

Bifurcations in a dynamical system describing formation of cloud droplets on atmospheric particulate matter

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 - ❖ 2017–2018: AETHON, Athens (H2020 “Innovation Associate”)

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Nonlinear Processes
in Geophysics



On the CCN (de)activation nonlinearities

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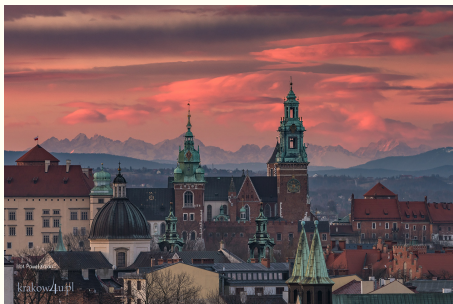
atmospheric particulate matter (PM) and clouds



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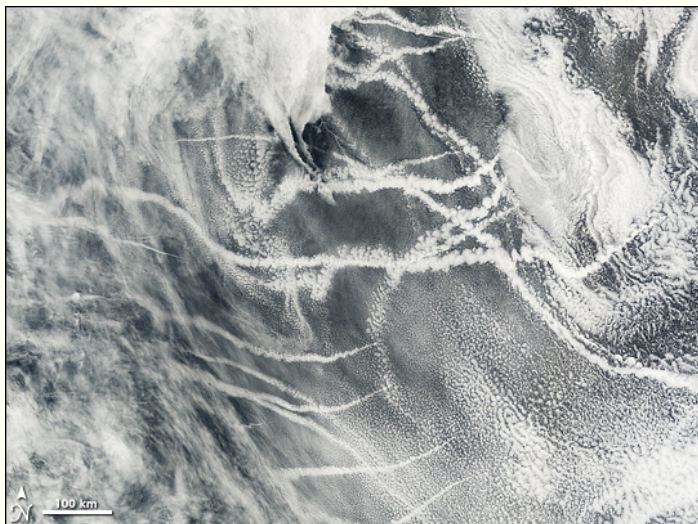


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no particles \rightsquigarrow no clouds

<https://www.youtube.com/watch?v=EneDwu0HrVg>

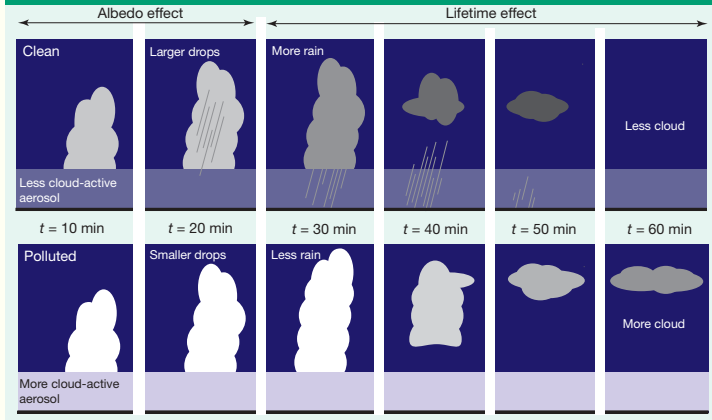
atmospheric particulate matter (PM) and clouds



NASA (<https://earthobservatory.nasa.gov/NaturalHazards/view.php?id=20248>)

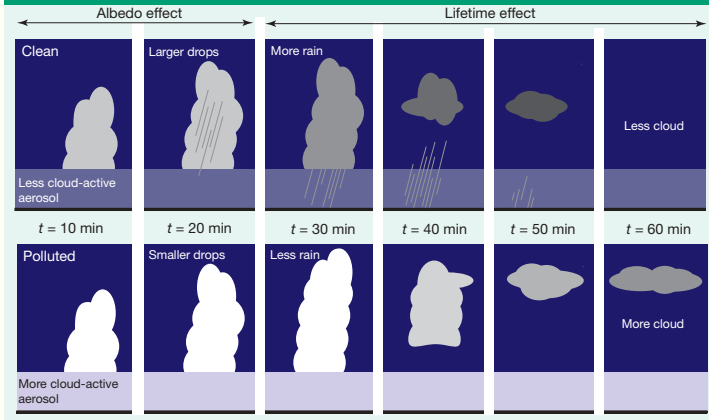
atmospheric particulate matter (PM) and clouds

Stevens and Feingold, 2009 (Nature)



atmospheric particulate matter (PM) and clouds

Stevens and Feingold, 2009 (Nature)



Stevens and Boucher, 2012 (Nature)

"there is something captivating about the idea that fine particulate matter, suspended almost invisibly in the atmosphere, holds the key to some of the greatest mysteries of climate science"

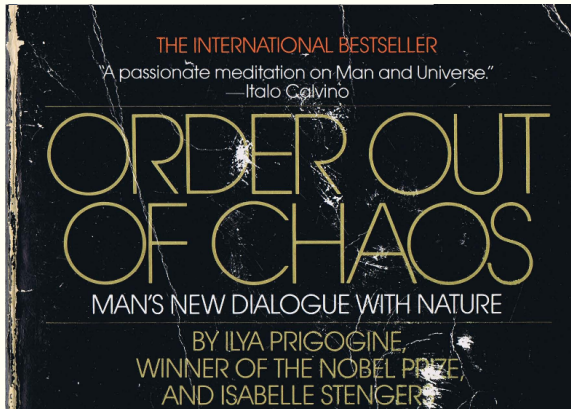
THE INTERNATIONAL BESTSELLER

"A passionate meditation on Man and Universe."
—Italo Calvino

ORDER OUT OF CHAOS

MAN'S NEW DIALOGUE WITH NATURE

BY ILYA PRIGOGINE
WINNER OF THE NOBEL PRIZE
AND ISABELLE STENGER



Prigogine and Stengers 1984

"Much of this book has centered around the relation between the microscopic and the macroscopic. One of the most important problems in evolutionary theory is the eventual feedback between macroscopic structures and microscopic events: macroscopic structures emerging from microscopic events would in turn lead to a modification of the microscopic mechanisms."

regime-transition (bifurcation) example from P&S 1984

ORDER OUT OF CHAOS 188

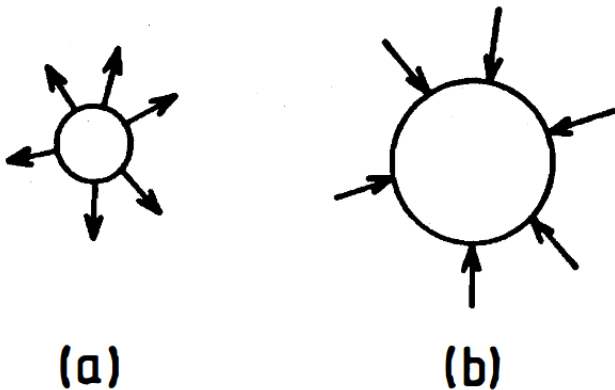
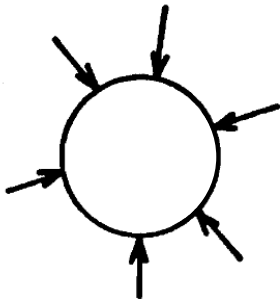


