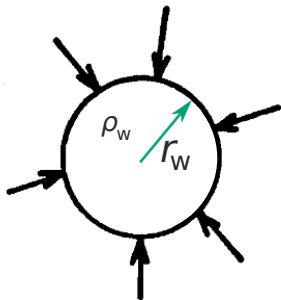
Fick's and Fourier's laws combined
spherical geometry



$$\dot{r}_w = \frac{1}{r_w} \frac{D_{\text{eff}}}{\rho_w} (\rho_v - \rho_o)$$

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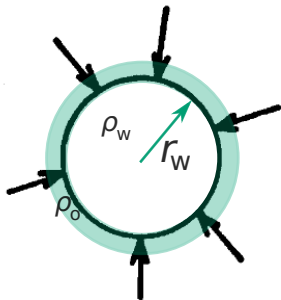
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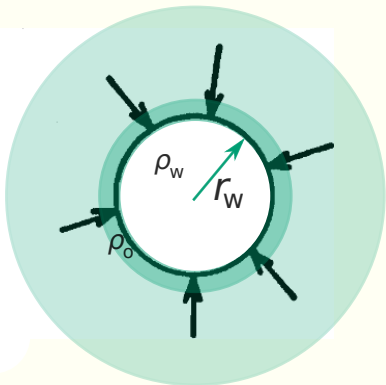


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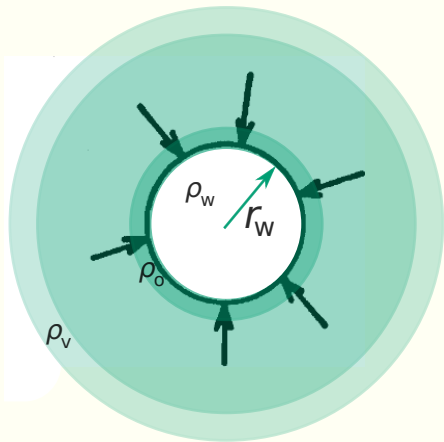
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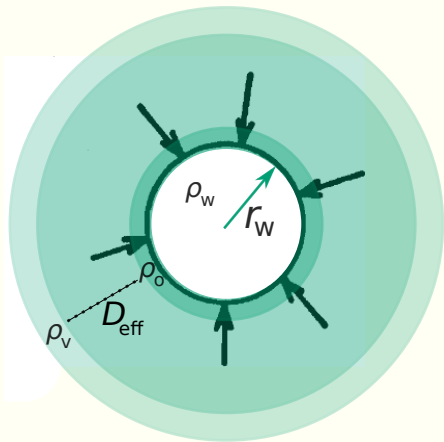
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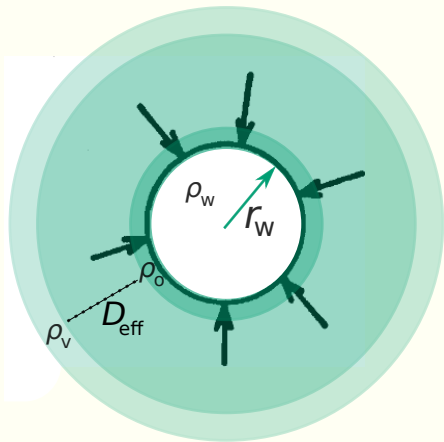
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non-dimensional numbers:

$$\text{RH} = \rho_v / \rho_{vs}$$

$$\text{RH}_{\text{eq}} = \rho_o / \rho_{vs}$$



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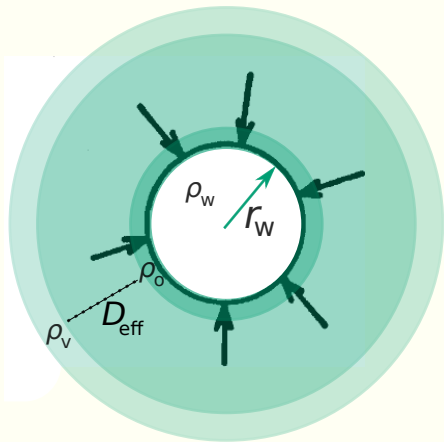
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droplet growth laws in a nutshell: Köhler curve

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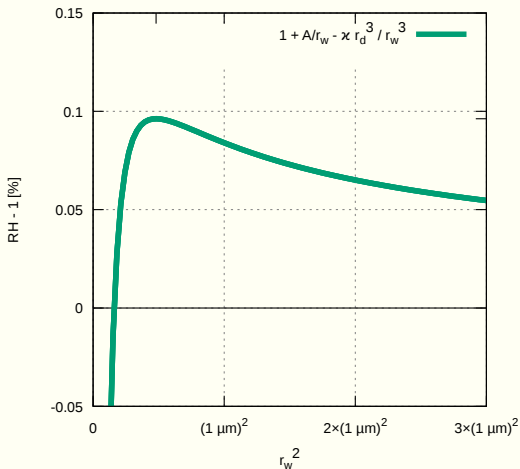
$$\begin{aligned} \text{RH}_{\text{eq}} &= \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A}{r_w}\right) \\ &\approx 1 + \frac{A}{r_w} - \frac{\kappa r_d^3}{r_w^3} \end{aligned}$$

droplet growth laws in a nutshell: Köhler curve

$$\dot{r}_w = \frac{1}{r_w} D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (RH - RH_{\text{eq}})$$

$$RH_{\text{eq}} = \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A}{r_w}\right)$$

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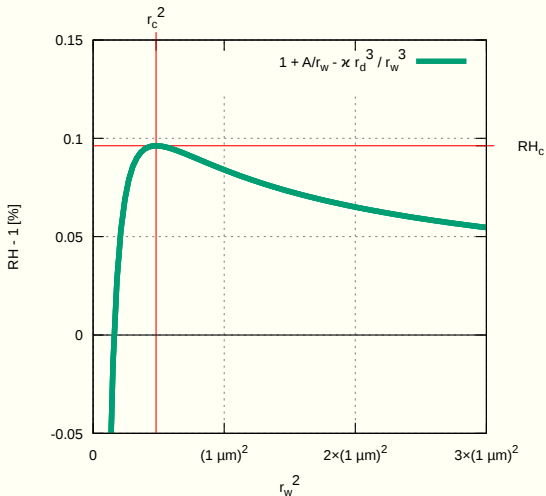


