Bifurcations in a dynamical system describing formation of cloud droplets on atmospheric particulate matter

Sylwester Arabas and Shin-ichiro Shima

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Nonlinear Processes in Geophysics
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On the CCN (de)activation nonlinearities

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atmospheric particulate matter (PM)

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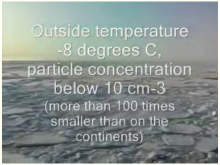

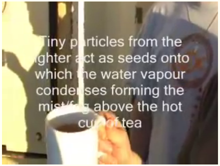

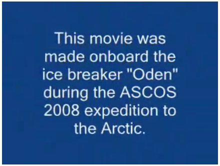

no particles $\rightsquigarrow$ no clouds

https://www.youtube.com/watch?v=EneDwu0HrVg

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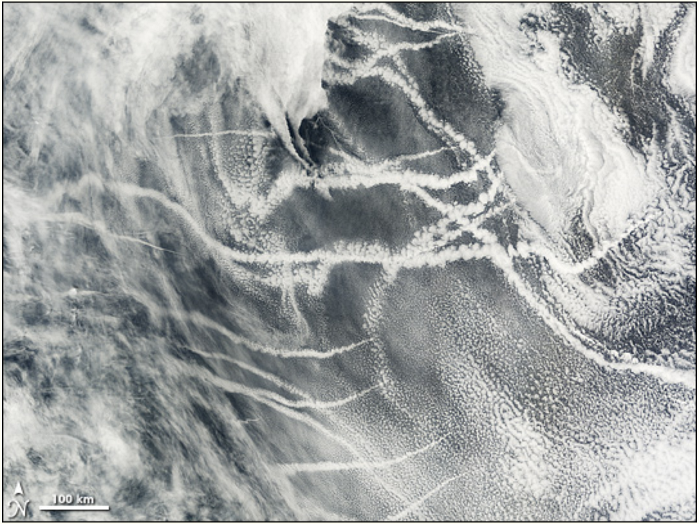
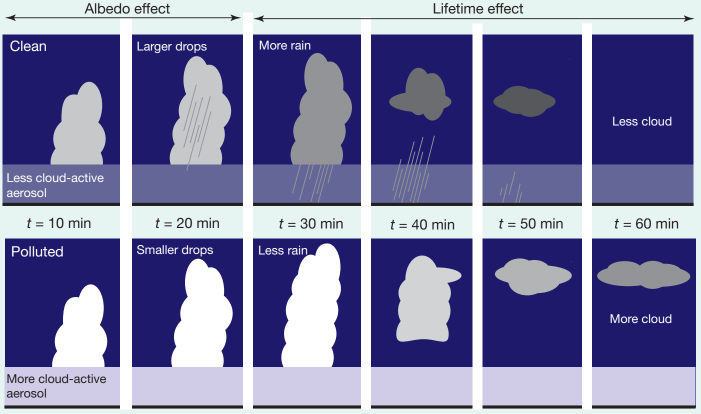


NASA (https://earthobservatory.nasa.gov/NaturalHazards/view.php?id=20248)

atmospheric particulate matter (PM)

Stevens and Feingold, 2009 (Nature)



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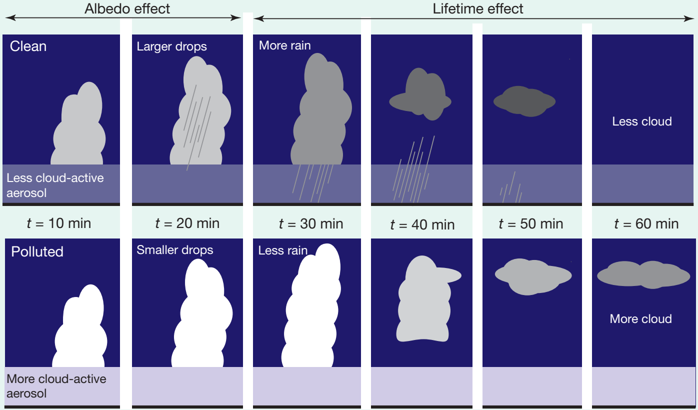
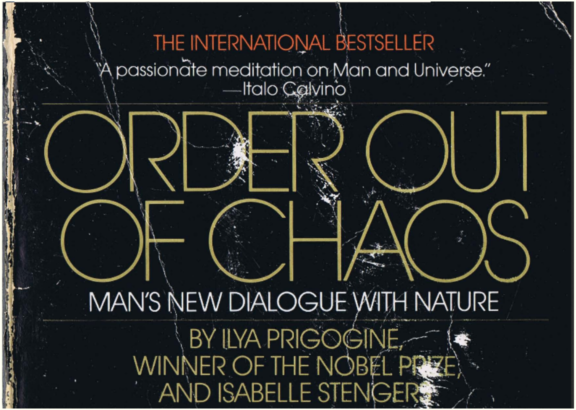


Stevens and Boucher, 2012 (Nature)

"there is something captivating about the idea that fine particulate matter, suspended almost invisibly in the atmosphere, holds the key to some of the greatest mysteries of climate science"