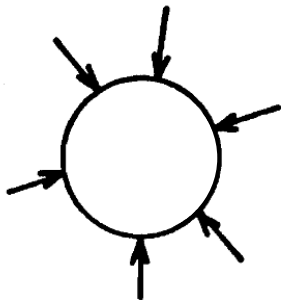
Figure 19. Nucleation of a liquid droplet in a supersaturated vapor. (a) droplet smaller than the critical size; (b) droplet larger than the critical size. The existence of the threshold has been experimentally verified for dissipative structures.

droplet growth laws in a nutshell: mass and heat diffusion



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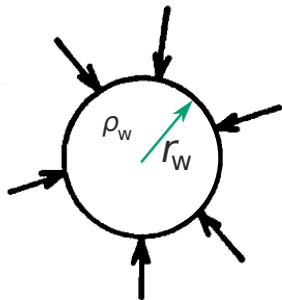
Fick's and Fourier's laws combined
spherical geometry



$$\dot{r}_w = \frac{1}{r_w} \frac{D_{\text{eff}}}{\rho_w} (\rho_v - \rho_o)$$

droplet growth laws in a nutshell: mass and heat diffusion

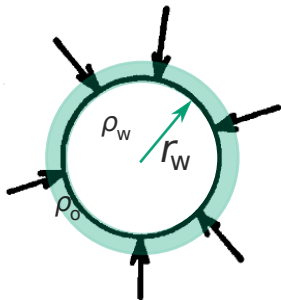
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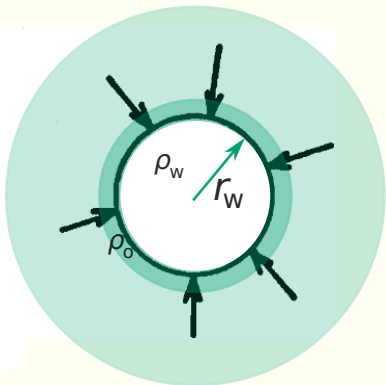


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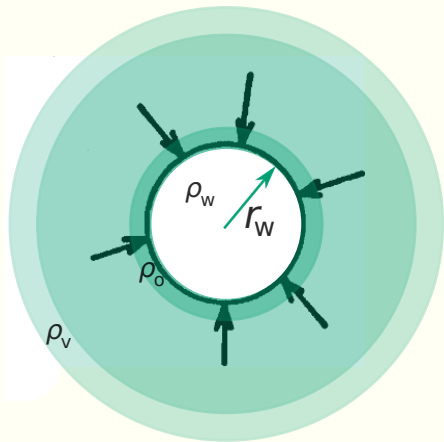
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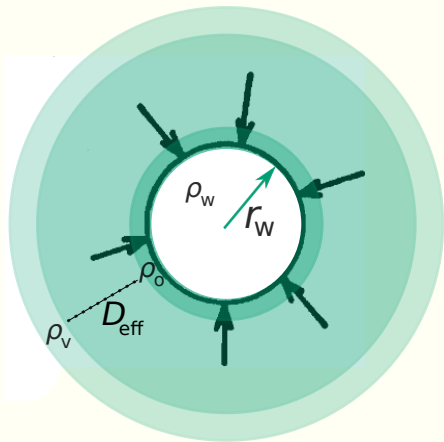
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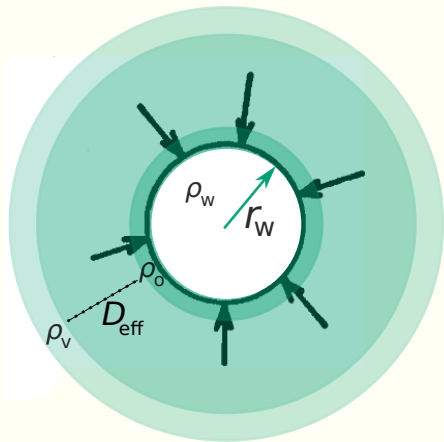
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non-dimensional numbers:

$$\text{RH} = \rho_v / \rho_{vs}$$

$$\text{RH}_{\text{eq}} = \rho_o / \rho_{vs}$$



droplet growth laws in a nutshell: mass and heat diffusion

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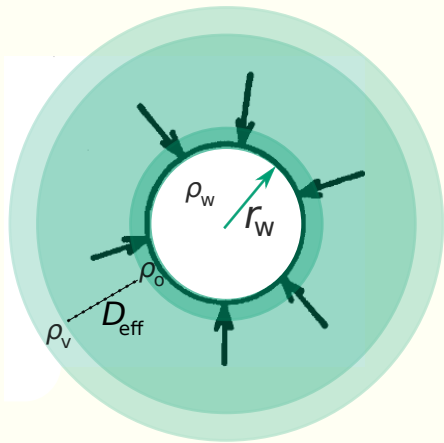
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droplet growth laws in a nutshell: Köhler curve

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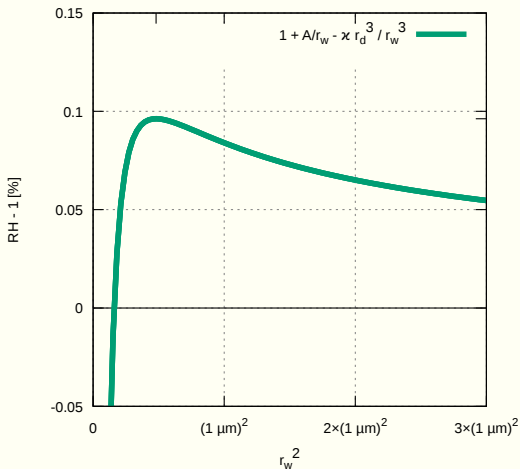
$$\begin{aligned} \text{RH}_{\text{eq}} &= \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A}{r_w}\right) \\ &\approx 1 + \frac{A}{r_w} - \frac{\kappa r_d^3}{r_w^3} \end{aligned}$$