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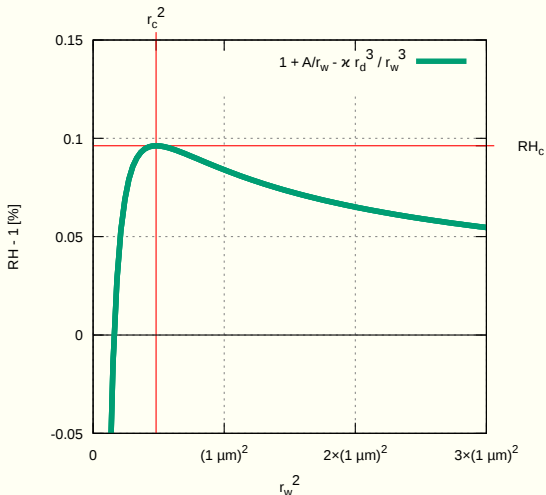


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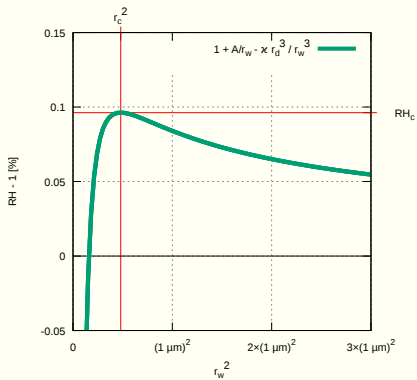


maximum at (r_c, RH_c) :

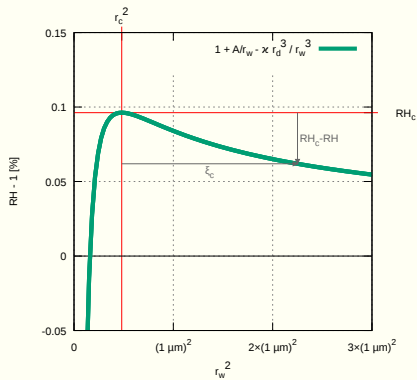
$$r_c = \sqrt{3\kappa r_d^3 / A}$$

$$RH_c = 1 + \frac{2A}{3r_c}$$

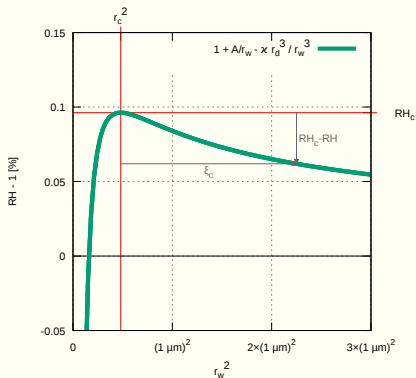
phase portrait of the system: flipped Köhler curve



phase portrait of the system: flipped Köhler curve



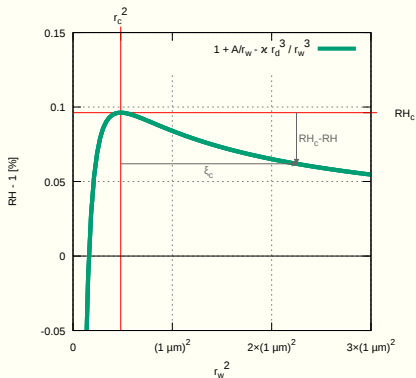
phase portrait of the system: flipped Köhler curve



$$\xi = r_w^2 + C$$

$$\dot{\xi} = 2D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (RH - RH_{\text{eq}}(\xi))$$

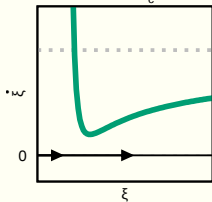
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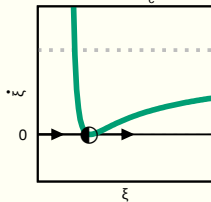
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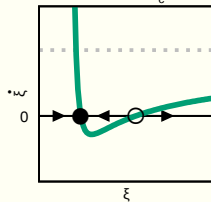
$RH > RH_c$



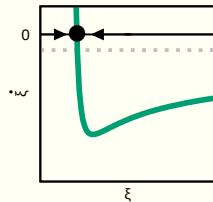
$RH = RH_c$



$1 < RH < RH_c$

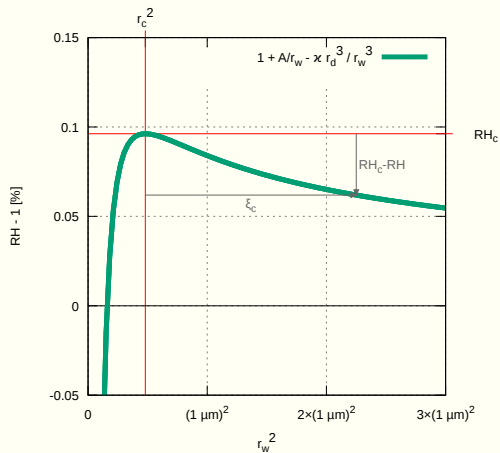


$RH < 1$



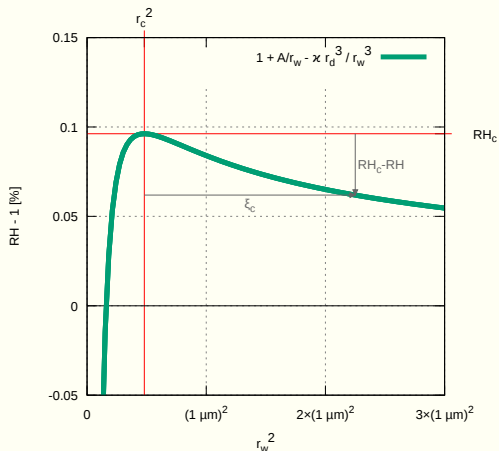
saddle-node bifurcation at Köhler curve maximum