THE INTERNATIONAL BESTSELLER
"A passionate meditation on Man and Universe."
—Italo Calvino
ORDER OUT OF CHAOS
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BY ILYA PRIGOGINE,
WINNER OF THE NOBEL PRIZE,
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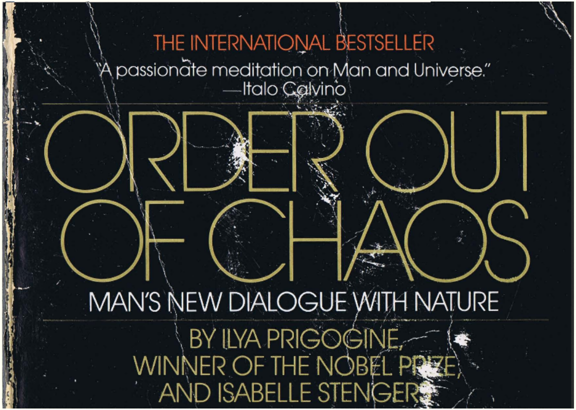


Prigogine and Stengers 1984

"Much of this book has centered around the relation between the microscopic and the macroscopic. One of the most important problems in evolutionary theory is the eventual feedback between macroscopic structures and microscopic events: macroscopic structures emerging from microscopic events would in turn lead to a modification of the microscopic mechanisms."

regime-transition (bifurcation) example from P&S 1984

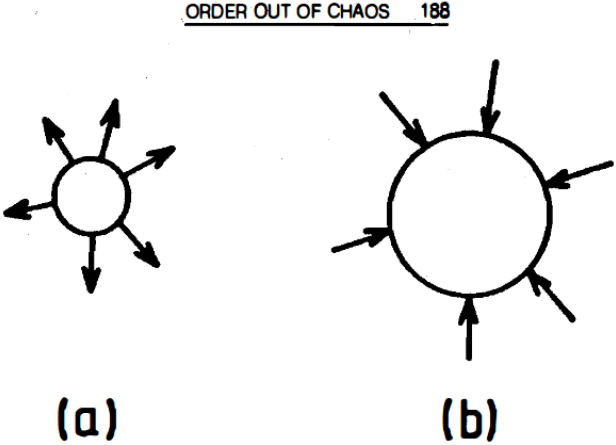
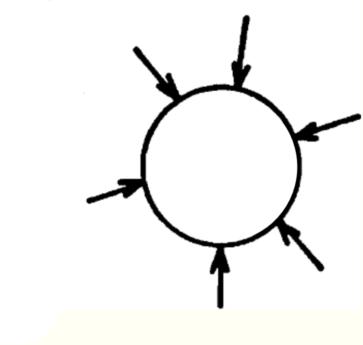


Figure 19. Nucleation of a liquid droplet in a supersaturated vapor. (a) droplet smaller than the critical size; (b) droplet larger than the critical size. The existence of the threshold has been experimentally verified for dissipative structures.

droplet growth laws in a nutshell: mass and heat diffusion



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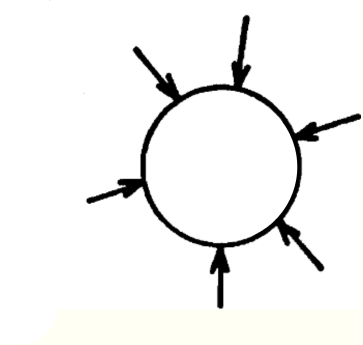

Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w}\frac{D_{eff}}{\rho_w}(\rho_v - \rho_\circ)$$

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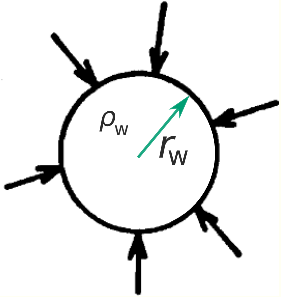

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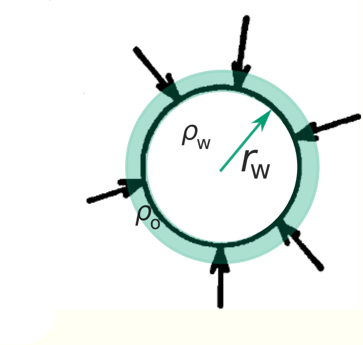

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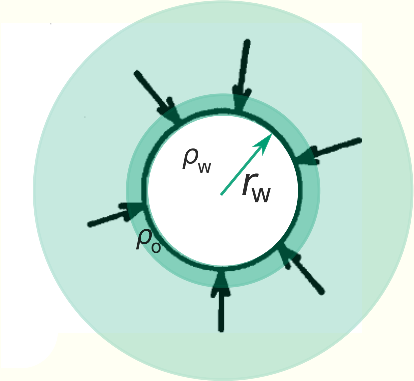

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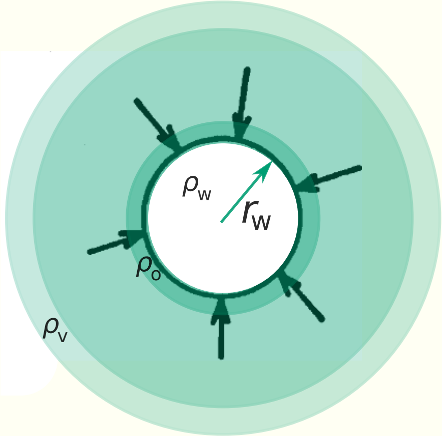

