Bifurcation analysis of CCN activation dynamics

Sylwester Arabas
Jagiellonian University, Kraków, Poland

Arabas & Shima 2017

Nonlin. Processes Geophys., 24, 535–542, 2017
https://doi.org/10.5194/npg-24-535-2017

Nonlinear Processes in Geophysics
Open Access
EGU

On the CCN (de)activation nonlinearities

Sylwester Arabas[1,2] **and Shin-ichiro Shima**[3]

[1]Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland
[2]Chatham Financial Corporation Europe, Cracow, Poland
[3]Graduate School of Simulation Studies, University of Hyogo, Kobe, Japan

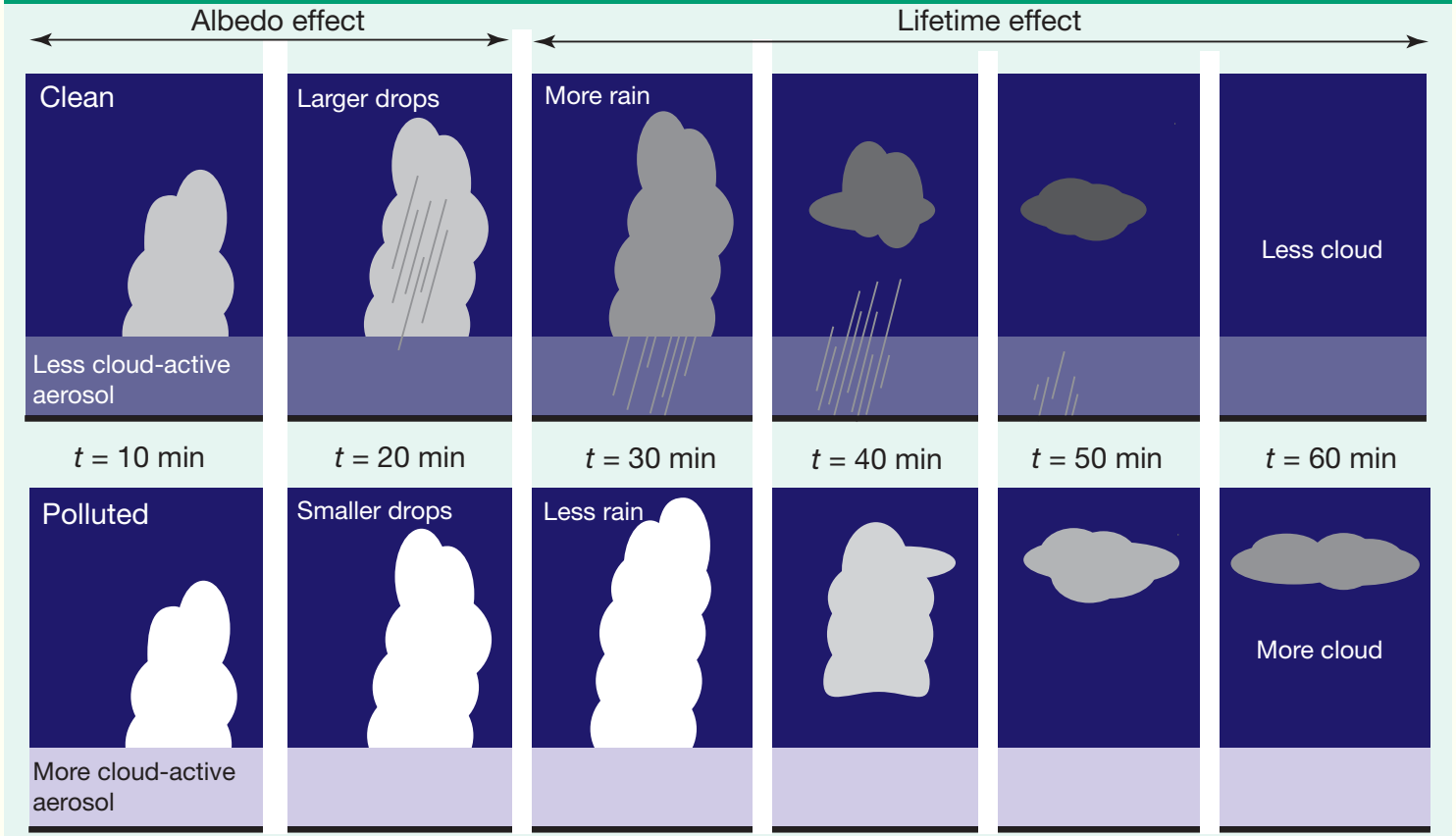
Correspondence to: Sylwester Arabas (sarabas@chathamfinancial.eu) and Shin-ichiro Shima (s_shima@sim.u-hyogo.ac.jp)

Received: 9 September 2016 – Discussion started: 4 October 2016
Revised: 23 May 2017 – Accepted: 24 July 2017 – Published: 5 September 2017

one-slide aerosol-cloud (micro-macro) interaction primer

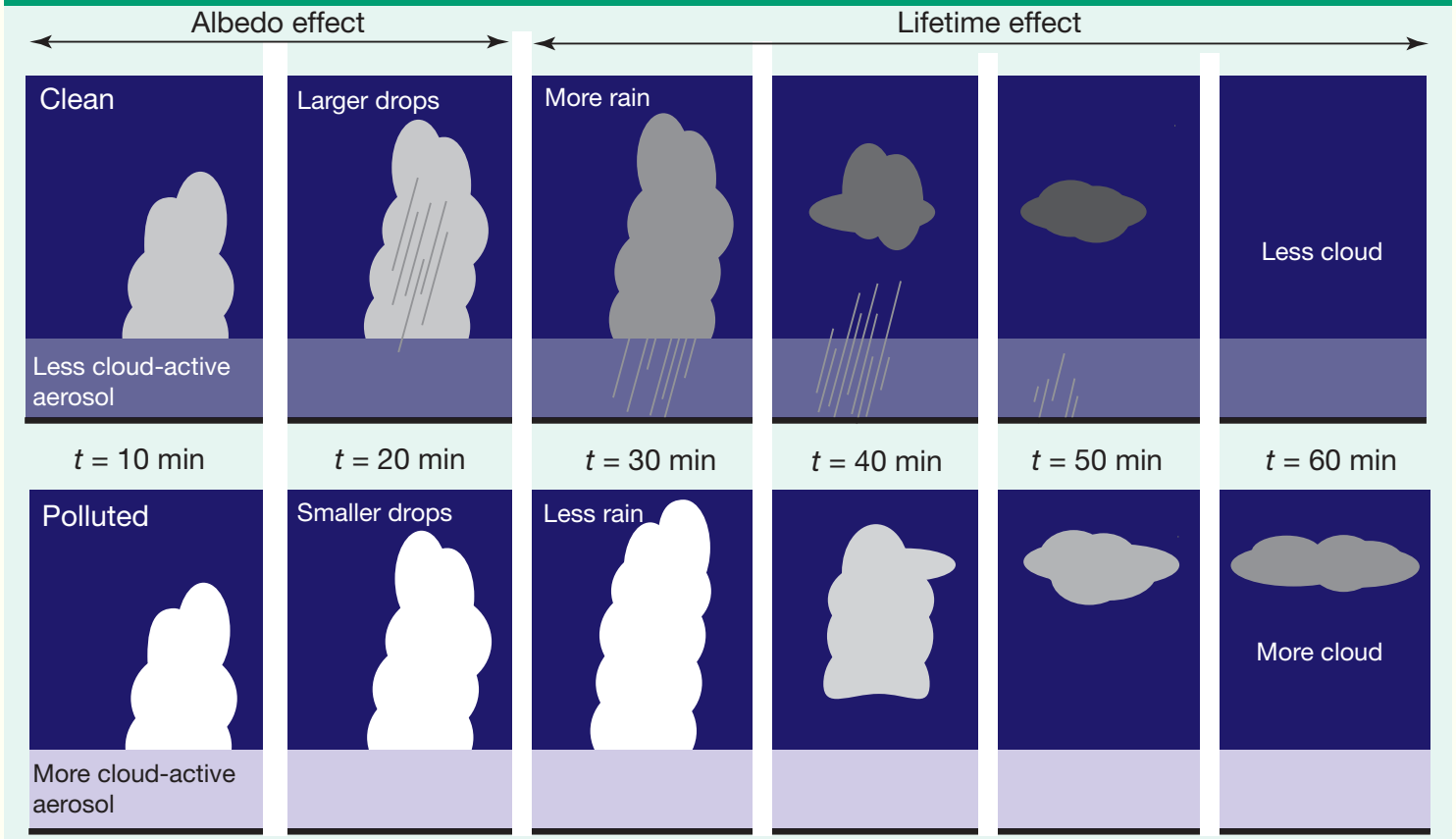
one-slide aerosol-cloud (micro-macro) interaction primer

Stevens and Feingold, 2009 (Nature)



one-slide aerosol-cloud (micro-macro) interaction primer

Stevens and Feingold, 2009 (Nature)