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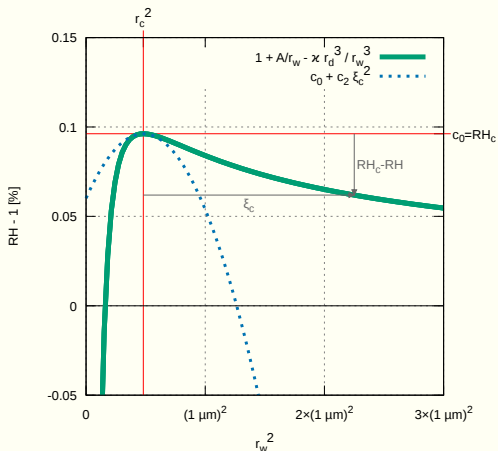
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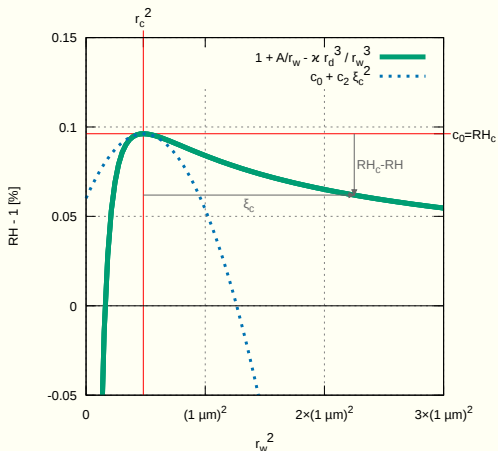
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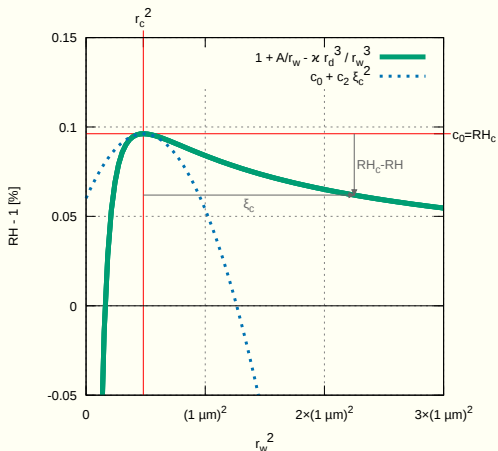


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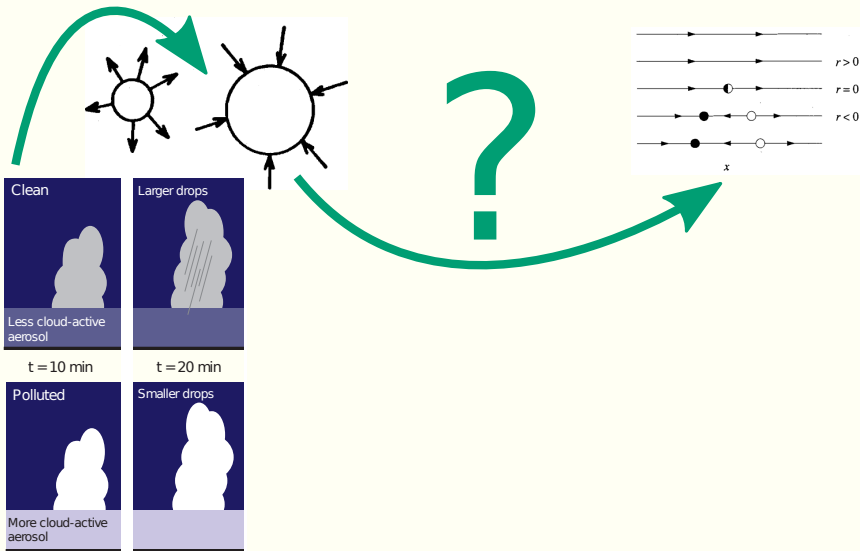
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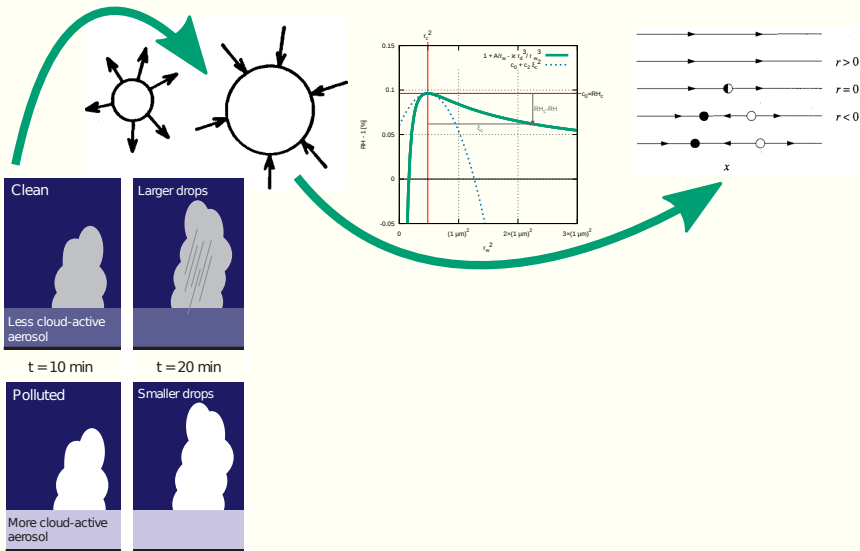
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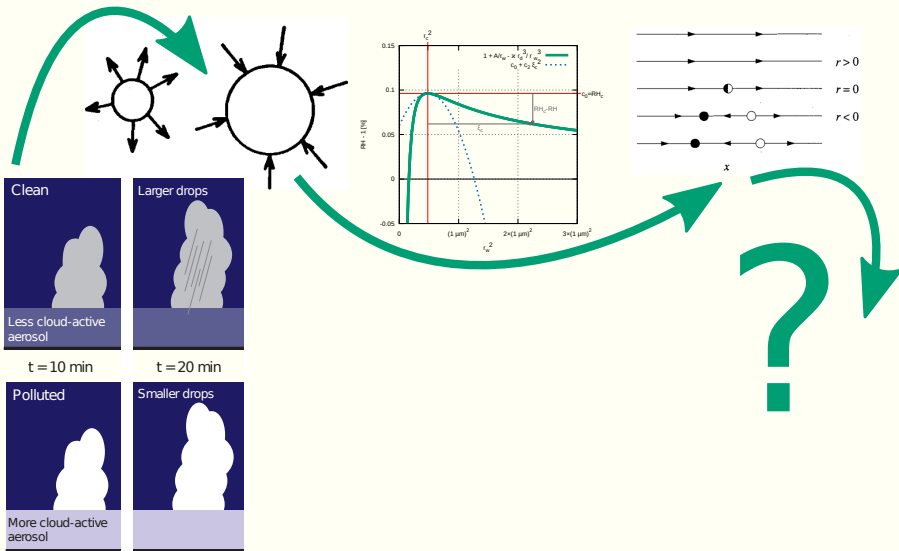
connecting the dots ...