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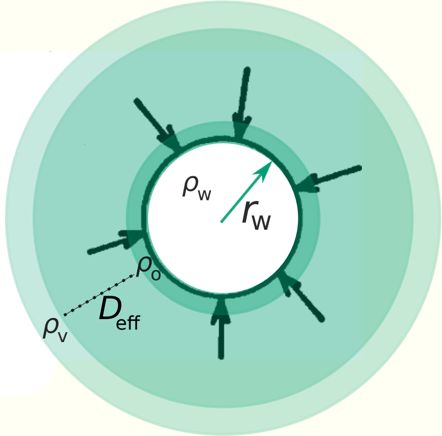

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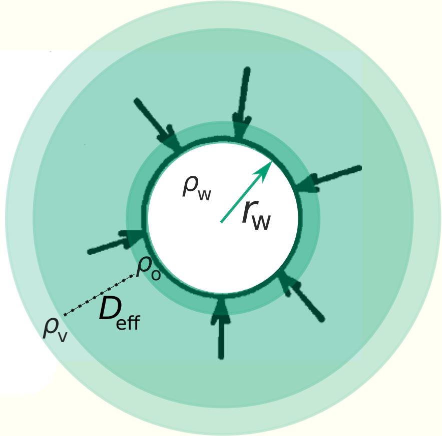

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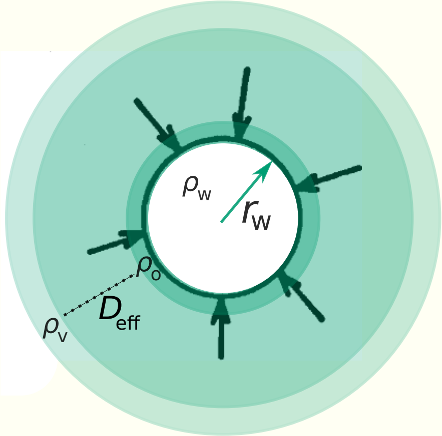

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$$\begin{aligned} \mathrm{RH_{eq}} &= \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1-\kappa)} \exp\left(\frac{A}{r_w}\right) \\ &\approx 1 + \frac{A}{r_\mathrm{w}} - \frac{\kappa r_\mathrm{d}^3}{r_\mathrm{w}^3} \end{aligned}$$

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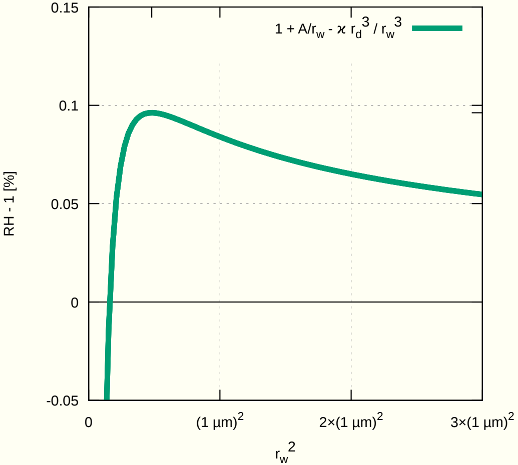

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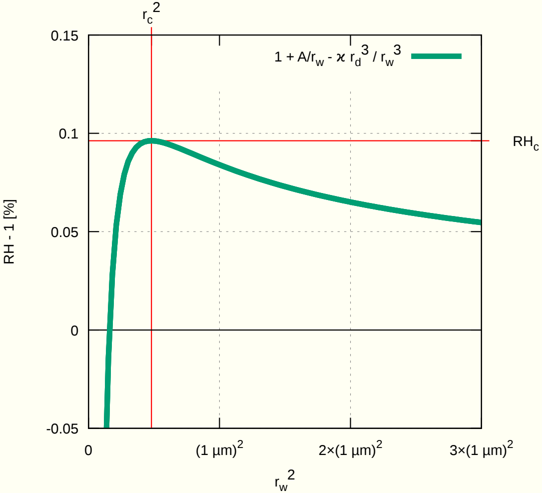

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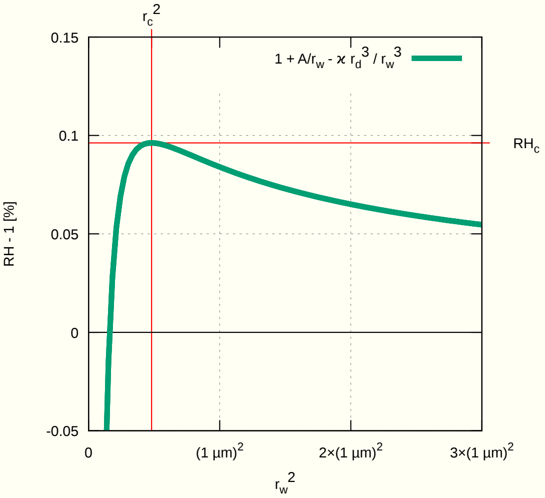
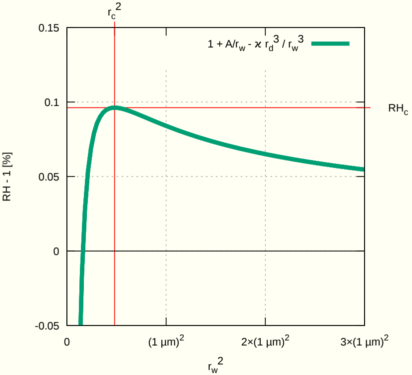