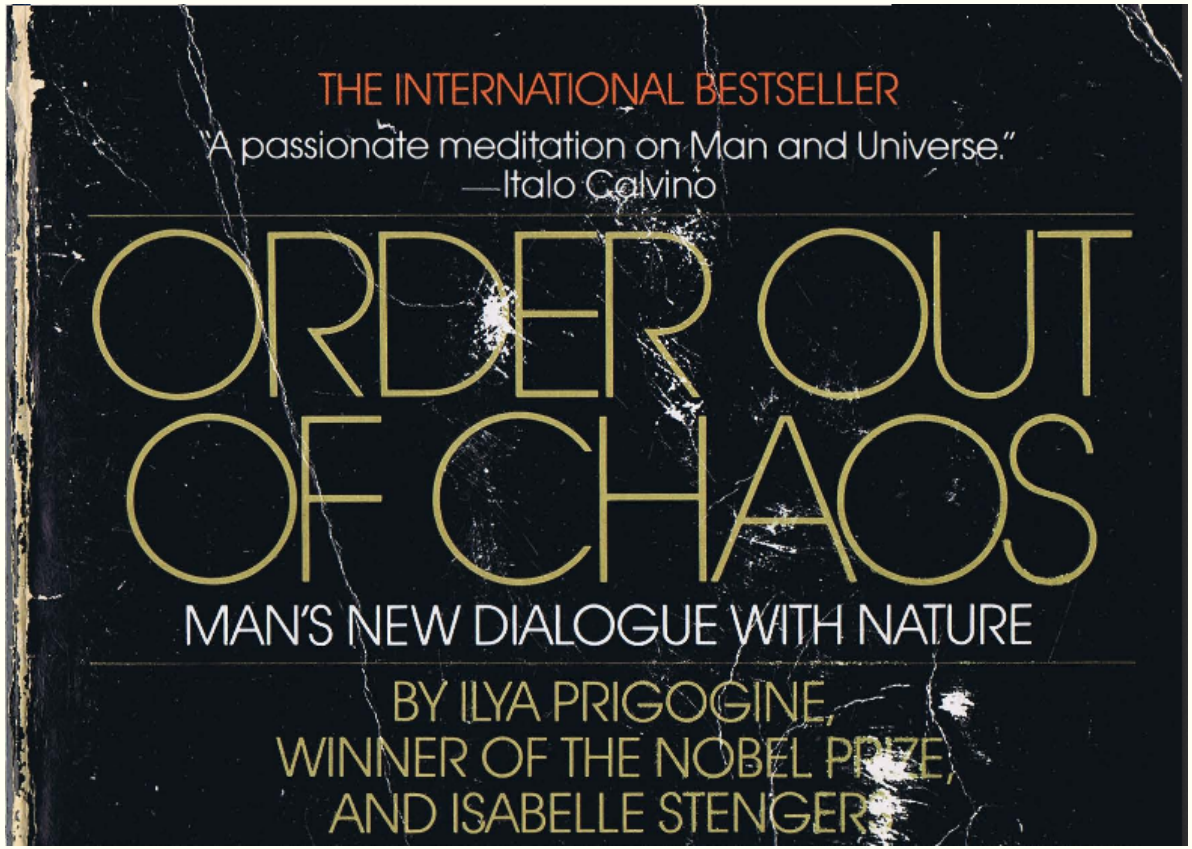


Stevens and Boucher, 2012 (Nature)

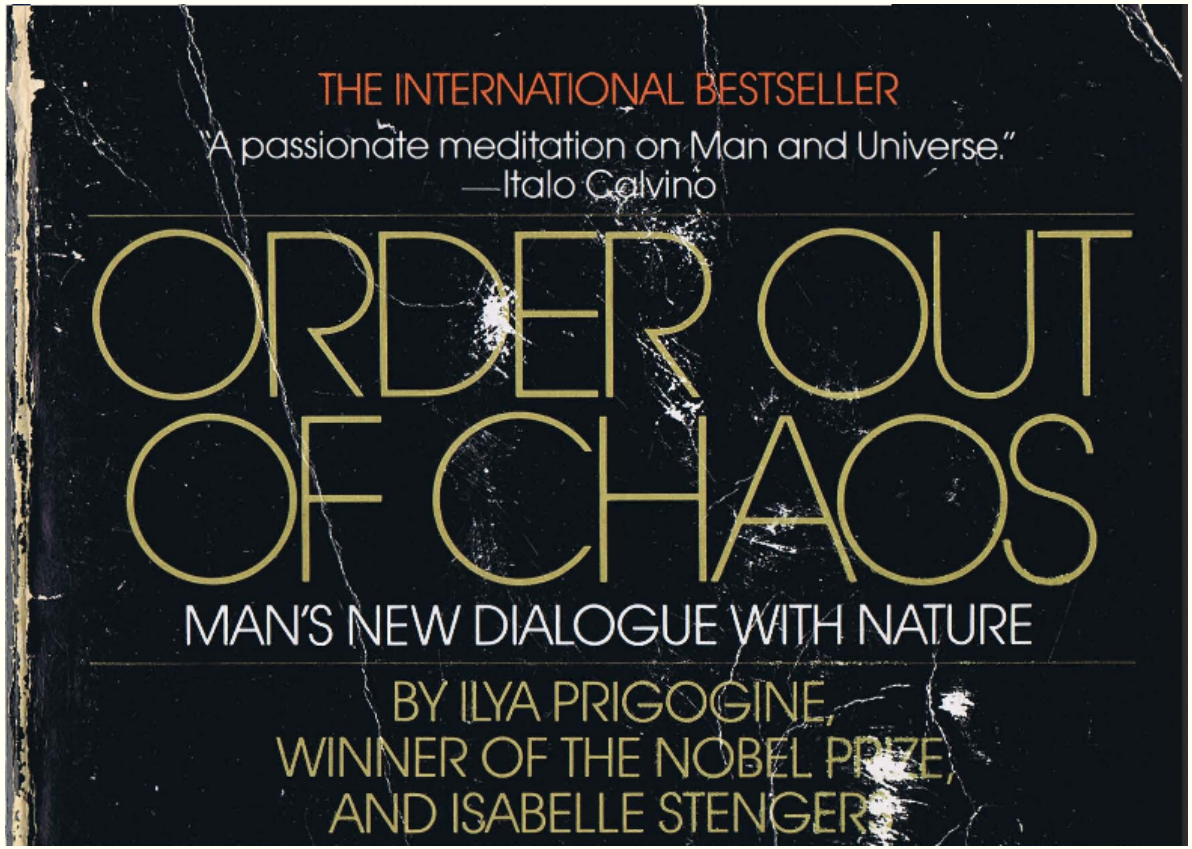
"there is something captivating about the idea that fine particulate matter, suspended almost invisibly in the atmosphere, holds the key to some of the greatest mysteries of climate science"

... others captivated by micro-macro interactions

... others captivated by micro-macro interactions



... others captivated by micro-macro interactions



Prigogine and Stengers 1984

"Much of this book has centered around the relation between the microscopic and the macroscopic. One of the most important problems in evolutionary theory is the eventual feedback between macroscopic structures and microscopic events: macroscopic structures emerging from microscopic events would in turn lead to a modification of the microscopic mechanisms."

regime-transition (bifurcation) example from P&S 1984

regime-transition (bifurcation) example from P&S 1984

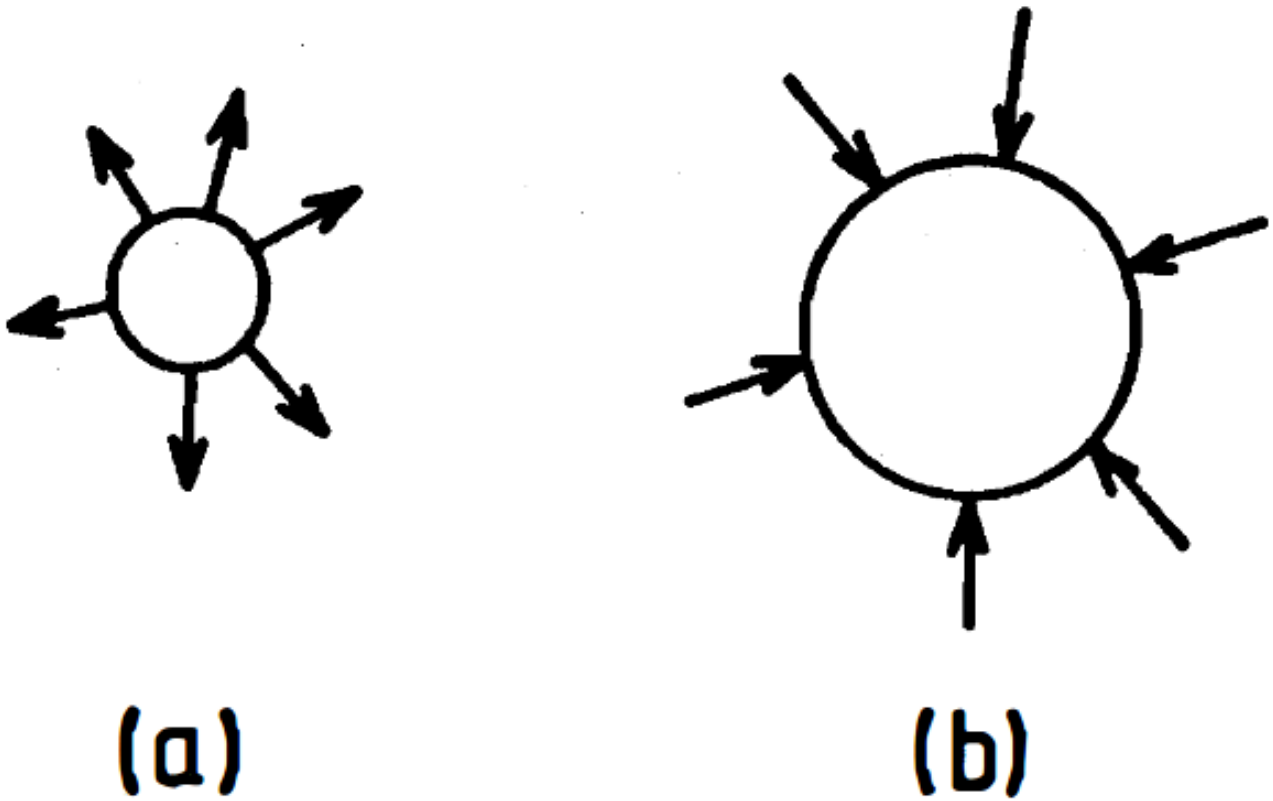


Figure 19. Nucleation of a liquid droplet in a supersaturated vapor. (a) droplet smaller than the critical size; (b) droplet larger than the critical size. The existence of the threshold has been experimentally verified for dissipative structures.