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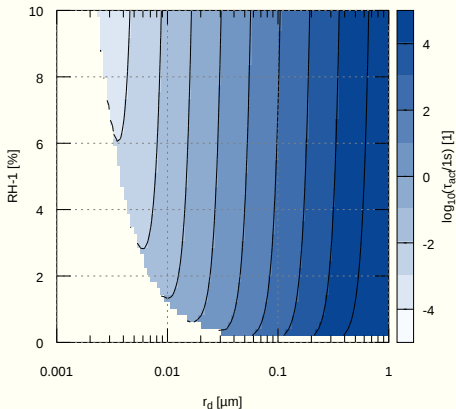
$$\begin{aligned}\tau_{\text{act}} &\approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c} \\ &= \frac{r_c^{5/2}}{\sqrt{A}} \frac{\rho_w/\rho_{vs}}{D_{\text{eff}}} \frac{\pi}{\sqrt{RH - RH_c}}\end{aligned}$$

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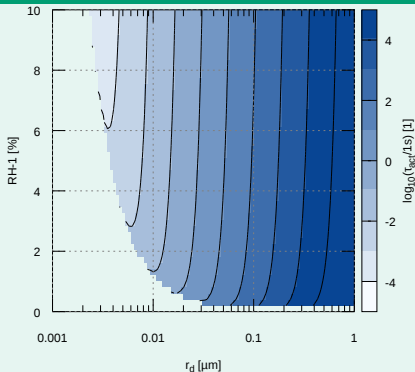
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activation timescale: analytic vs. numerical

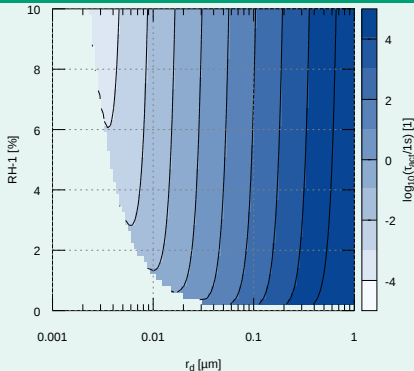
Arabas & Shima 2017



note: axes ranges vs. close-to-equilibrium assumption

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Hoffmann, 2016 (MWR)

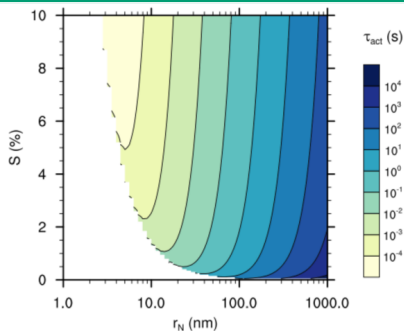
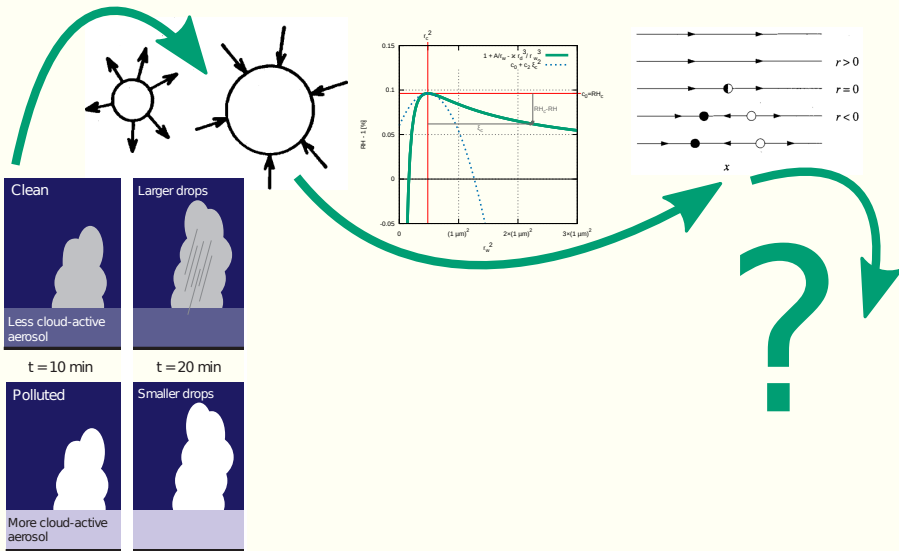


FIG. 2. The activation time scale τ_{act} as a function of dry aerosol radius r_N and supersaturation S . For values of $S < S_{\text{crit}}$ (white areas), τ_{act} does not exist.

$$r \frac{dr}{dt} = \left(S - \frac{A}{r} + \frac{Br_N^3}{r^3} \right) / (F_k + F_D), \quad (10)$$

The second time scale is associated with the activation of particles, for which Köhler theory is essential. This makes an analytic solution for (10) impossible. Numerically calculated values of τ_{act} measuring the time needed for a wetted aerosol to grow beyond its critical radius $r_{\text{crit}} = \sqrt{3Br_N^3/A}$ are given in Fig. 2 as a function of

connecting the dots ...