maximum at (r_c, RH_c):

$$r_c = \sqrt{3\kappa r_d^3 / A}$$
$$\mathrm{RH}_c = 1 + \frac{2A}{3r_c}$$

phase portrait of the system: flipped Köhler curve



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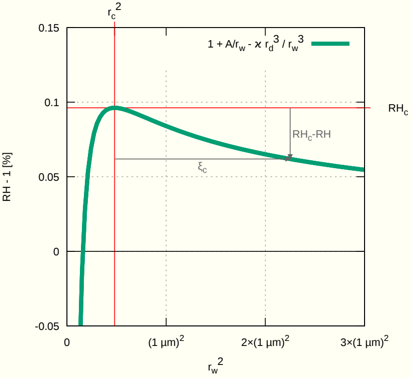

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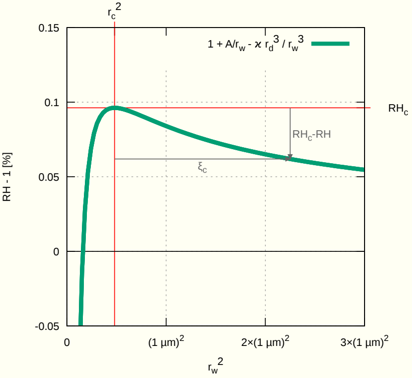

$$\xi = r_{\mathrm{w}}^2 + C$$

$$\dot{\xi} = 2D_{\mathrm{eff}} \frac{\rho_{\mathrm{vs}}}{\rho_{\mathrm{w}}} \left(\mathrm{RH} - \mathrm{RH}_{\mathrm{eq}}(\xi)\right)$$

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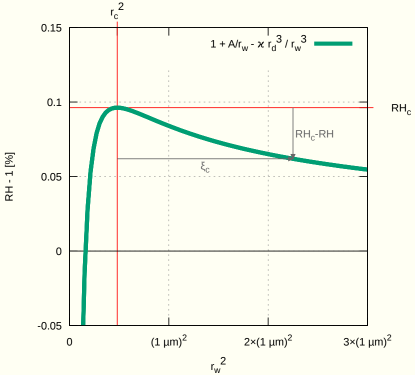

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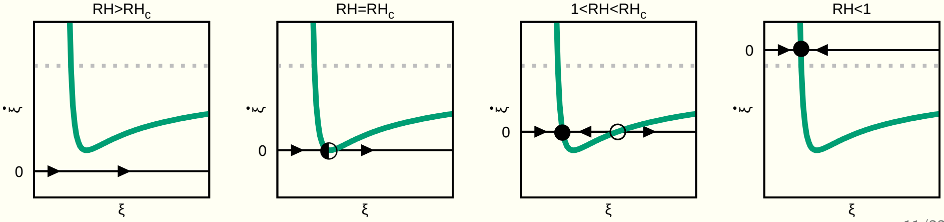

saddle-node bifurcation at Köhler curve maximum

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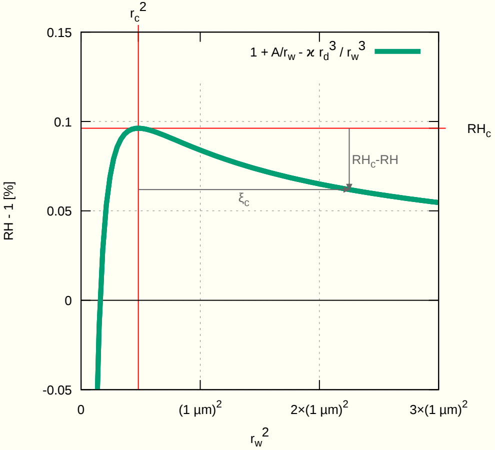
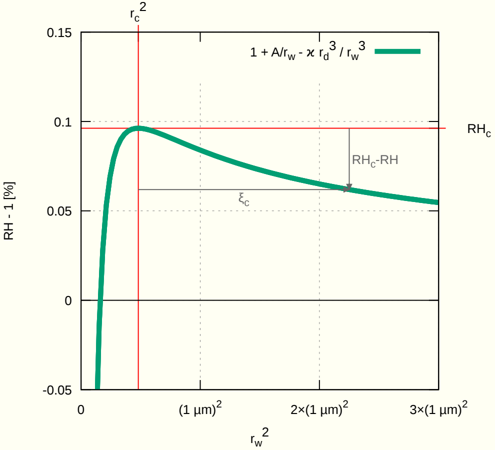


saddle-node bifurcation at Köhler curve maximum

$$\mathrm{RH}_{\mathrm{eq}}(\xi_c) = c_0 + \cancel{c_1\xi_c} + c_2\xi_c^2 + \ldots$$