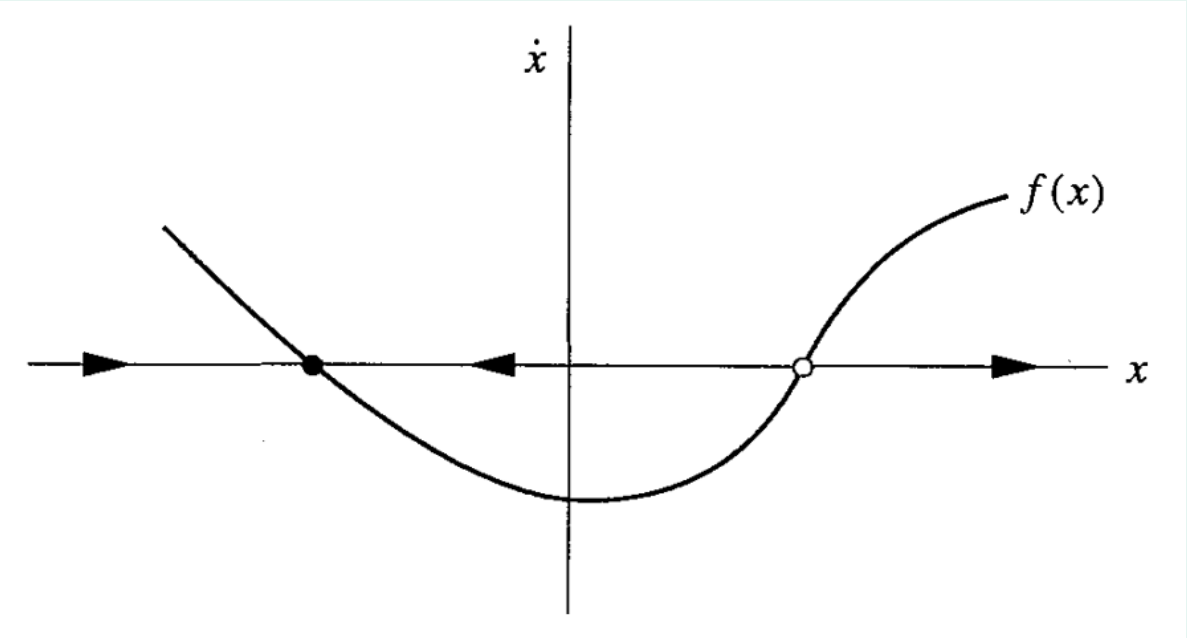
two-slide bifurcation analysis primer (1/2)

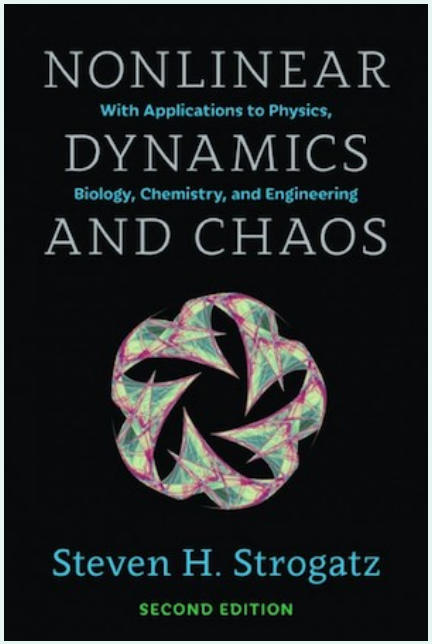
two-slide bifurcation analysis primer (1/2)

Strogatz 2014 (sect. 2.2): fixed points and stability

graphical (qualitative) analysis
of a non-linear one-dimensional dynamical system:

$$\dot{x} = f(x)$$





two-slide bifurcation analysis primer (2/2)

Strogatz 2014 (sect. 3.1): saddle-node bifurcation

prototypical example of saddle-node bifurcation:

$$\dot{x} = r + x^2$$

r: parameter (distinct regimes if positive, negative or zero)

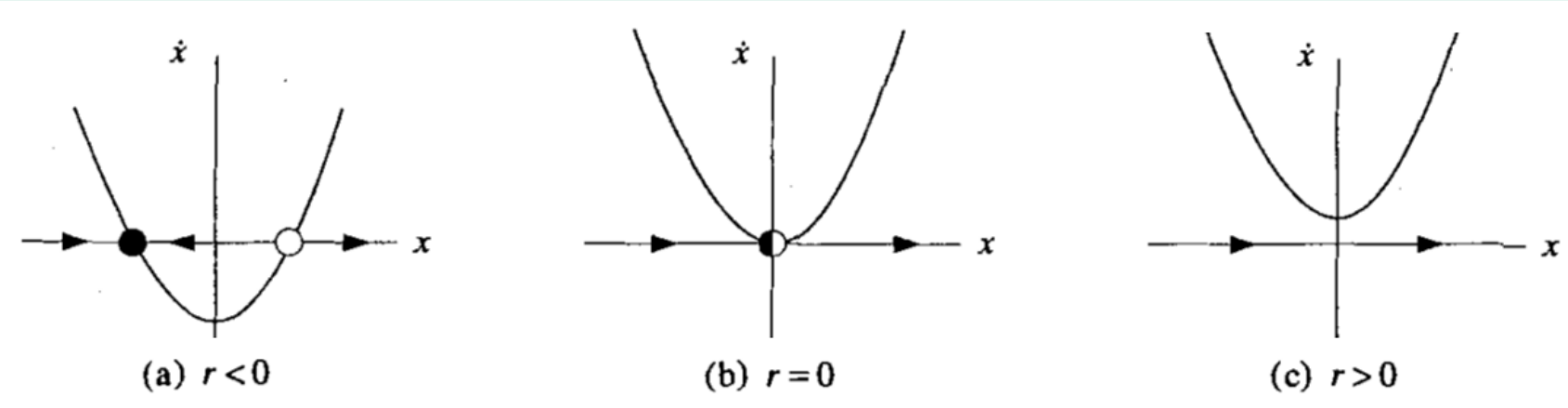
two-slide bifurcation analysis primer (2/2)

Strogatz 2014 (sect. 3.1): saddle-node bifurcation

prototypical example of saddle-node bifurcation:

$$\dot{x} = r + x^2$$

r: parameter (distinct regimes if positive, negative or zero)



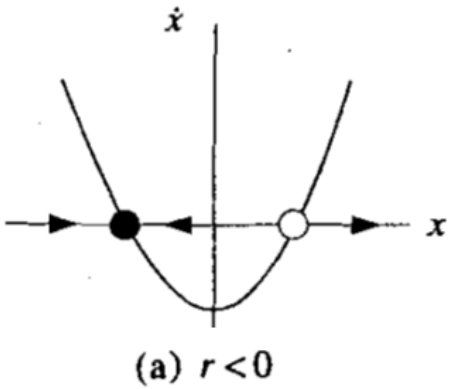
two-slide bifurcation analysis primer (2/2)

Strogatz 2014 (sect. 3.1): saddle-node bifurcation

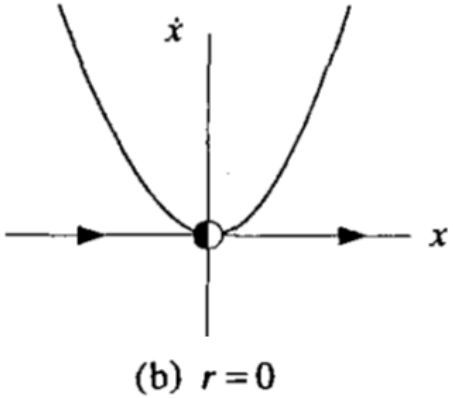
prototypical example of saddle-node bifurcation:

$$\dot{x} = r + x^2$$

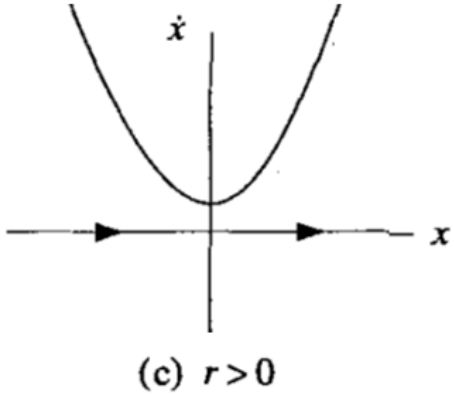
r: parameter (distinct regimes if positive, negative or zero)