RH-coupled system & particle concentration as parameter

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simple moisture budget (const T,p):

$$\text{RH} \approx \frac{\dot{\rho}_v}{\rho_{vs}} = -N \underbrace{\frac{4\pi\rho_w}{3\rho_{vs}}}_{\alpha} 3r_w^2 \dot{r}_w$$

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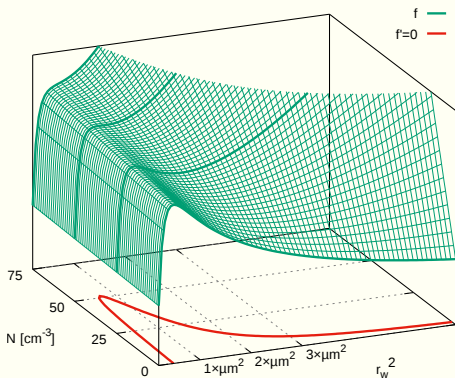
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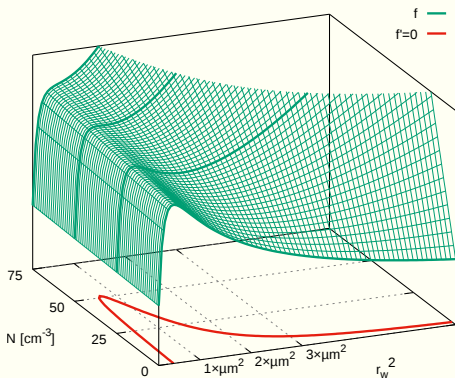
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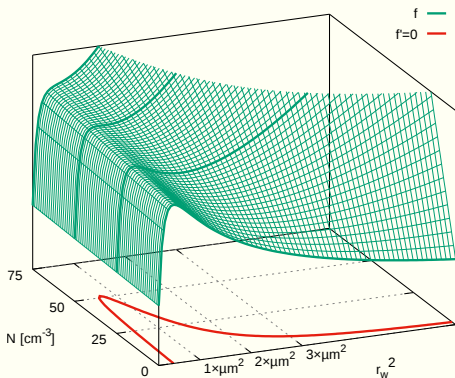
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$$\text{sgn}(f') = \text{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3\right)$$

bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

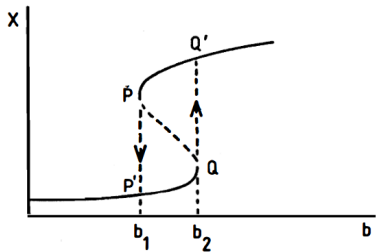


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter b first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while b grows. But at $b=b_2$, there will be a discontinuity: The system jumps from Q to Q' , on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till $b=b_1$, when it will jump down to P' . Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

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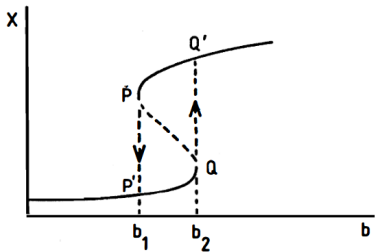
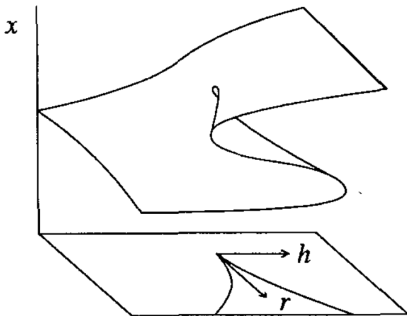


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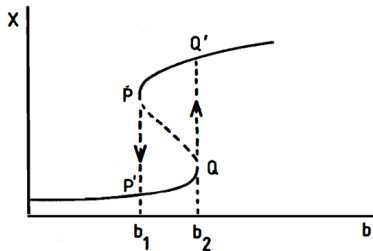
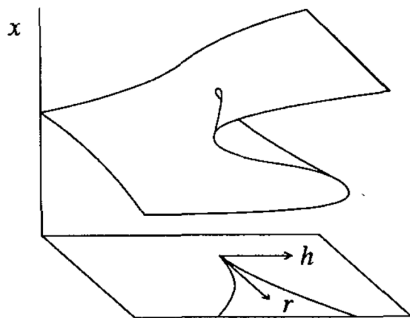


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\rightsquigarrow "jumps", hysteretic behaviour (r_w , RH) for small enough N , close to equilibrium (slow process)

hysteresis: activation/deactivation cycle



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■ nomenclature:

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vertically displaced (velocity w , hydrostatic background) adiabatic parcel:
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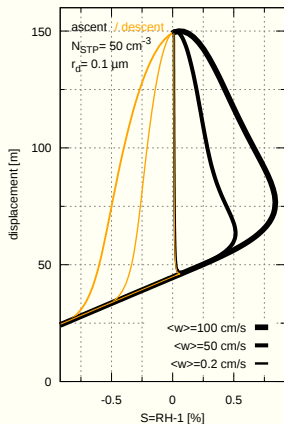
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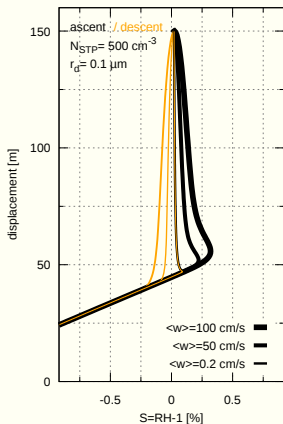
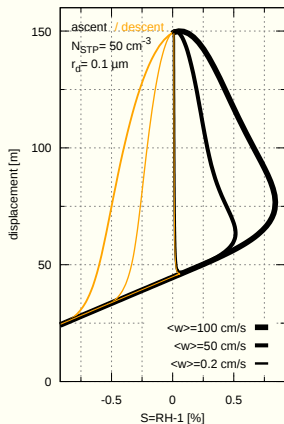
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parcel model: numerical integration (sinusoidal w)