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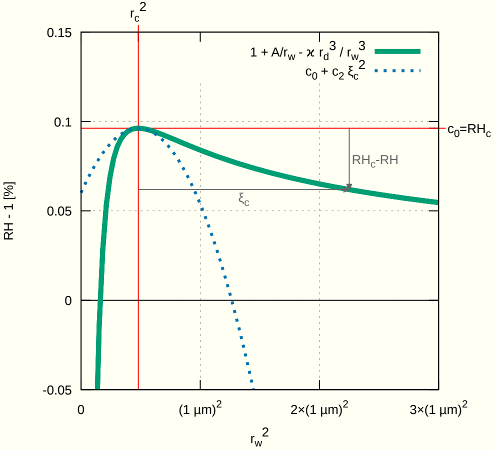

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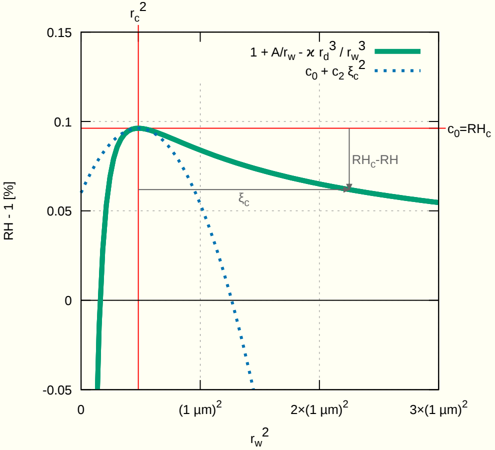

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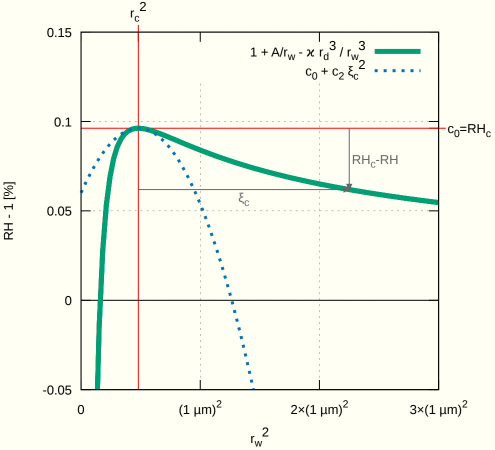
$$\dot{x} = r + x^2$$



coalescence in the saddle-node bottleneck

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coalescence of the fixed points is associated with a passage through a *bottleneck* (e.g., Strogatz 2014),

key observation: time of passage through the parabolic *bottleneck* dominates all other timescales

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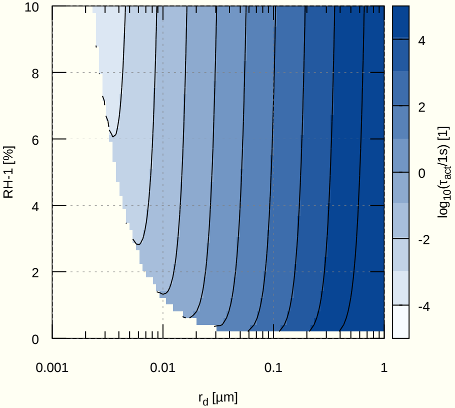

activation timescale: analytic vs. numerical

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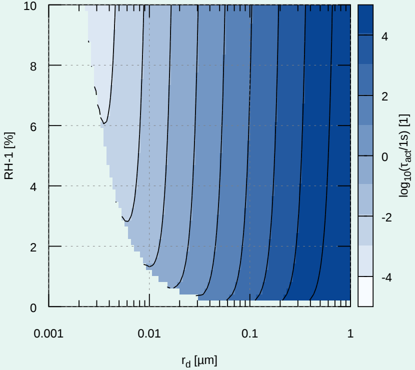

note: axes ranges vs. close-to-equilibrium assumption

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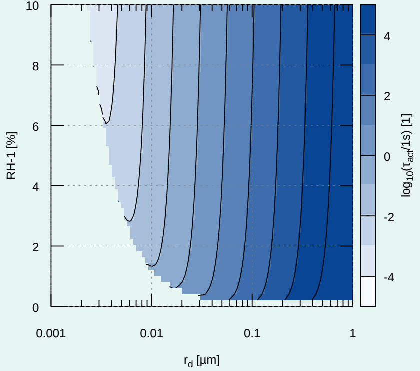

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Hoffmann, 2016 (MWR)

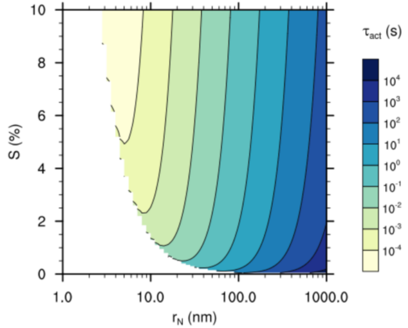

FIG. 2. The activation time scale τ_{act} as a function of dry aerosol radius r_N and supersaturation S. For values of $S < S_{crit}$ (white areas), τ_{act} does not exist.