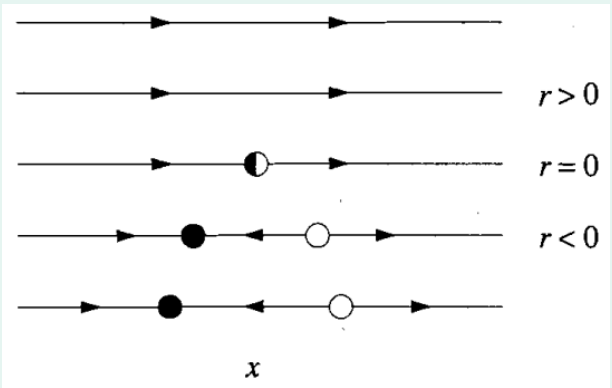
(a) $r<0$



(b) $r=0$



(c) $r>0$



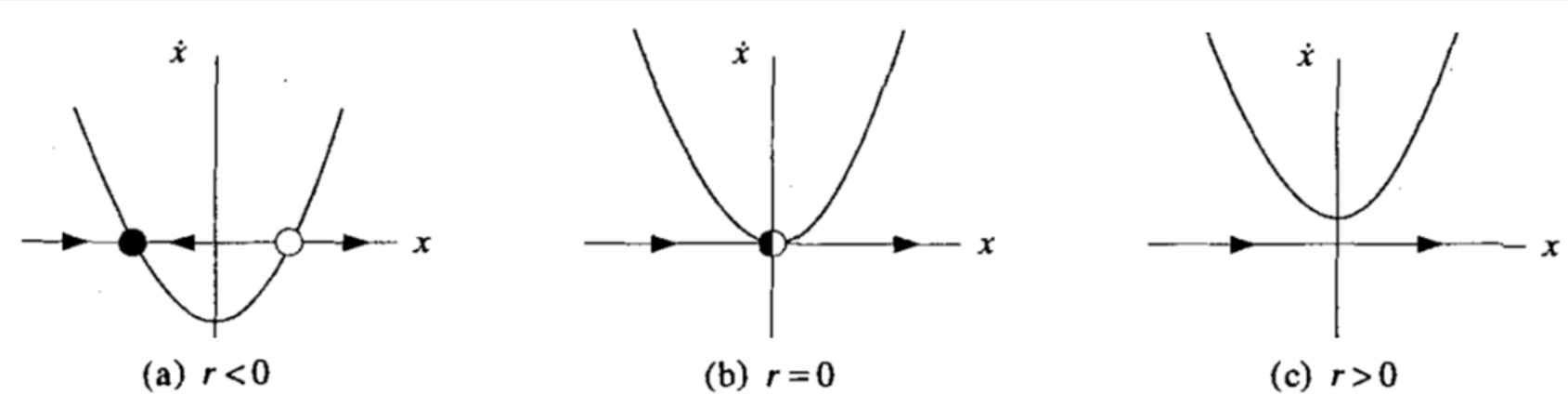
two-slide bifurcation analysis primer (2/2)

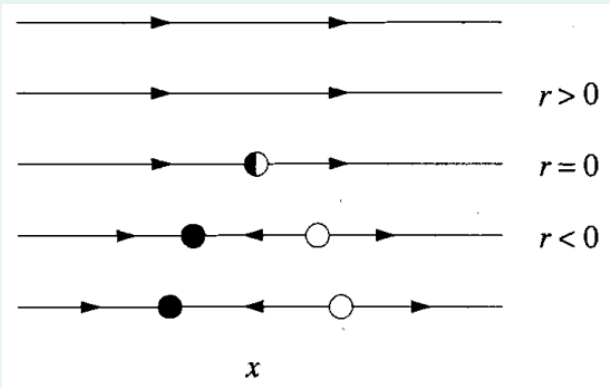
Strogatz 2014 (sect. 3.1): saddle-node bifurcation

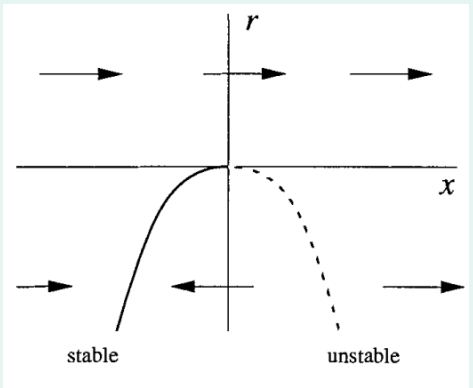
prototypical example of saddle-node bifurcation:

$$\dot{x} = r + x^2$$

r: parameter (distinct regimes if positive, negative or zero)







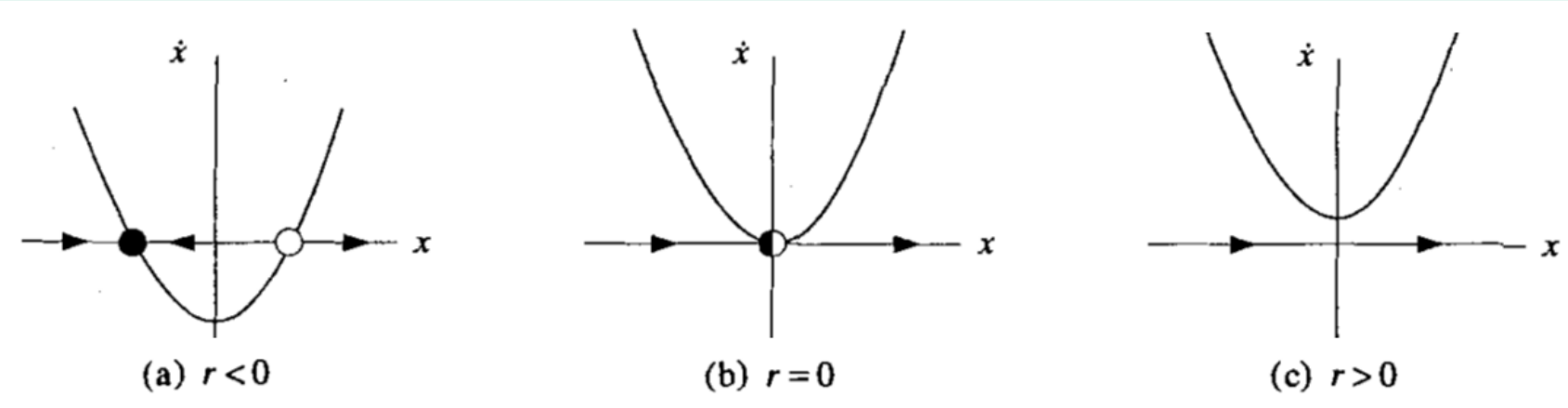
two-slide bifurcation analysis primer (2/2)

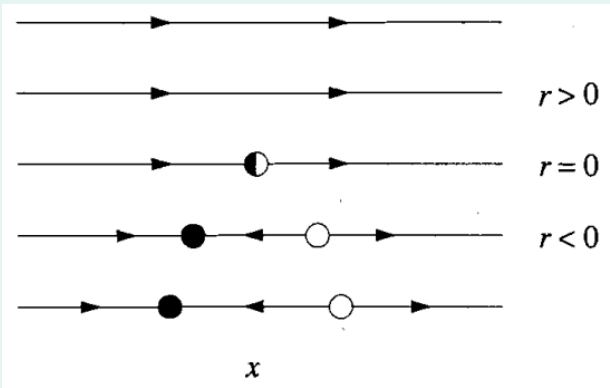
Strogatz 2014 (sect. 3.1): saddle-node bifurcation

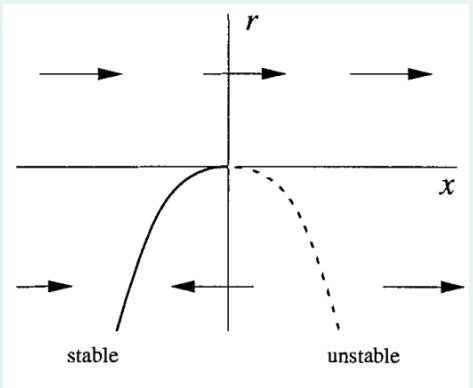
prototypical example of saddle-node bifurcation:

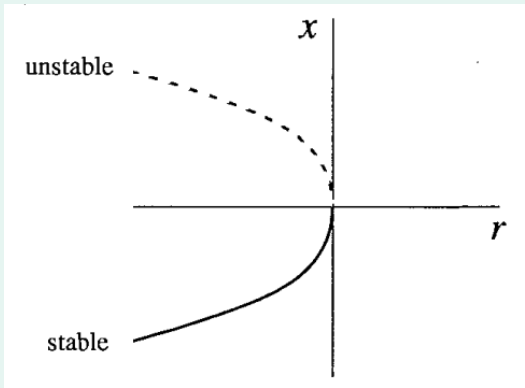
$$\dot{x} = r + x^2$$

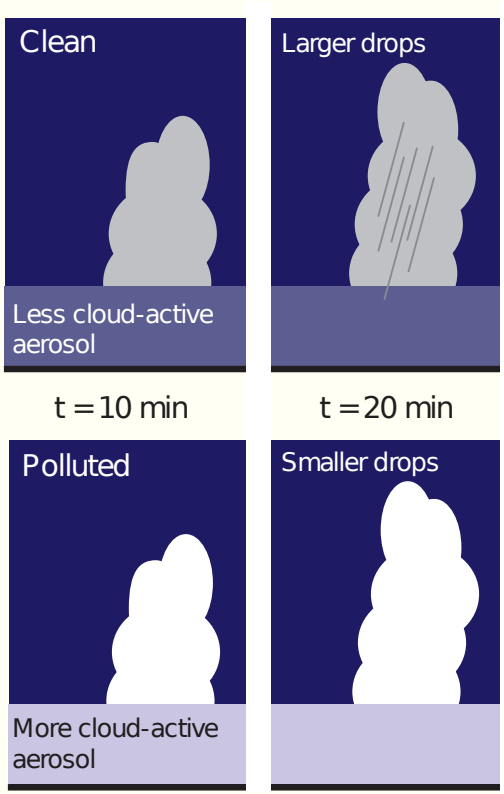
r: parameter (distinct regimes if positive, negative or zero)

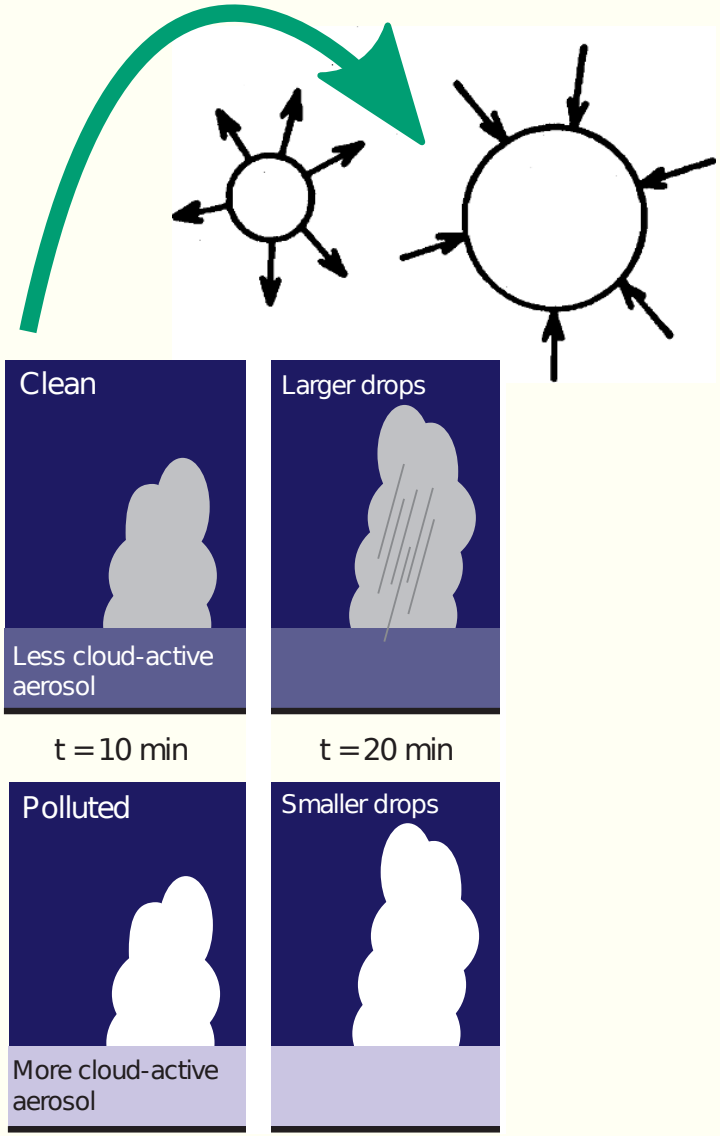


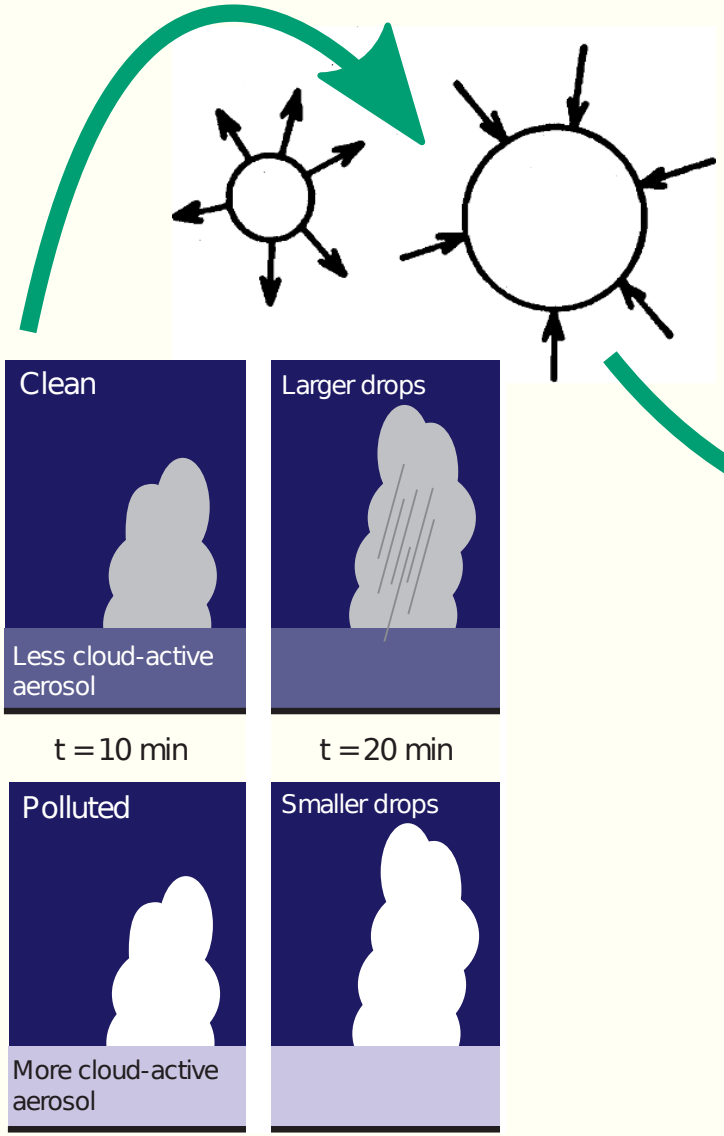


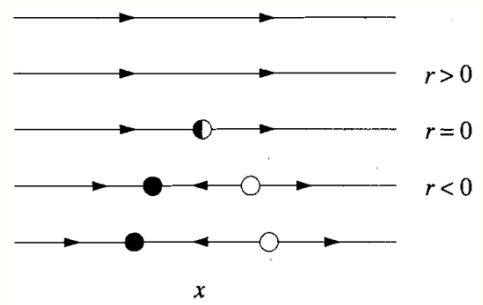


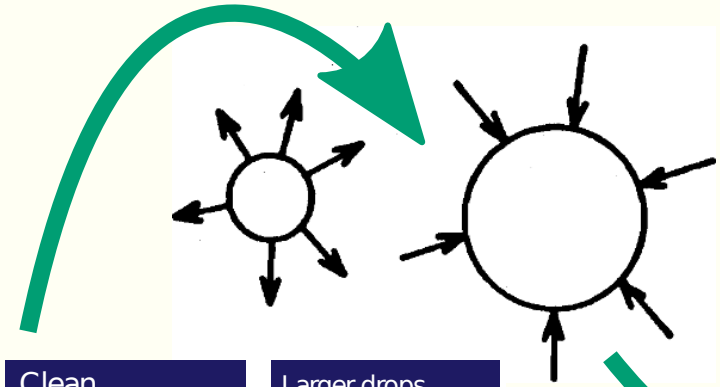


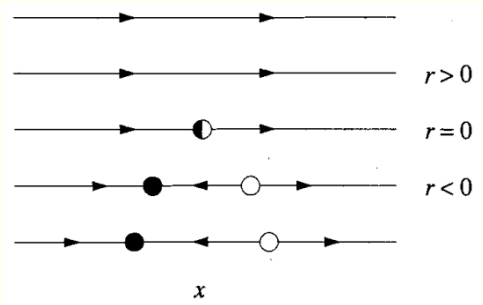

Clean
Larger drops
Less cloud-active aerosol
t = 10 min
t = 20 min
Polluted
Smaller drops
More cloud-active aerosol

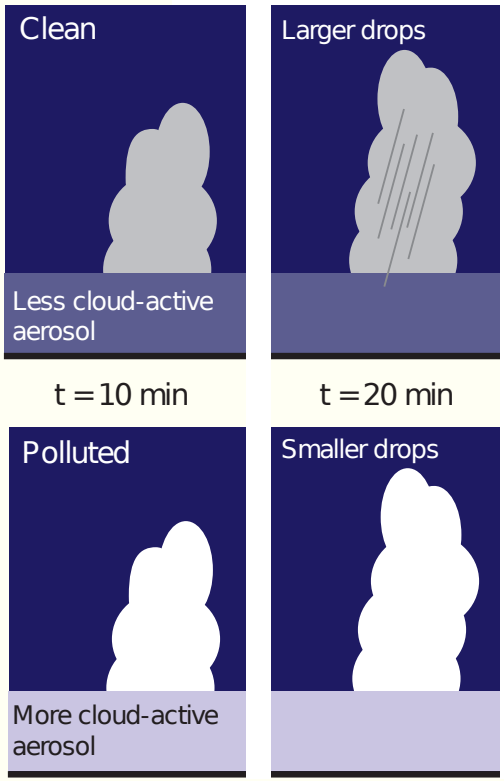

Clean
Larger drops
Less cloud-active aerosol
t = 10 min
t = 20 min
Polluted
Smaller drops
More cloud-active aerosol