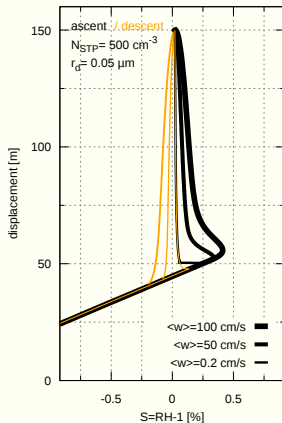
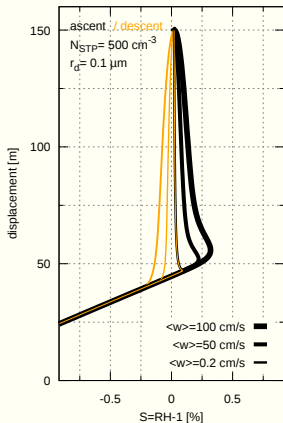
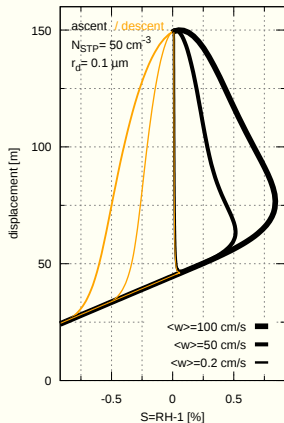
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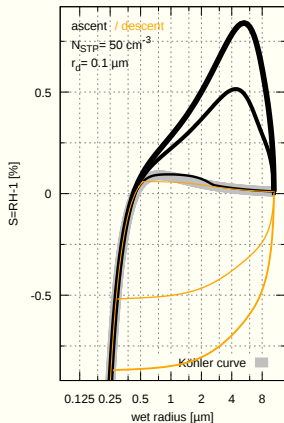
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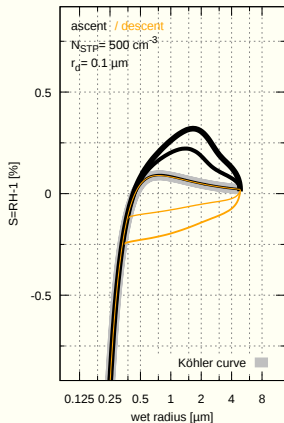
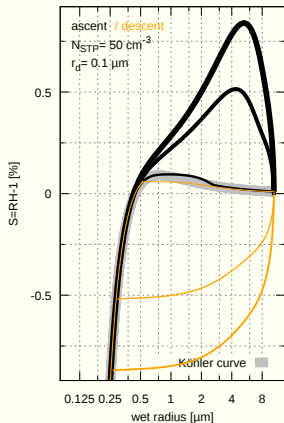
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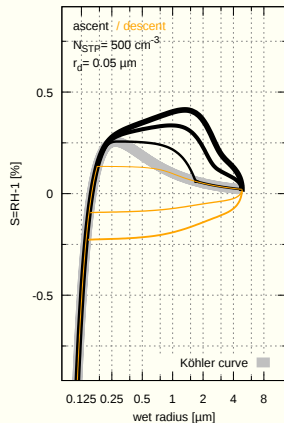
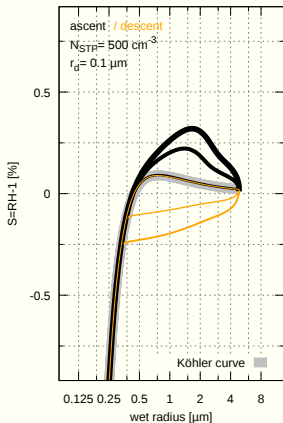
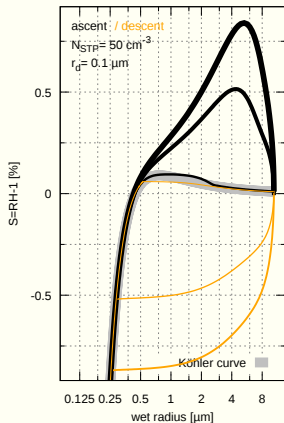
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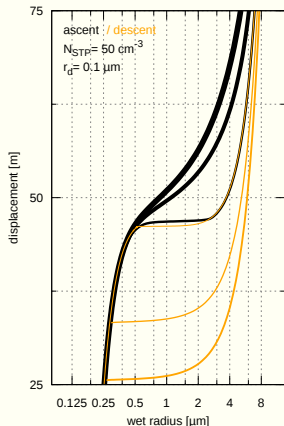
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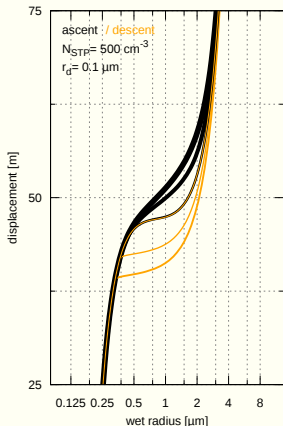
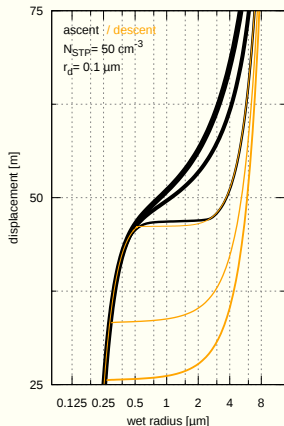
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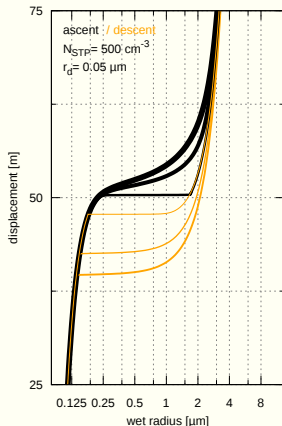
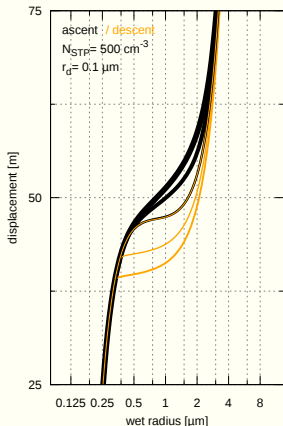
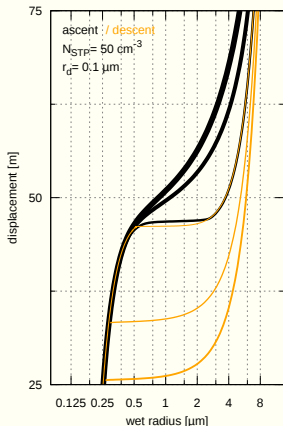
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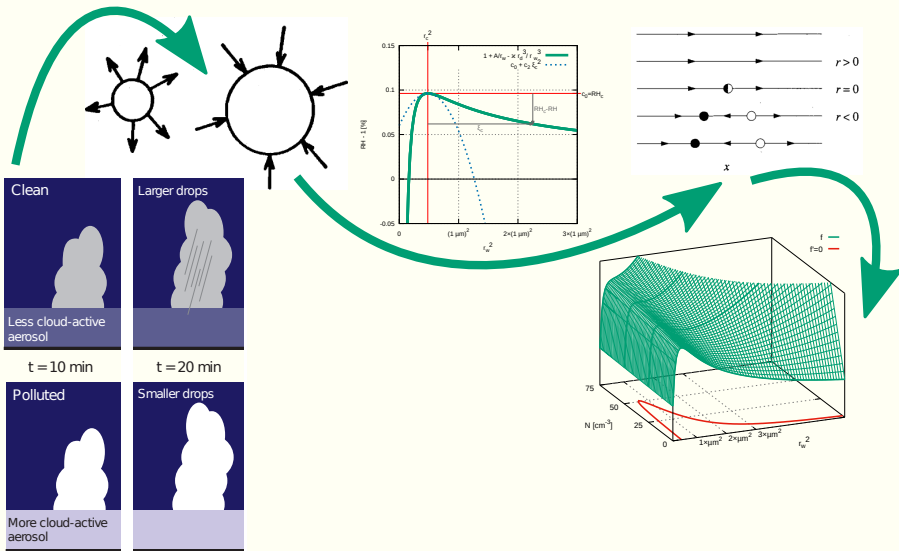
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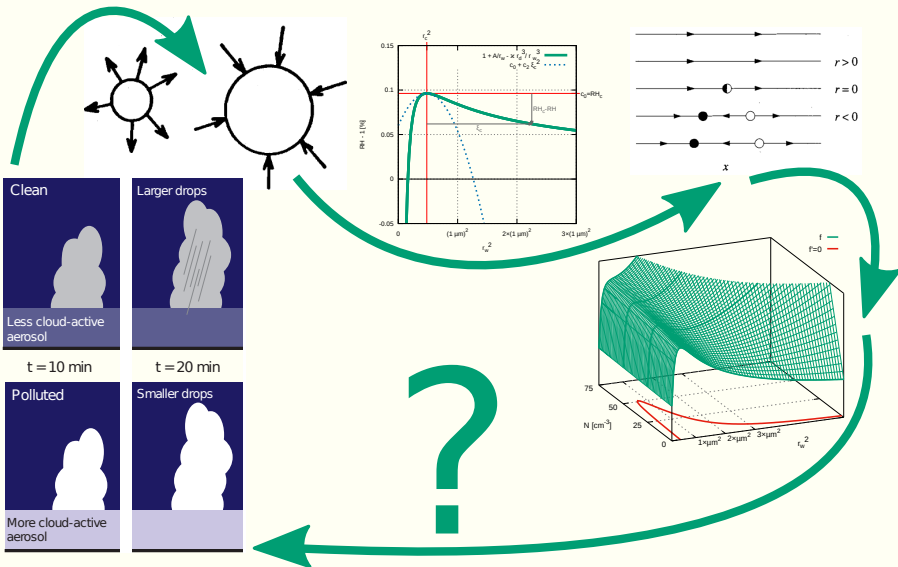