$$r\frac{dr}{dt} = \left(S - \frac{A}{r} + \frac{Br_N^3}{r^3}\right) \Big/ (F_k + F_D), \qquad (10)$$

The second time scale is associated with the activation of particles, for which Köhler theory is essential. This makes an analytic solution for (10) impossible. Numerically calculated values of τ_{act} measuring the time needed for a wetted aerosol to grow beyond its critical radius $r_{crit} = \sqrt{3Br_N^3/A}$ are given in Fig. 2 as a function of

RH-coupled system & particle concentration as parameter

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simple moisture budget (const T,p):

$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_{\mathrm{v}}}{\rho_{\mathrm{vs}}} = -N \underbrace{\frac{4\pi\rho_{\mathrm{w}}}{3\rho_{\mathrm{vs}}}}_{\alpha} 3r_{\mathrm{w}}^2\dot{r}_{\mathrm{w}}$$

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new phase portrait:

$$\dot{\xi} \sim (\mathrm{RH}_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_\mathrm{d}^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_{f}$$

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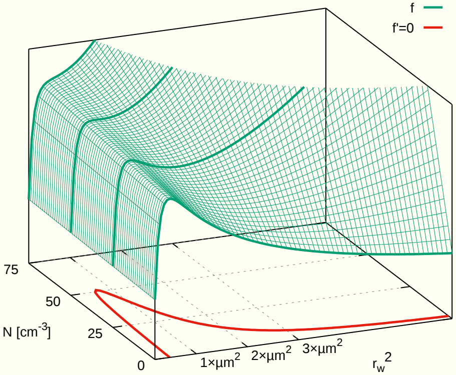

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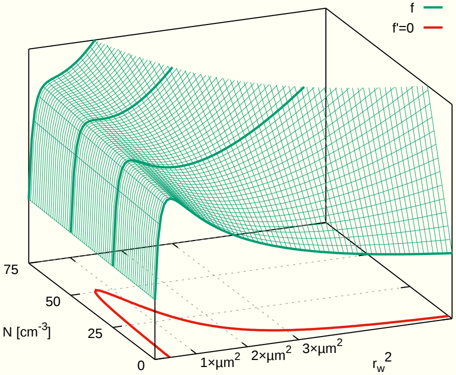

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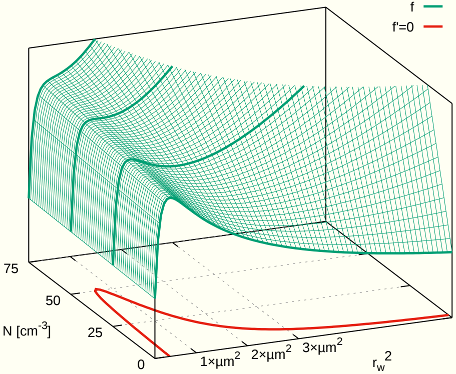

$$\mathrm{sgn}(f') = \mathrm{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3\right)$$

bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

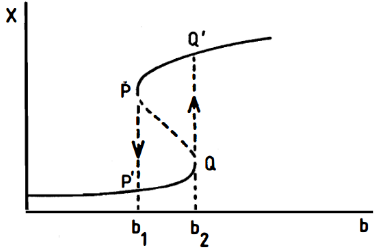

Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter b first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while b grows. But at $b = b_2$, there will be a discontinuity: The system jumps from Q to Q', on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till $b = b_1$, when it will jump down to P'. Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

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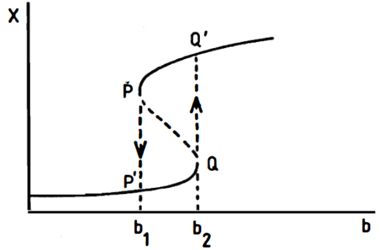

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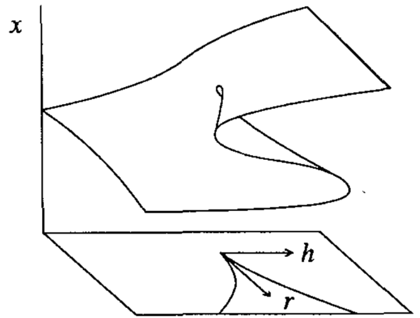

"cusp catastrophe"

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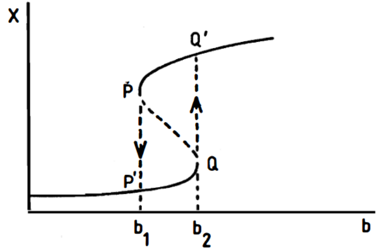

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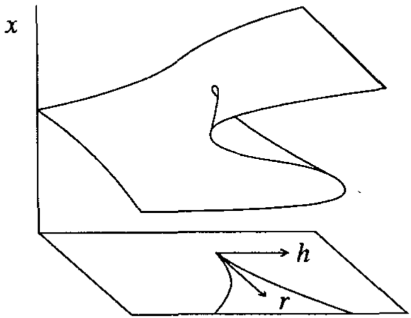