Clean
Larger drops
Less cloud-active aerosol
t = 10 min
t = 20 min
Polluted
Smaller drops
More cloud-active aerosol

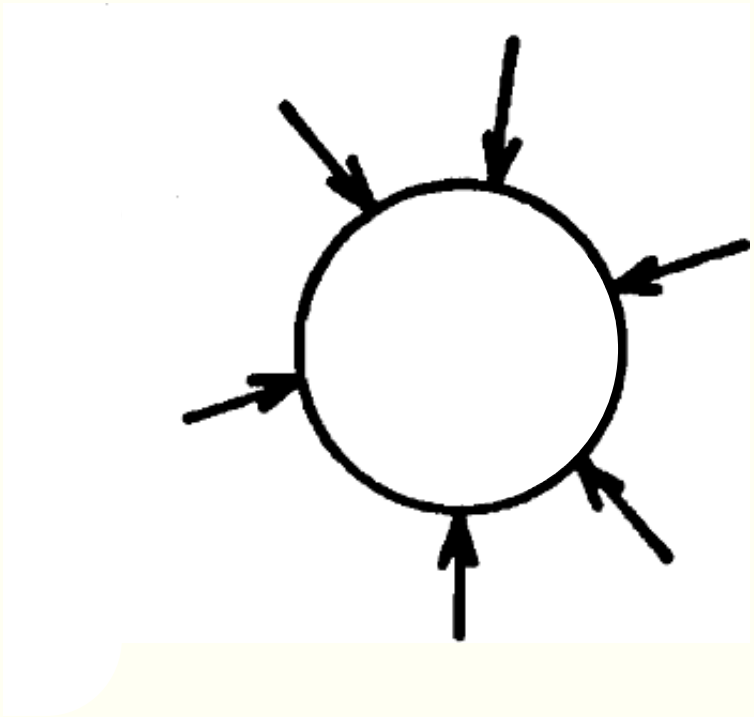

r > 0
r = 0
r < 0
x



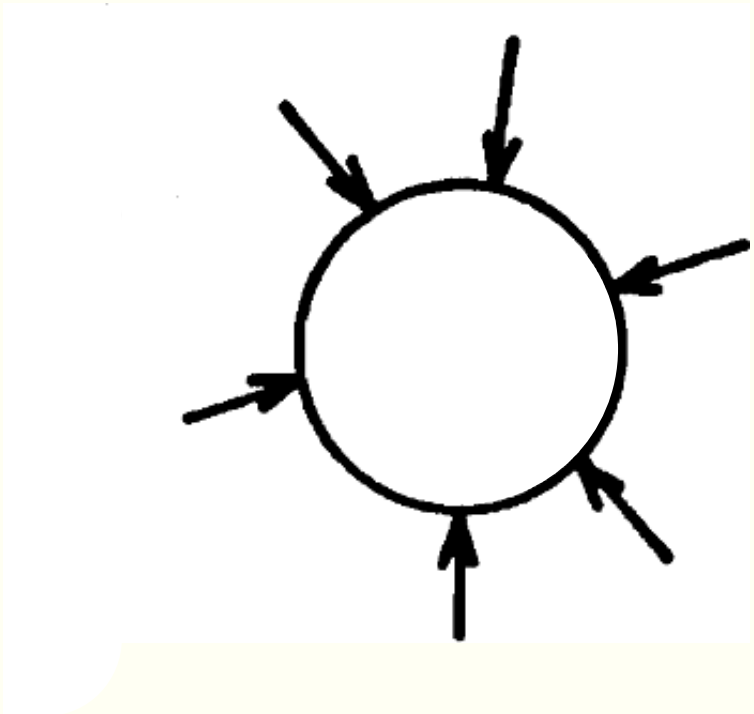

r > 0
r = 0
r < 0
x


Clean
Less cloud-active aerosol
t = 10 min
Larger drops
t = 20 min
Polluted
More cloud-active aerosol
Smaller drops

droplet growth laws in a nutshell: mass and heat diffusion



droplet growth laws in a nutshell: mass and heat diffusion

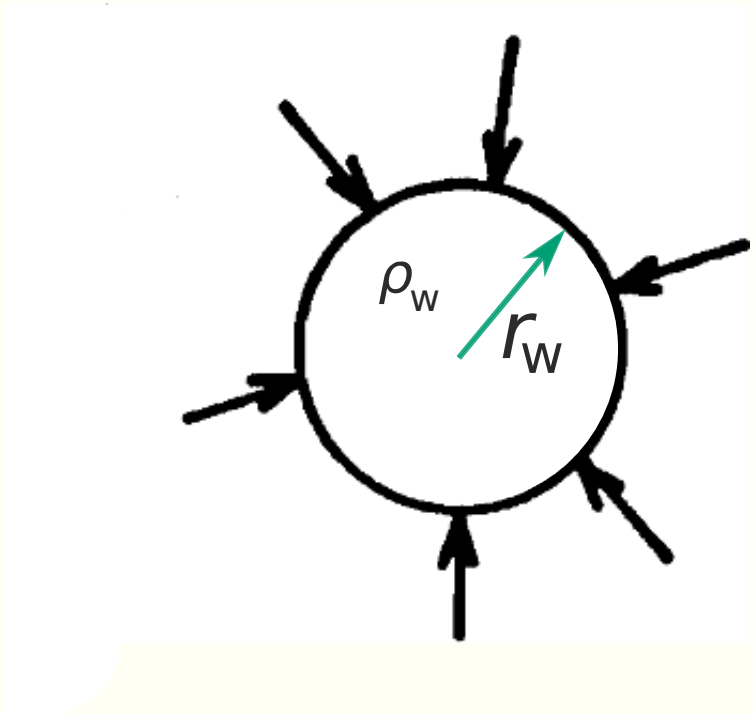


Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_{\mathrm{w}} = \frac{1}{r_{\mathrm{w}}} \frac{D_{\mathrm{eff}}}{\rho_{\mathrm{w}}} \left(\rho_{\mathrm{v}} - \rho_{\circ}\right)$$

droplet growth laws in a nutshell: mass and heat diffusion

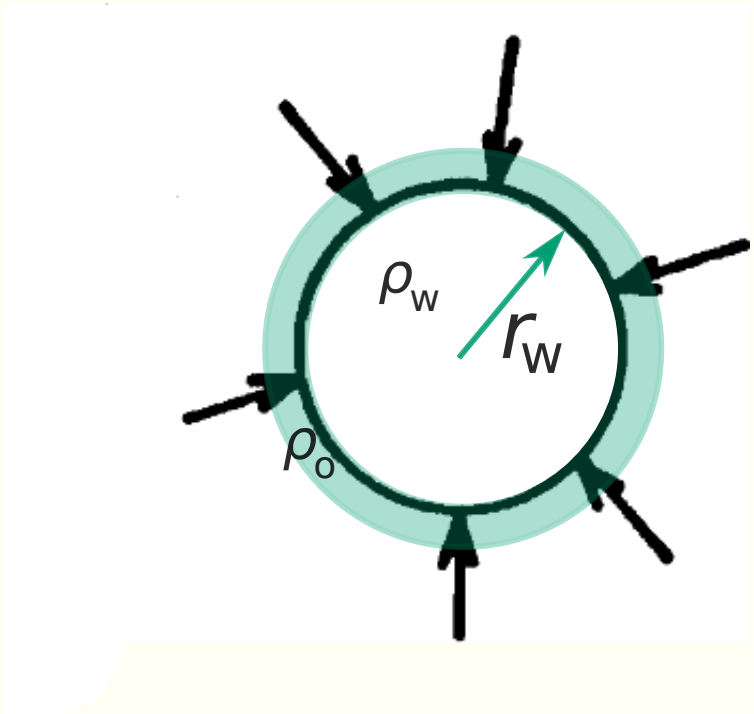


Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{eff}}{\rho_w} (\rho_v - \rho_\circ)$$