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connecting the dots ...



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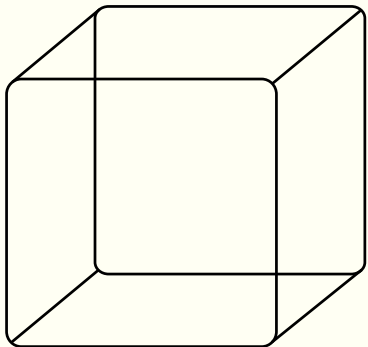
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particle-based μ -physics schemes for LES!
(Lagrangian Cloud Models / Super-Droplet Models)

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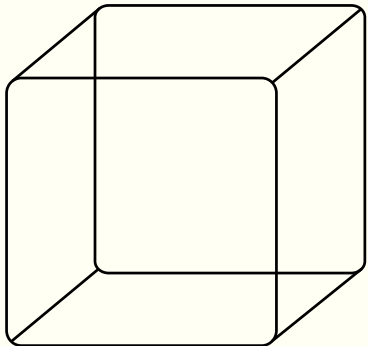
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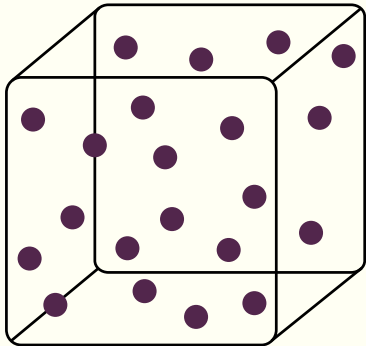
particle-based μ -physics for LES