"cusp catastrophe"

⇝ "jumps", hysteretic behaviour (r_w, RH) for small enough N, close to equilibrium (slow process)

lifting the constant T-p assumptions: parcel model

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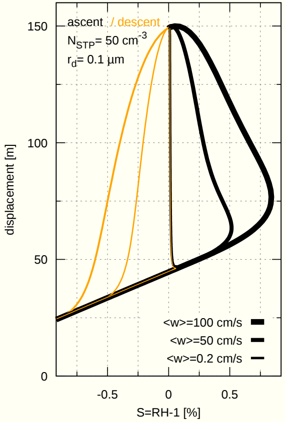
parcel model: numerical integration (sinusoidal w)



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open source code (based on libcloudph++) as electronic paper supplement

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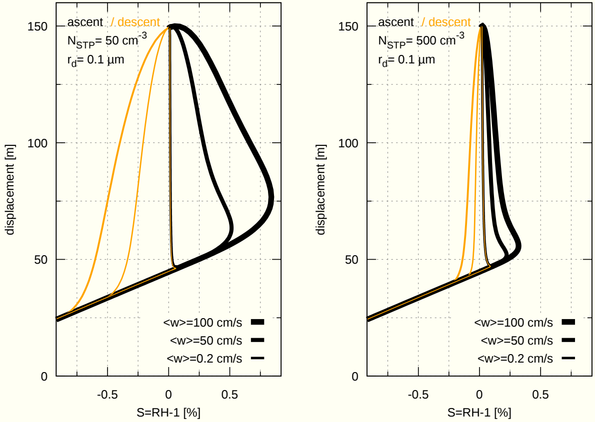

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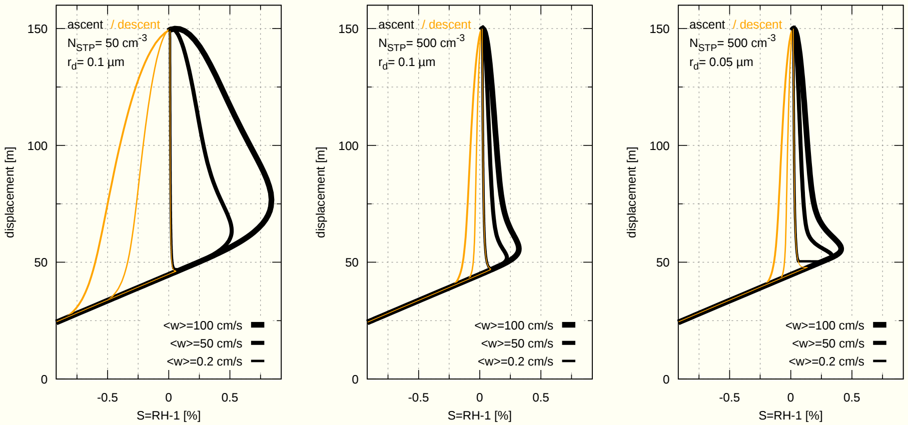

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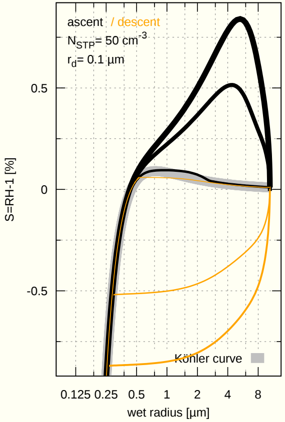

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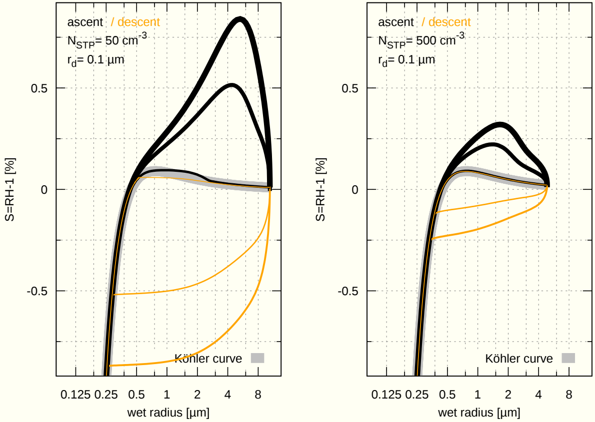

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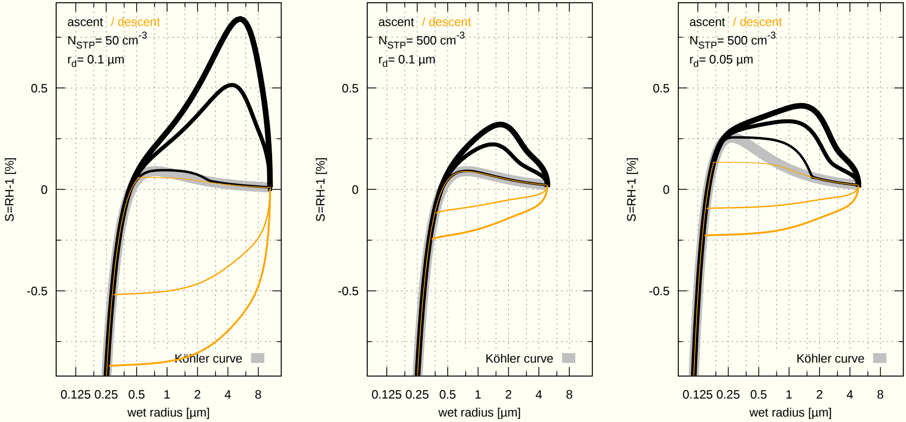