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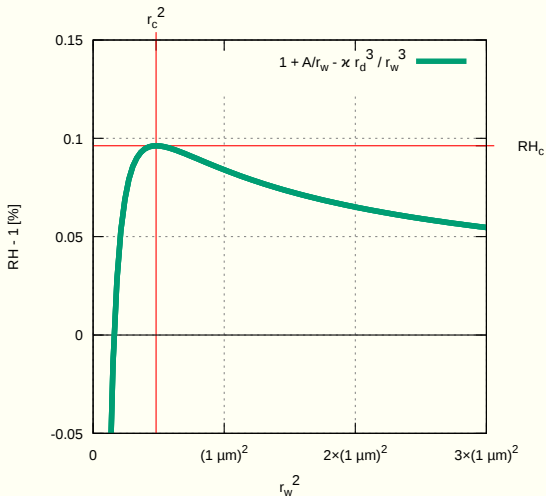


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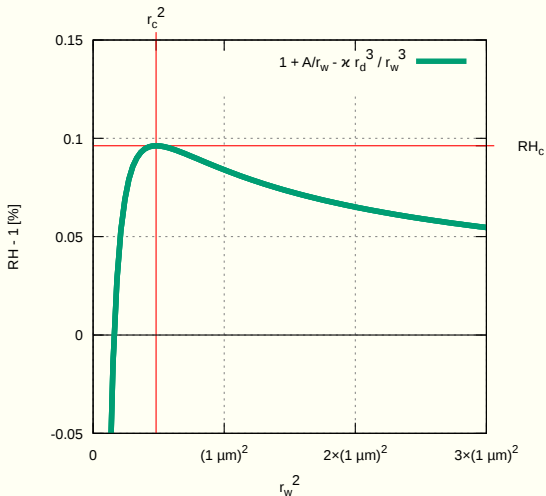


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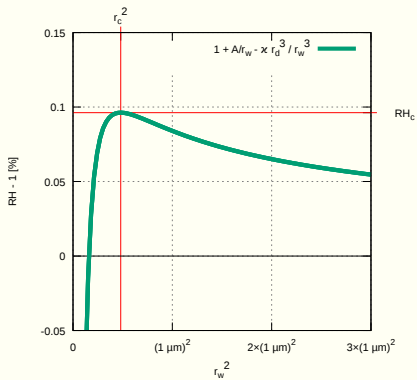


maximum at (r_c, RH_c) :

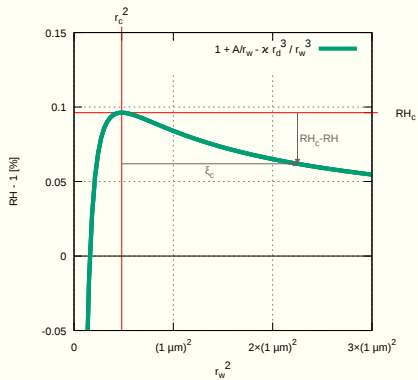
$$r_c = \sqrt{3\kappa r_d^3 / A}$$

$$RH_c = 1 + \frac{2A}{3r_c}$$

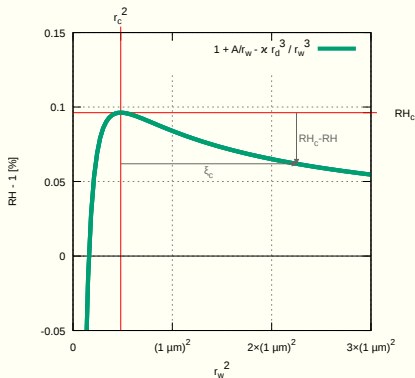
phase portrait of the system: flipped Köhler curve



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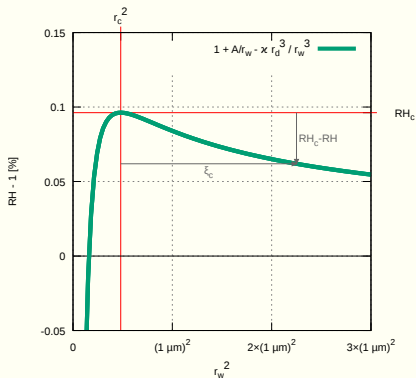
phase portrait of the system: flipped Köhler curve



$$\xi = r_w^2 + C$$

$$\dot{\xi} = 2D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (RH - RH_{\text{eq}}(\xi))$$

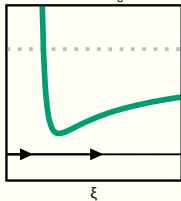
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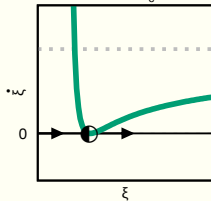
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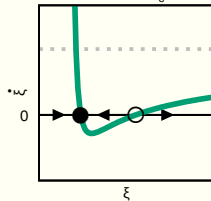
$RH > RH_c$



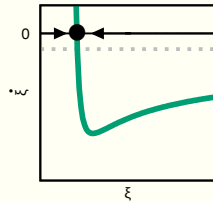
$RH = RH_c$



$1 < RH < RH_c$

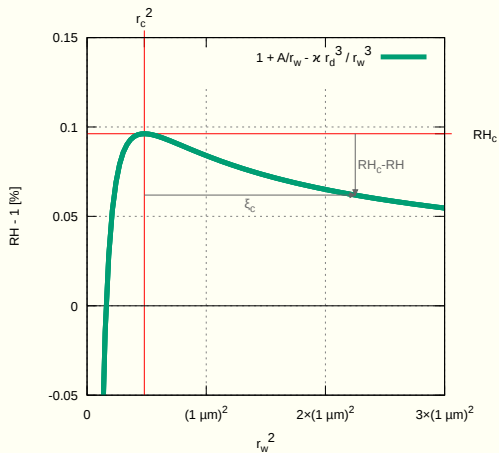


$RH < 1$



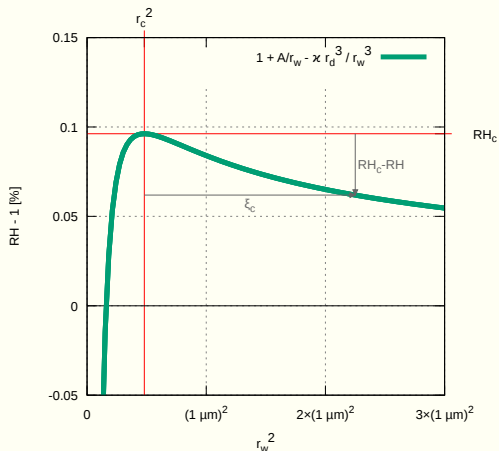
saddle-node bifurcation at Köhler curve maximum