droplet growth laws in a nutshell: mass and heat diffusion

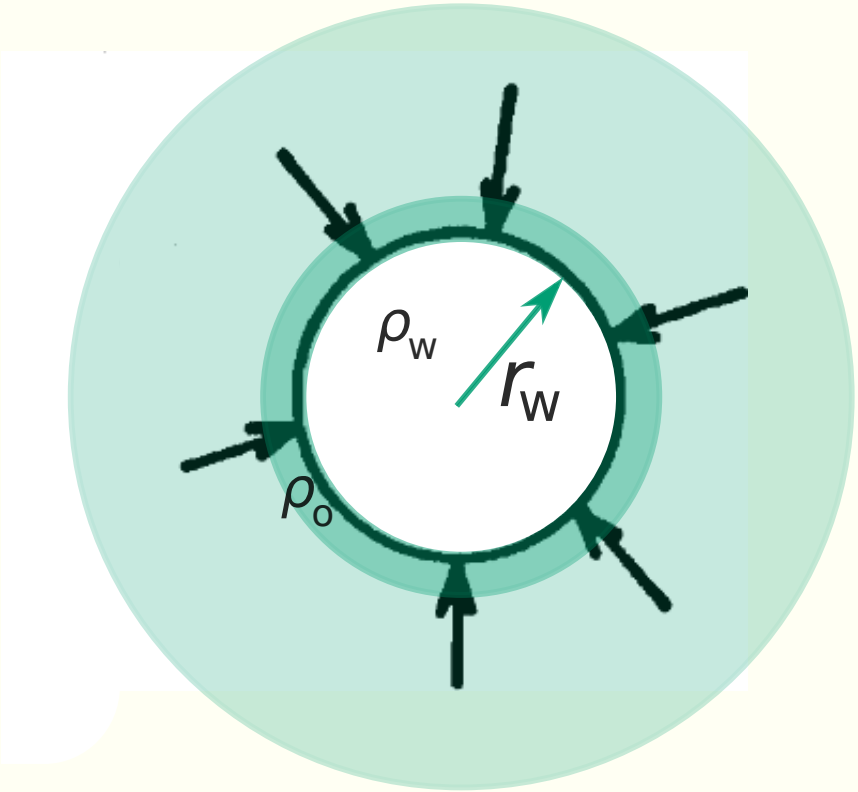


Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{eff}}{\rho_w} (\rho_v - \rho_\circ)$$

droplet growth laws in a nutshell: mass and heat diffusion

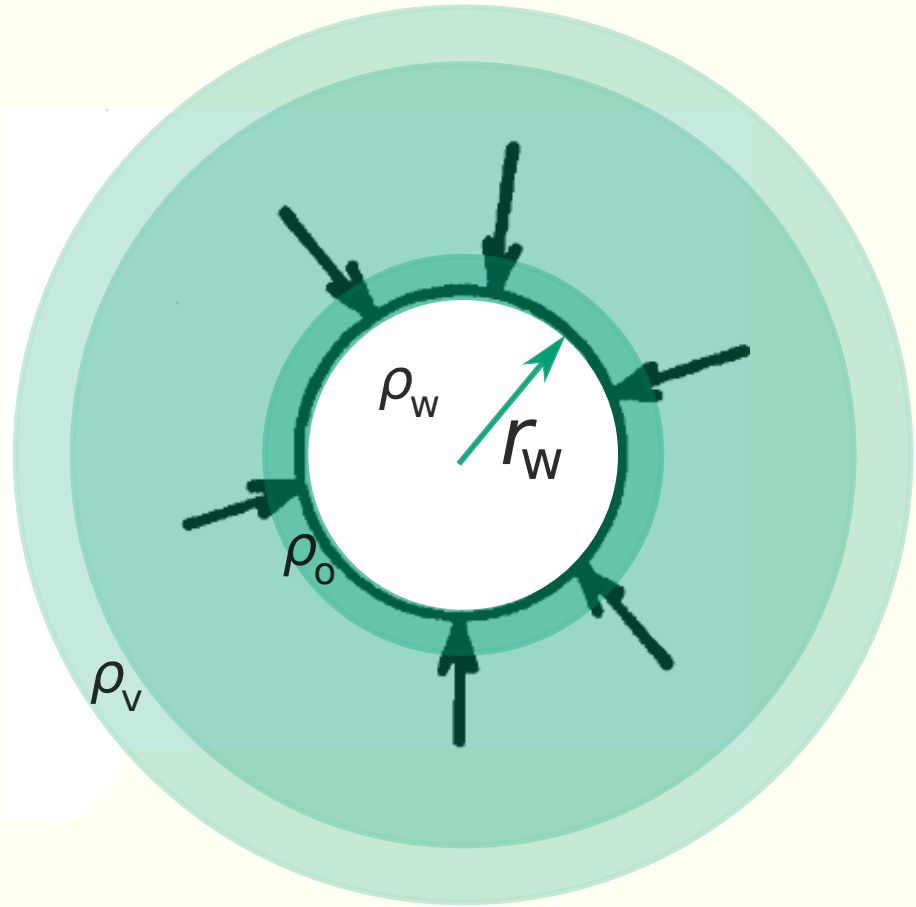


Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{eff}}{\rho_w} (\rho_v - \rho_\circ)$$

droplet growth laws in a nutshell: mass and heat diffusion

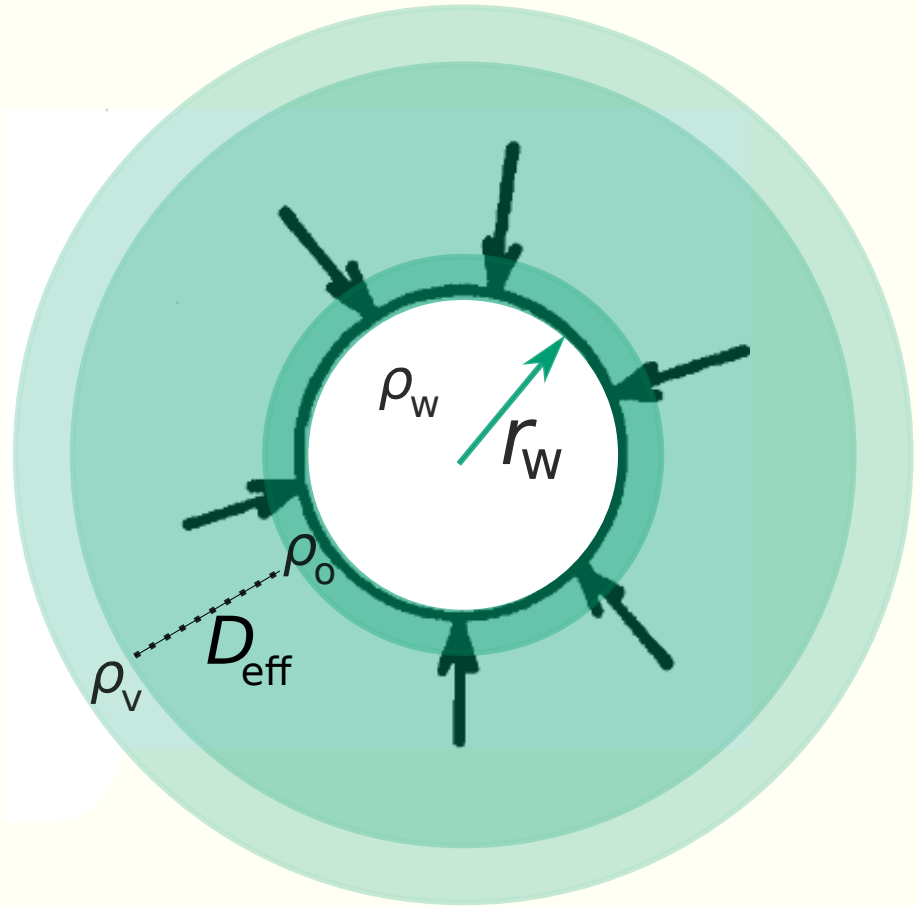


Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{eff}}{\rho_w} (\rho_v - \rho_\circ)$$

droplet growth laws in a nutshell: mass and heat diffusion

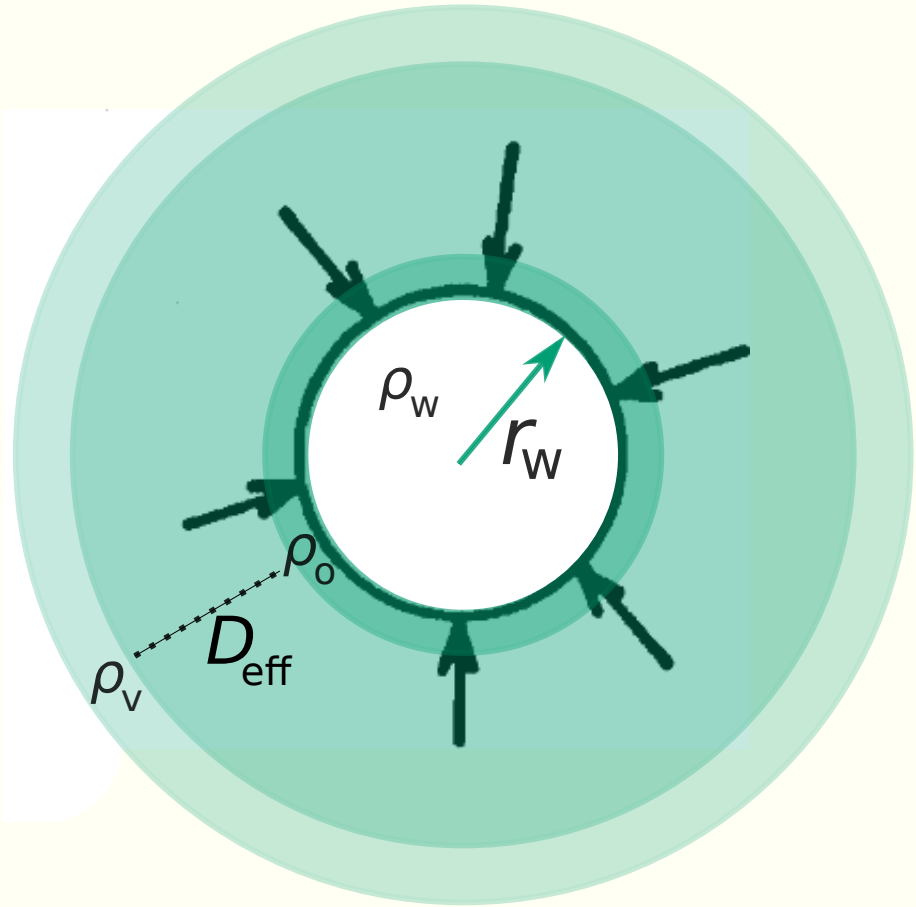


Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{eff}}{\rho_w} (\rho_v - \rho_\circ)$$