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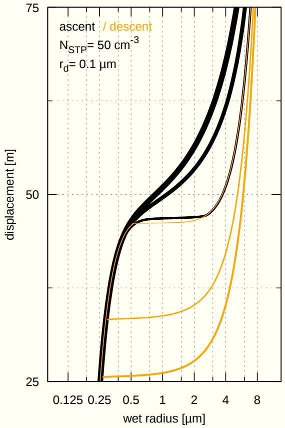

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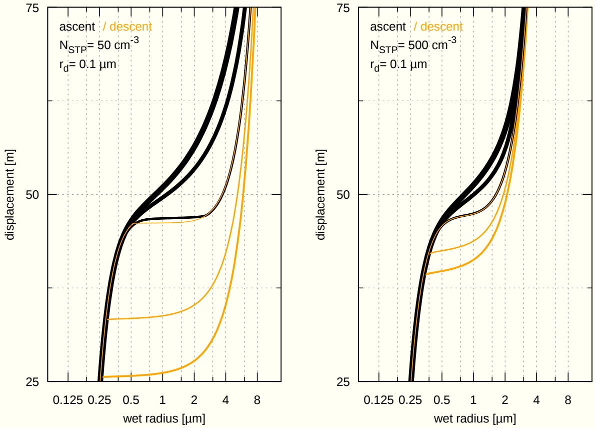

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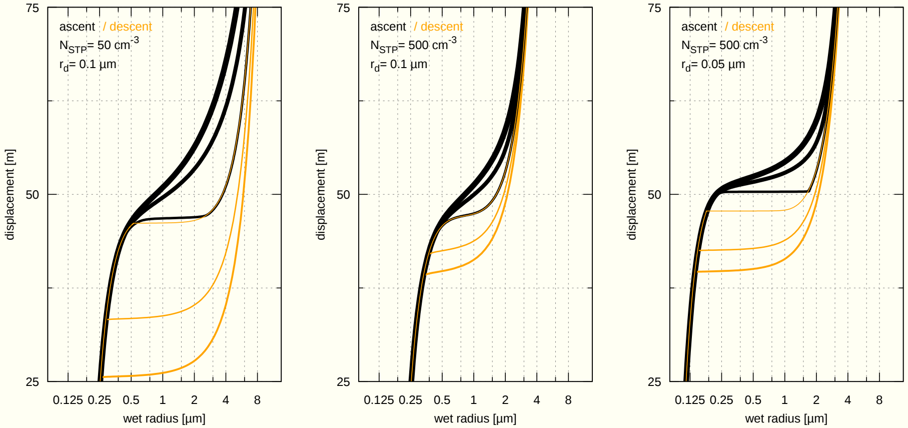

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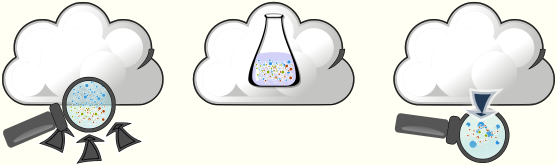

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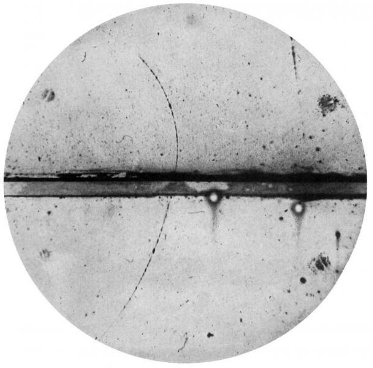

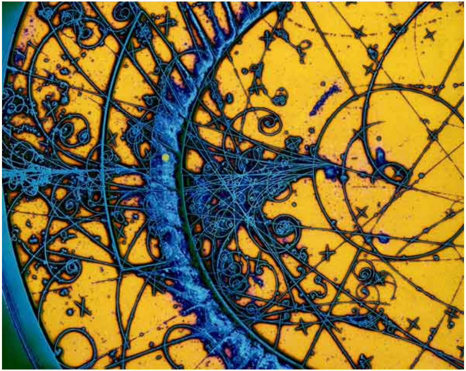

Wilson & bubble chambers

https://home.cern/about/updates/2015/06/seeing-invisible-event-displays-particle-physics

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- extensions:
  – bi-/poly- modal/disperse spectra (spectrum width!),
  – activated/unactivated partitioning (excitable behaviour!),
  – beyond Köhler curve (charge, surfactants, non-soluble aerosol, ...)

Thank you for your attention!

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