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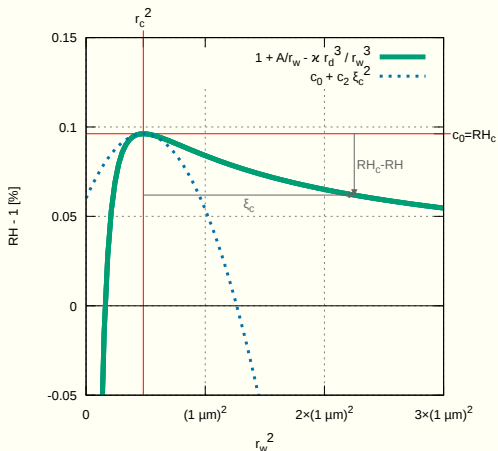
saddle-node bifurcation at Köhler curve maximum

$$RH_{eq}(\xi_c) = c_0 + c_1 \xi_c + c_2 \xi_c^2 + \dots$$



saddle-node bifurcation at Köhler curve maximum

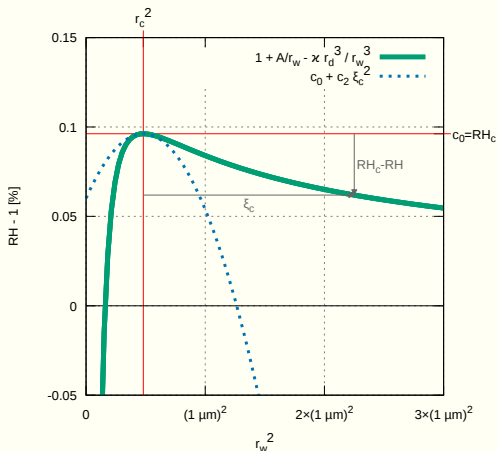
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$$\dot{\xi}_c \Big|_{\xi_c \rightarrow 0} \sim \frac{RH - RH_c}{A/(4r_c^5)} + \xi_c^2$$

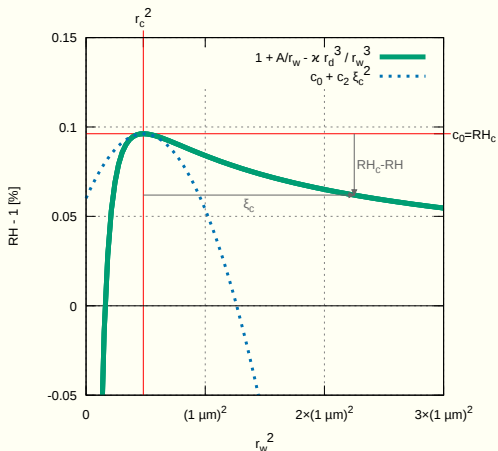


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$$\dot{x} = r + x^2$$



coalescence in the saddle-node bottleneck

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coalescence of the fixed points is associated with a passage through a *bottleneck* (e.g., Strogatz 2014),

key observation: time of passage through the parabolic *bottleneck* dominates all other timescales

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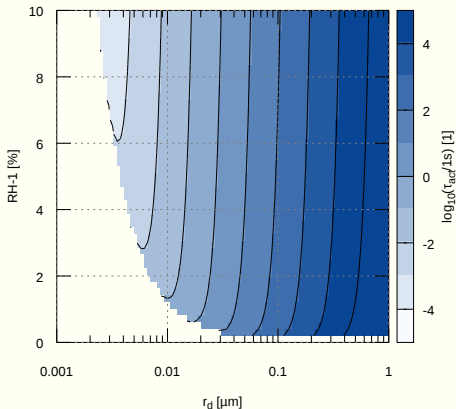
$$\begin{aligned}\tau_{\text{act}} &\approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c} \\ &= \frac{r_c^{5/2}}{\sqrt{A}} \frac{\rho_w/\rho_{vs}}{D_{\text{eff}}} \frac{\pi}{\sqrt{RH - RH_c}}\end{aligned}$$

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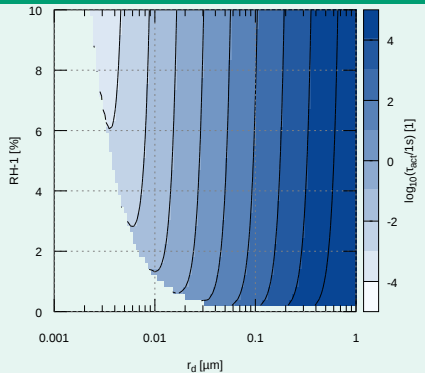
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activation timescale: analytic vs. numerical

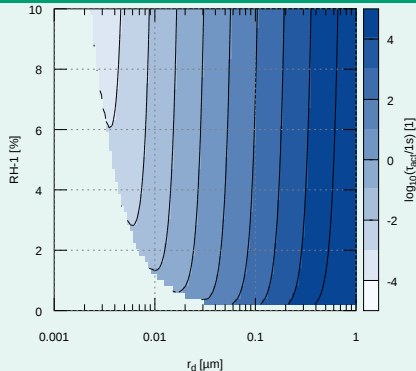
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note: axes ranges vs. close-to-equilibrium assumption

activation timescale: analytic vs. numerical

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Hoffmann, 2016 (MWR)

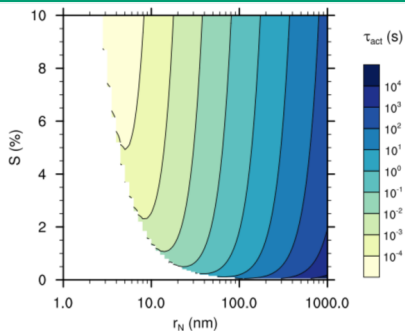


FIG. 2. The activation time scale τ_{act} as a function of dry aerosol radius r_N and supersaturation S . For values of $S < S_{\text{crit}}$ (white areas), τ_{act} does not exist.

$$r \frac{dr}{dt} = \left(S - \frac{A}{r} + \frac{Br_N^3}{r^3} \right) / (F_k + F_D), \quad (10)$$

The second time scale is associated with the activation of particles, for which Köhler theory is essential. This makes an analytic solution for (10) impossible. Numerically calculated values of τ_{act} measuring the time needed for a wetted aerosol to grow beyond its critical radius $r_{\text{crit}} = \sqrt{3Br_N^3/A}$ are given in Fig. 2 as a function of

RH-coupled system & particle concentration as parameter

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simple moisture budget (const T,p):

$$\text{RH} \approx \frac{\dot{\rho}_v}{\rho_{vs}} = -N \underbrace{\frac{4\pi\rho_w}{3\rho_{vs}}}_{\alpha} 3r_w^2 \dot{r}_w$$

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integrating in time:

$$RH = RH_0 - \alpha N r_w^3$$

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new phase portrait:

$$\dot{\xi} \sim (\text{RH}_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_d^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_f$$