- ❖ “information carriers” in LES domain
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particle=aerosol/cloud/rain
- ❖ attributes:

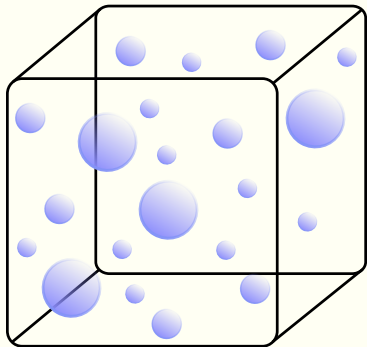


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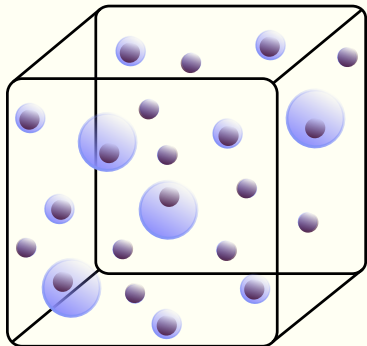
- ❖ “information carriers” in LES domain
- ❖ ab-initio approach:
particle=aerosol/cloud/rain
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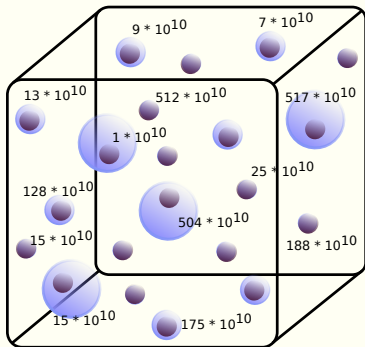
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 - ❖ wet radius

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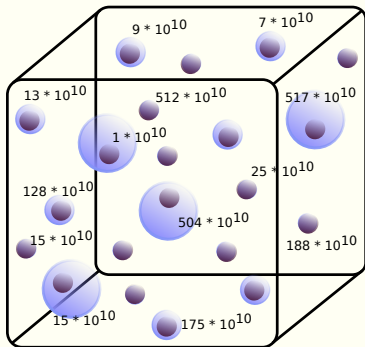
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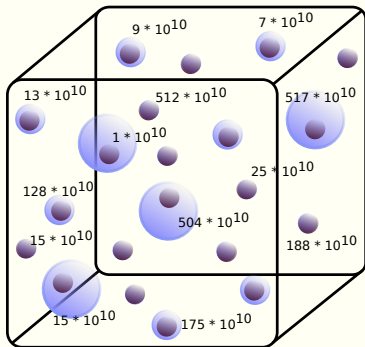
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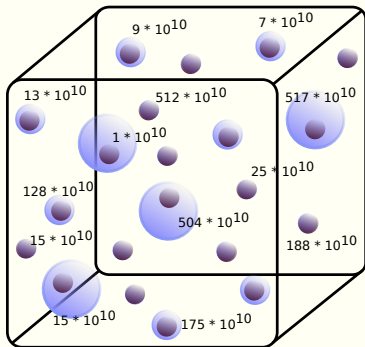
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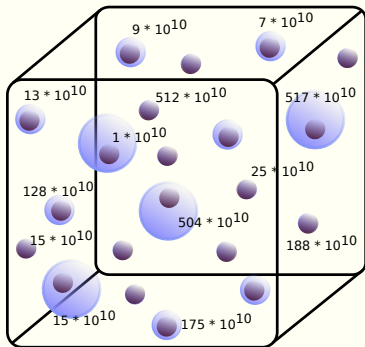
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(aqueous chemistry!)

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- ❖ each particle: **monodisperse!**
- ❖ each timestep: **constant RH!**

particle-based μ -physics for LES

Seminal works: [Shima et al. 2009](#), [Andrejczuk et al. 2010](#)
(3D simulations of atmospheric aerosol-cloud-precipitation system)

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particle-based μ -physics for LES

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- ❏ UWLCM⁷ from Univ. Warsaw (Grabowski et al. 2018).

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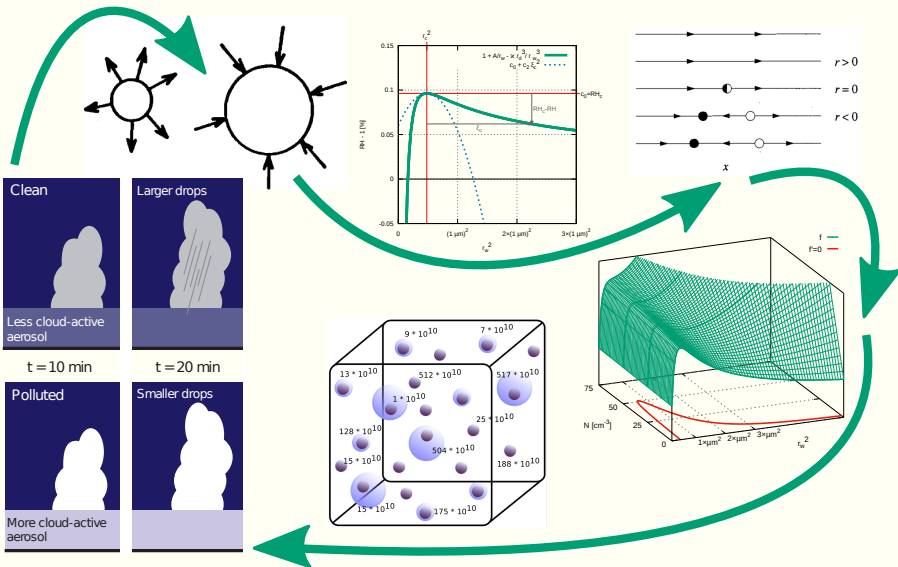
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⁶<http://scale.aics.riken.jp/>

⁷<http://github.com/igfuw/UWLCM>

connecting the dots ...



conclusions, takeaways, prospects

- CCN (de)activation as a bifurcating dynamical system

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- ❖ extensions: fluctuations, bi-/poly-disperse spectra, non-cloud appl.

Thank you for your attention!

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