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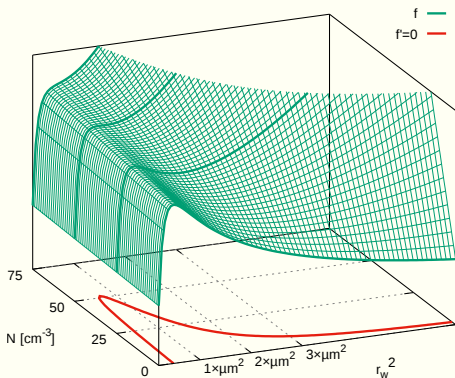
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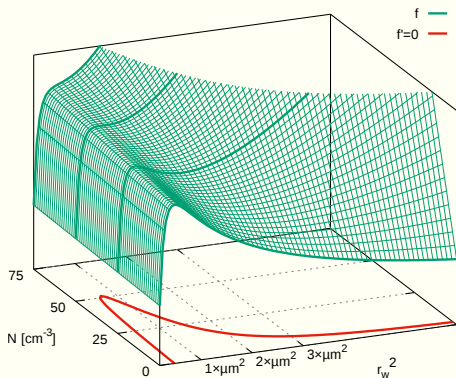
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regime-controlling params: RH, N



RH-coupled system & particle concentration as parameter

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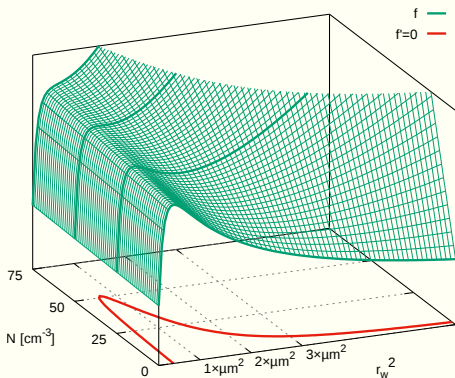
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$$RH = RH_0 - \alpha N r_w^3$$

new phase portrait:

$$\dot{\xi} \sim (RH_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_d^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_f$$

regime-controlling params: RH, N



$$\text{sgn}(f') = \text{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3\right)$$

bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

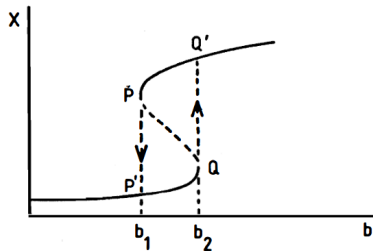


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter b first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while b grows. But at $b=b_2$, there will be a discontinuity: The system jumps from Q to Q' , on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till $b=b_1$, when it will jump down to P' . Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

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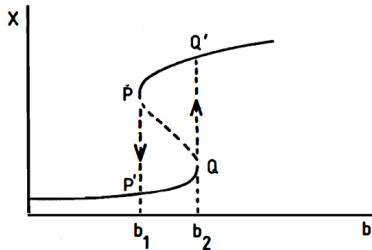
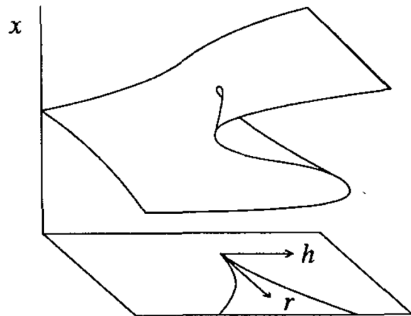


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Strogatz 2014



"cusp catastrophe"

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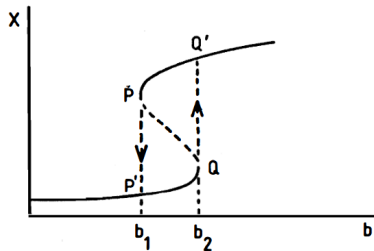
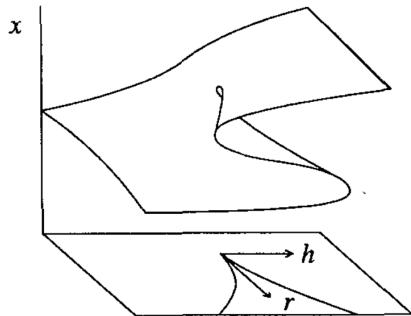


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"cusp catastrophe"

↪ "jumps", hysteretic behaviour (r_w , RH) for small enough N , close to equilibrium (slow process)

hysteresis: activation/deactivation cycle



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■ nomenclature:

hysteresis: activation/deactivation cycle



- ❖ nomenclature:
 - ❖ CCN activation
 - ❖ (heterogeneous) nucleation

hysteresis: activation/deactivation cycle



❖ nomenclature:

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- ❖ aerosol processing by clouds (aqueous chemistry, coalescence)

lifting the constant T-p assumptions: parcel model

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vertically displaced (velocity w , hydrostatic background) adiabatic parcel:
(q : mixing ratio, p_d : bgnd pressure, ρ_d bgnd density, g, l_v, c_{pd} : constants)

$$\begin{bmatrix} \dot{p}_d \\ \dot{T} \\ \dot{r}_w \end{bmatrix} = \begin{bmatrix} -\rho_d g w \\ (\dot{p}_d/\rho_d - \dot{q}l_v)/c_{pd} \\ (D_{\text{eff}}/\rho_w)(\rho_v - \rho_o)/r_w \end{bmatrix}$$

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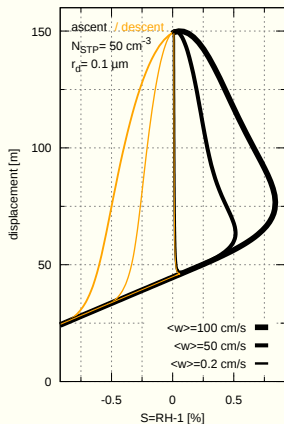
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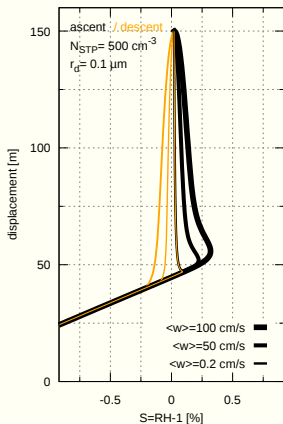
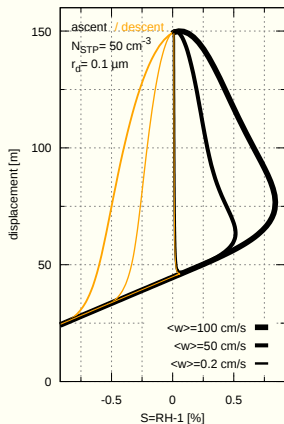
- ❑ $w \rightarrow 0$ (and hence $\dot{p}_d \approx 0$) i.e., slow, close-to-equilibrium evolution of the system relevant to fixed-point analysis (by some means pertinent to formation of non-convective clouds such as fog)
- ❑ $N \rightarrow 0$ (and hence $\dot{q} \approx 0$) i.e., weak coupling between particle size evolution and ambient thermodynamics (pertinent to the case of low particle concentration).

parcel model: numerical integration (sinusoidal w)



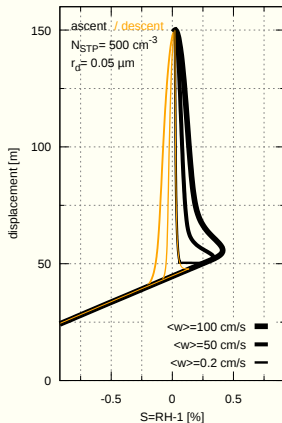
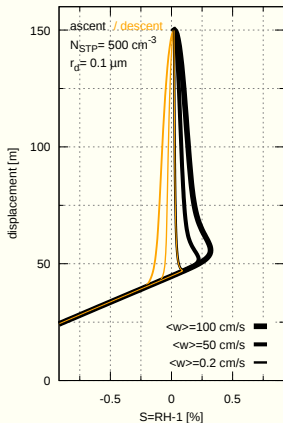
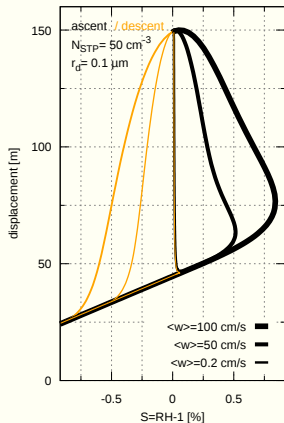
integration using CVODE adaptive solver
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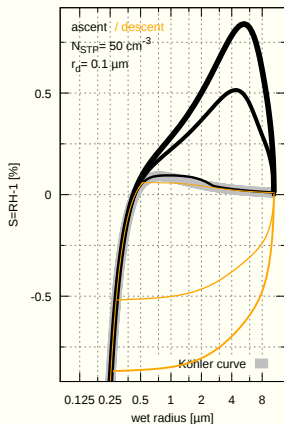
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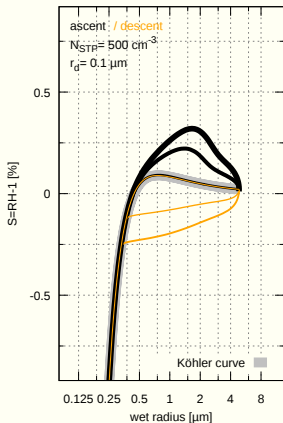
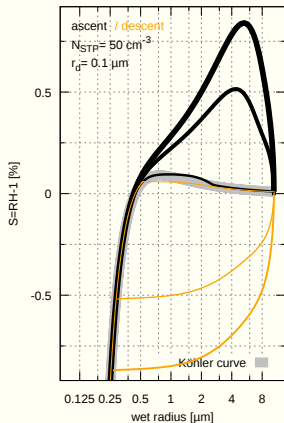
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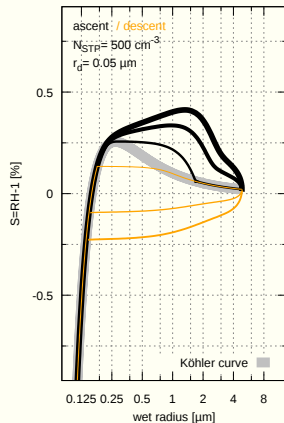
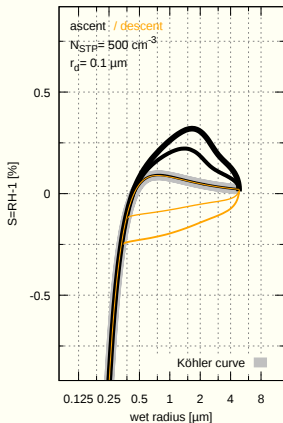
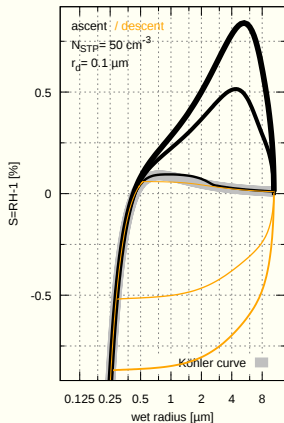
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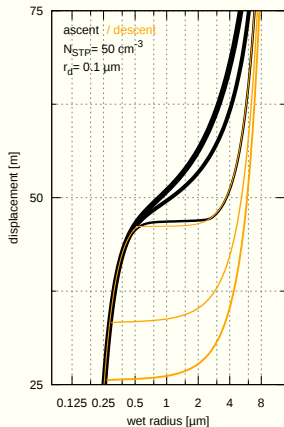
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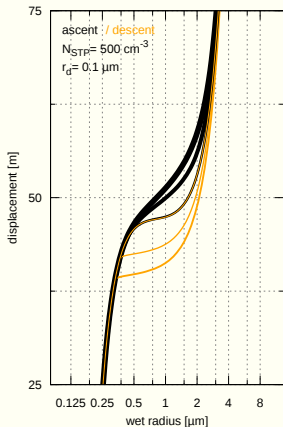
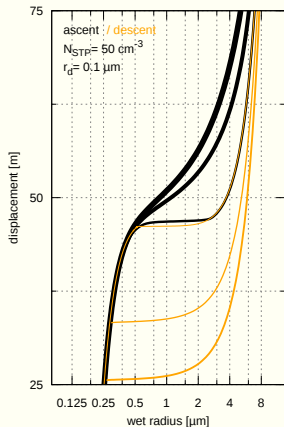
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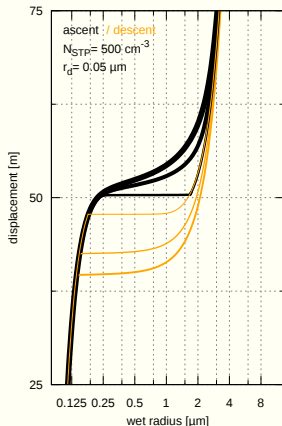
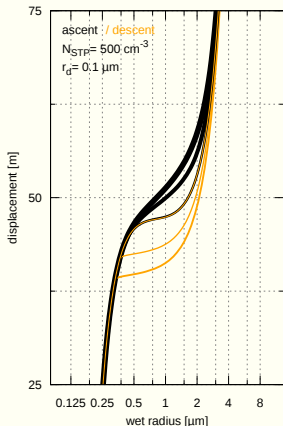
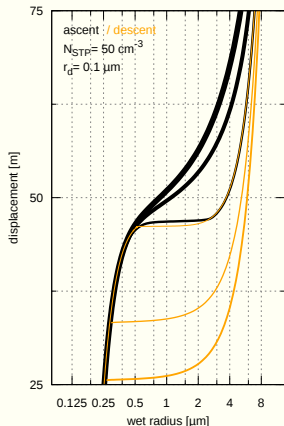
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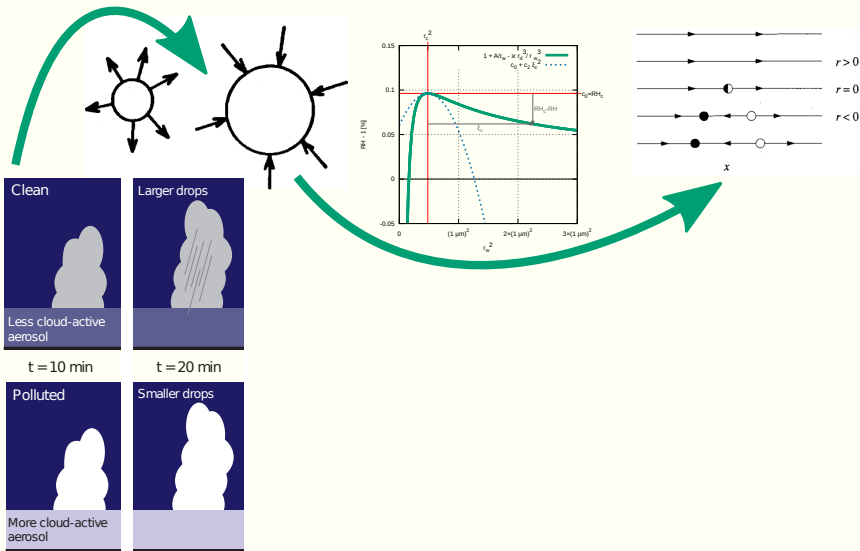
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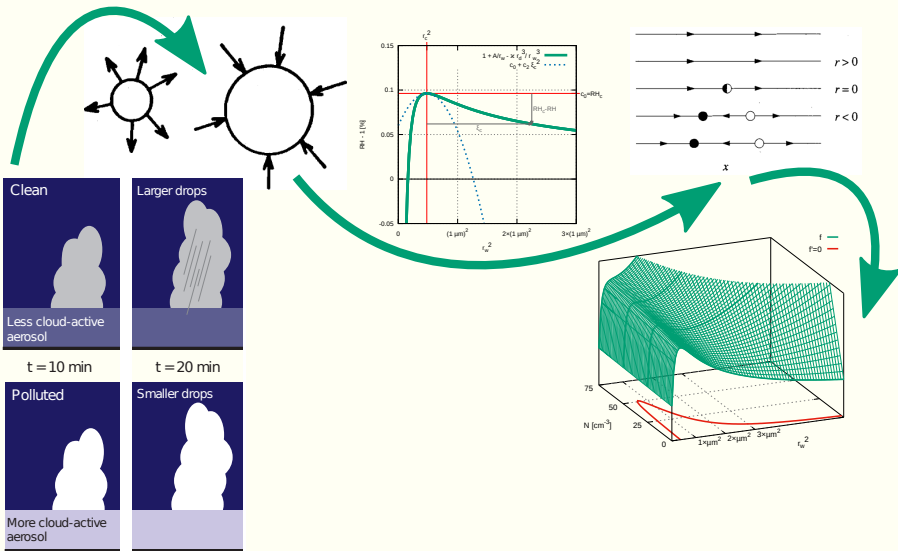


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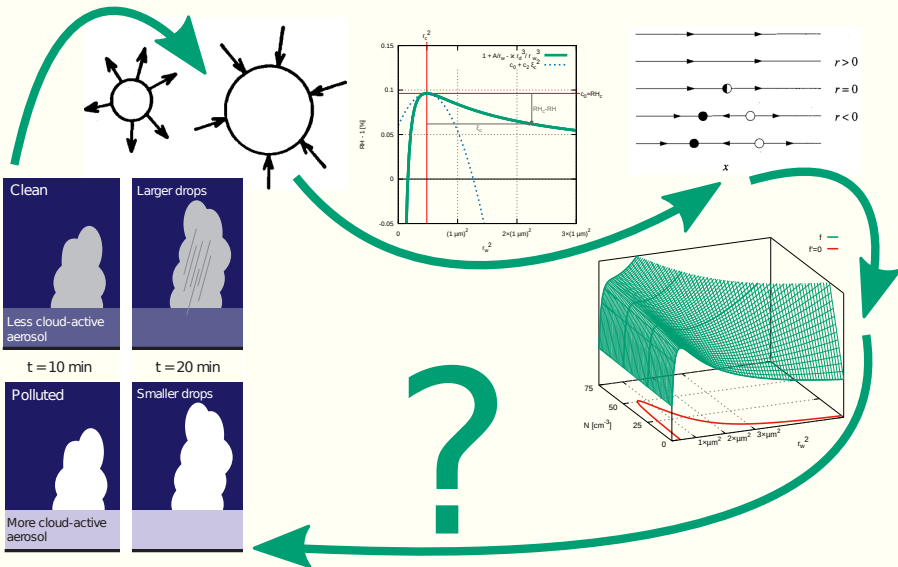
connecting the dots ...



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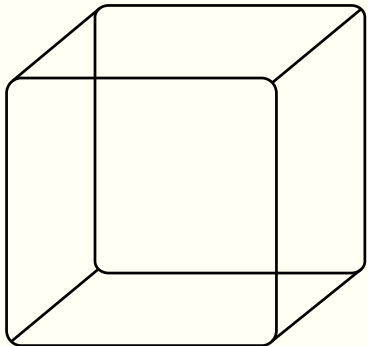


connecting the dots ...



- “information carriers” in LES domain

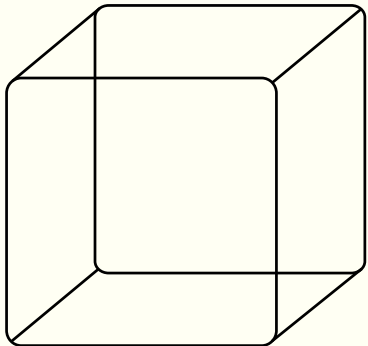
particle-based μ -physics for LES



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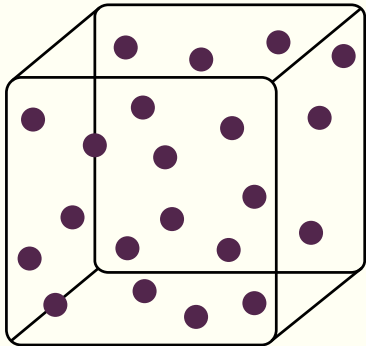
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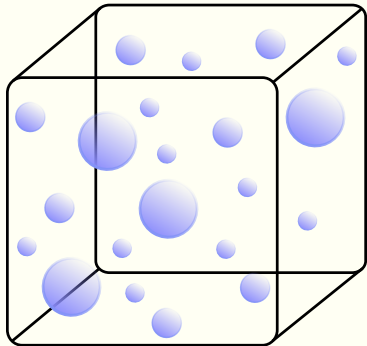


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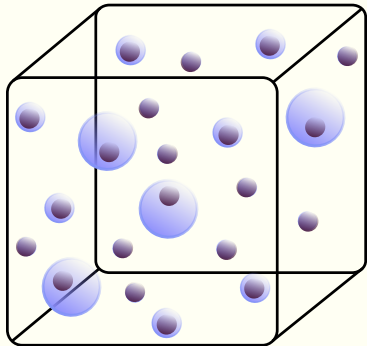
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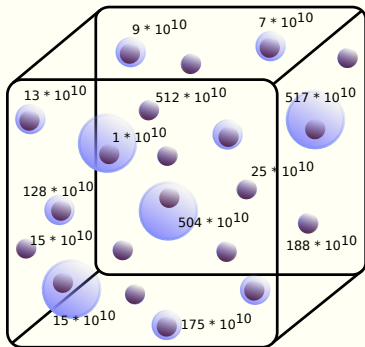
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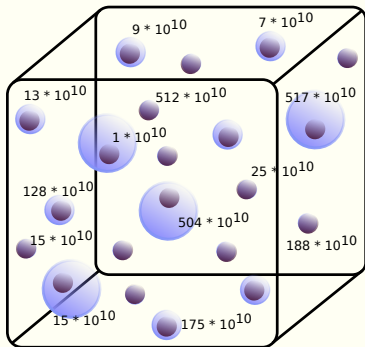
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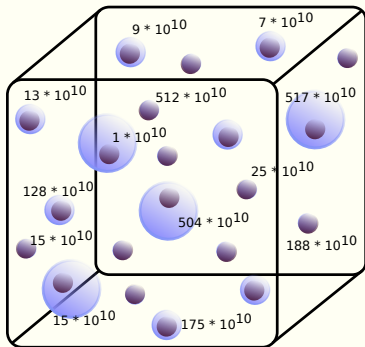
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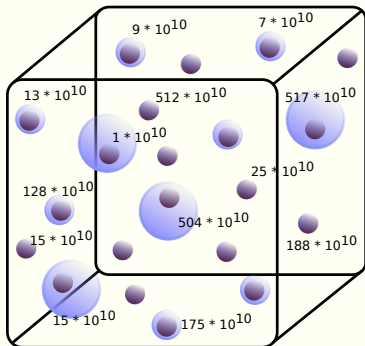
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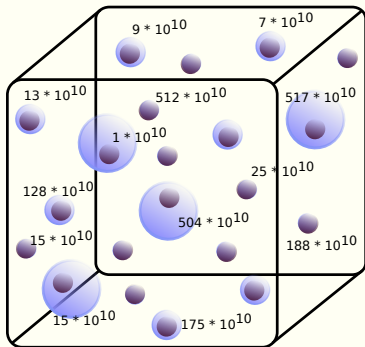
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(aqueous chemistry!)

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- ❖ each timestep: **constant RH!**

particle-based μ -physics for LES

Seminal works: [Shima et al. 2009](#), [Andrejczuk et al. 2010](#)
(3D simulations of atmospheric aerosol-cloud-precipitation system)

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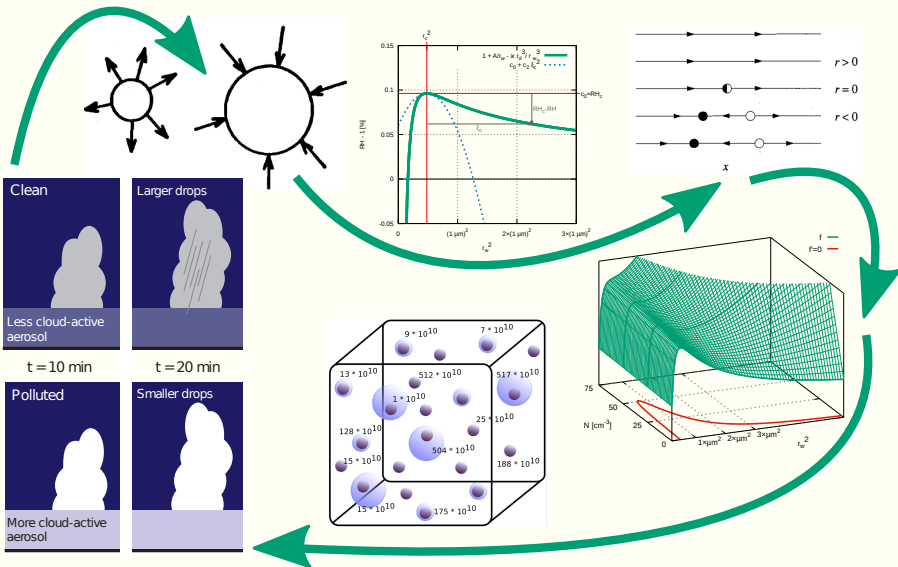
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 - guidance for numerical scheme design (particle-based μ -physics)

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- CCN (de)activation as a bifurcating dynamical system:
 - analytical results: conditions for hysteretic behaviour, timescales
 - guidance for numerical scheme design (particle-based μ -physics)
- applicability beyond cloud physics:
 - e.g., charge-induced activation (\rightsquigarrow Wilson chamber),
 - modelling operation of air-pollution monitoring instruments

conclusions, takeaways, prospects

- aerosol-cloud interactions:
 - formation of droplets on aerosol,
 - cloud processing of aerosol,
 - wet deposition
- CCN (de)activation as a bifurcating dynamical system:
 - analytical results: conditions for hysteretic behaviour, timescales
 - guidance for numerical scheme design (particle-based μ -physics)
- applicability beyond cloud physics:
 - e.g., charge-induced activation (\rightsquigarrow Wilson chamber),
 - modelling operation of air-pollution monitoring instruments
- extensions:
 - bi-/poly- modal/disperse spectra (spectrum width!),
 - activated/unactivated partitioning (excitable behaviour!),
 - beyond Köhler curve (charge, surfactants, non-soluble aerosol, ...)

Thank you for your attention!

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