droplet growth laws in a nutshell: mass and heat diffusion



Fick's and Fourier's laws combined

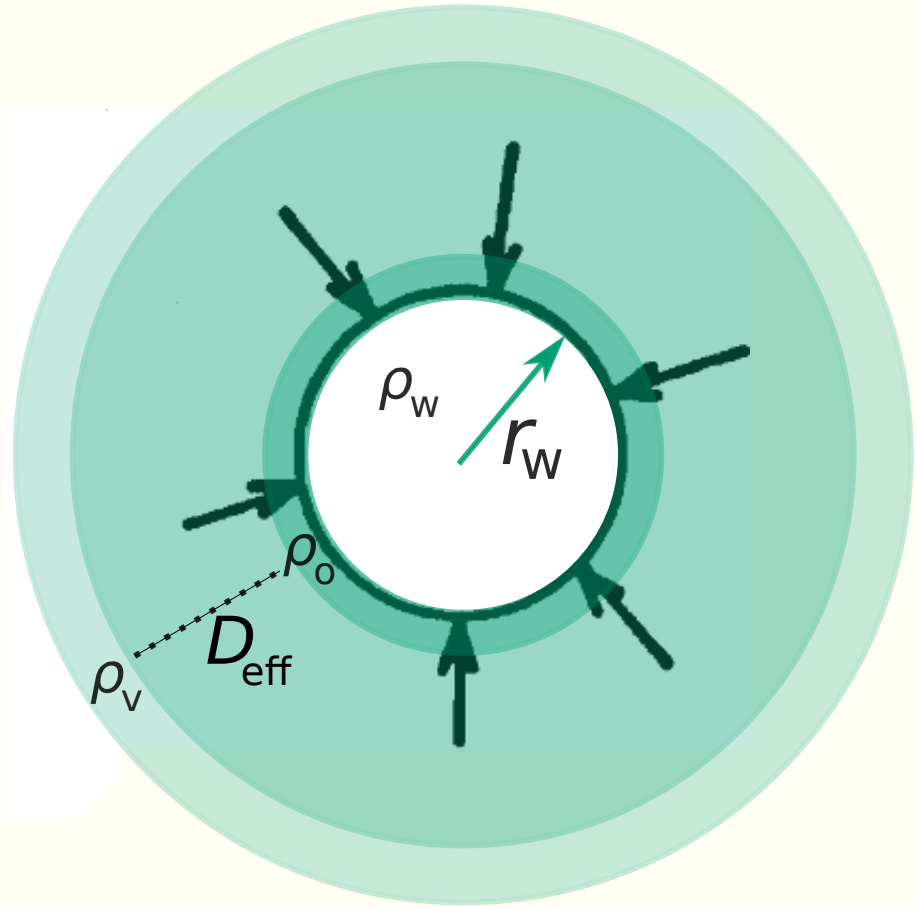
spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{eff}}{\rho_w} (\rho_v - \rho_\circ)$$

non-dimensional numbers:

$$RH = \rho_v / \rho_{vs}$$
$$RH_{eq} = \rho_\circ / \rho_{vs}$$

droplet growth laws in a nutshell: mass and heat diffusion



Fick's and Fourier's laws combined

spherical geometry

$$\dot{r}_w = \frac{1}{r_w}\frac{D_{eff}}{\rho_w}(\rho_v - \rho_\circ)$$

non-dimensional numbers:

$$RH = \rho_v/\rho_{vs}$$
$$RH_{eq} = \rho_\circ/\rho_{vs}$$

$$\dot{r}_w = \frac{1}{r_w} D_{eff}\frac{\rho_{vs}}{\rho_w}\left(RH - RH_{eq}\right)$$

droplet growth laws in a nutshell: Köhler curve

$$\dot{r}_{\mathrm{w}} = \frac{1}{r_{\mathrm{w}}} D_{\mathrm{eff}} \frac{\rho_{\mathrm{vs}}}{\rho_{\mathrm{w}}} \left(\mathrm{RH} - \mathrm{RH}_{\mathrm{eq}}\right)$$

droplet growth laws in a nutshell: Köhler curve

$$\dot{r}_\mathrm{w} = \frac{1}{r_\mathrm{w}} D_\mathrm{eff} \frac{\rho_\mathrm{vs}}{\rho_\mathrm{w}} \left(\mathrm{RH} - \mathrm{RH}_\mathrm{eq}\right)$$

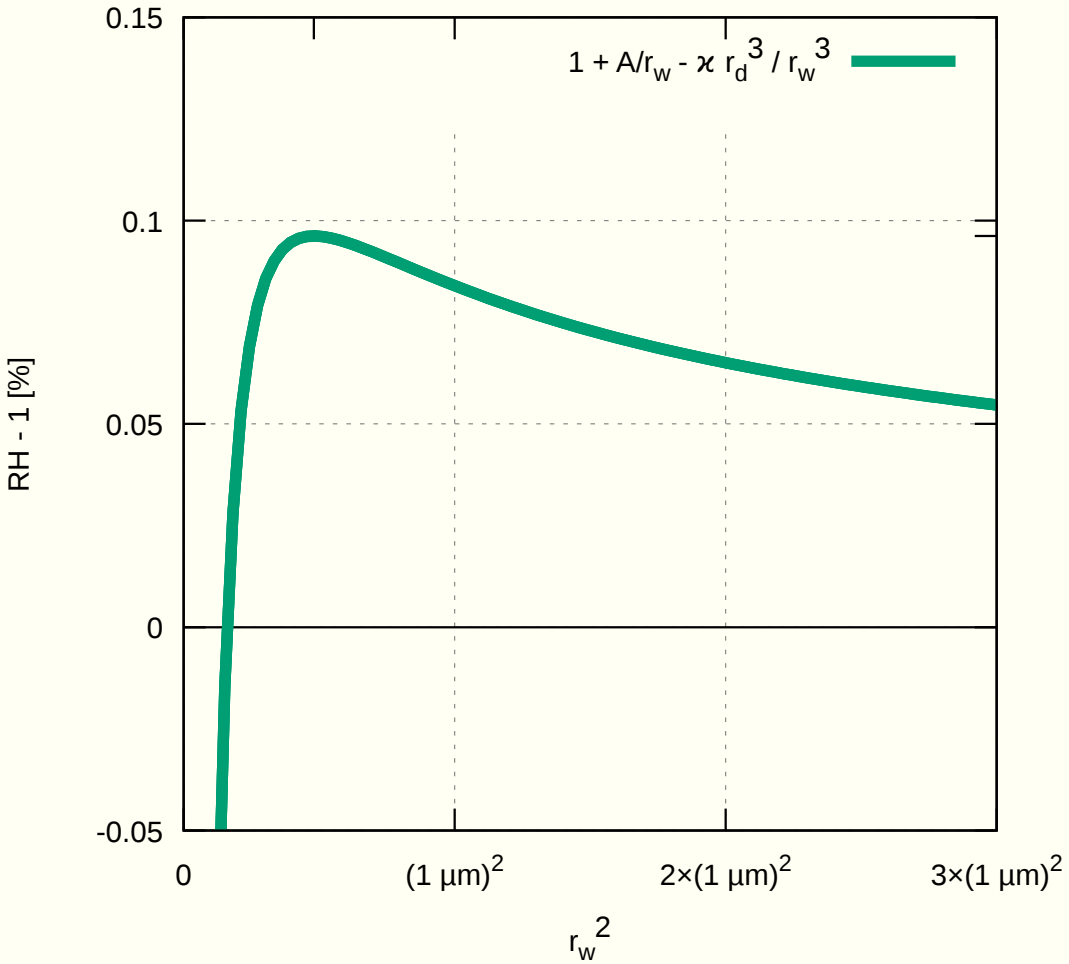
$$\begin{aligned} \mathrm{RH}_\mathrm{eq} &= \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1-\kappa)} \exp\left(\frac{A}{r_w}\right) \\ &\approx 1 + \frac{A}{r_\mathrm{w}} - \frac{\kappa r_\mathrm{d}^3}{r_\mathrm{w}^3} \end{aligned}$$

droplet growth laws in a nutshell: Köhler curve

$$\dot{r}_\mathrm{w} = \frac{1}{r_\mathrm{w}} D_\mathrm{eff} \frac{\rho_\mathrm{vs}}{\rho_\mathrm{w}} \left(\mathrm{RH} - \mathrm{RH}_\mathrm{eq}\right)$$

$$\mathrm{RH}_\mathrm{eq} = \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1-\kappa)} \exp\left(\frac{A}{r_w}\right)$$

$$\approx 1 + \frac{A}{r_\mathrm{w}} - \frac{\kappa r_\mathrm{d}^3}{r_\mathrm{w}^3}$$

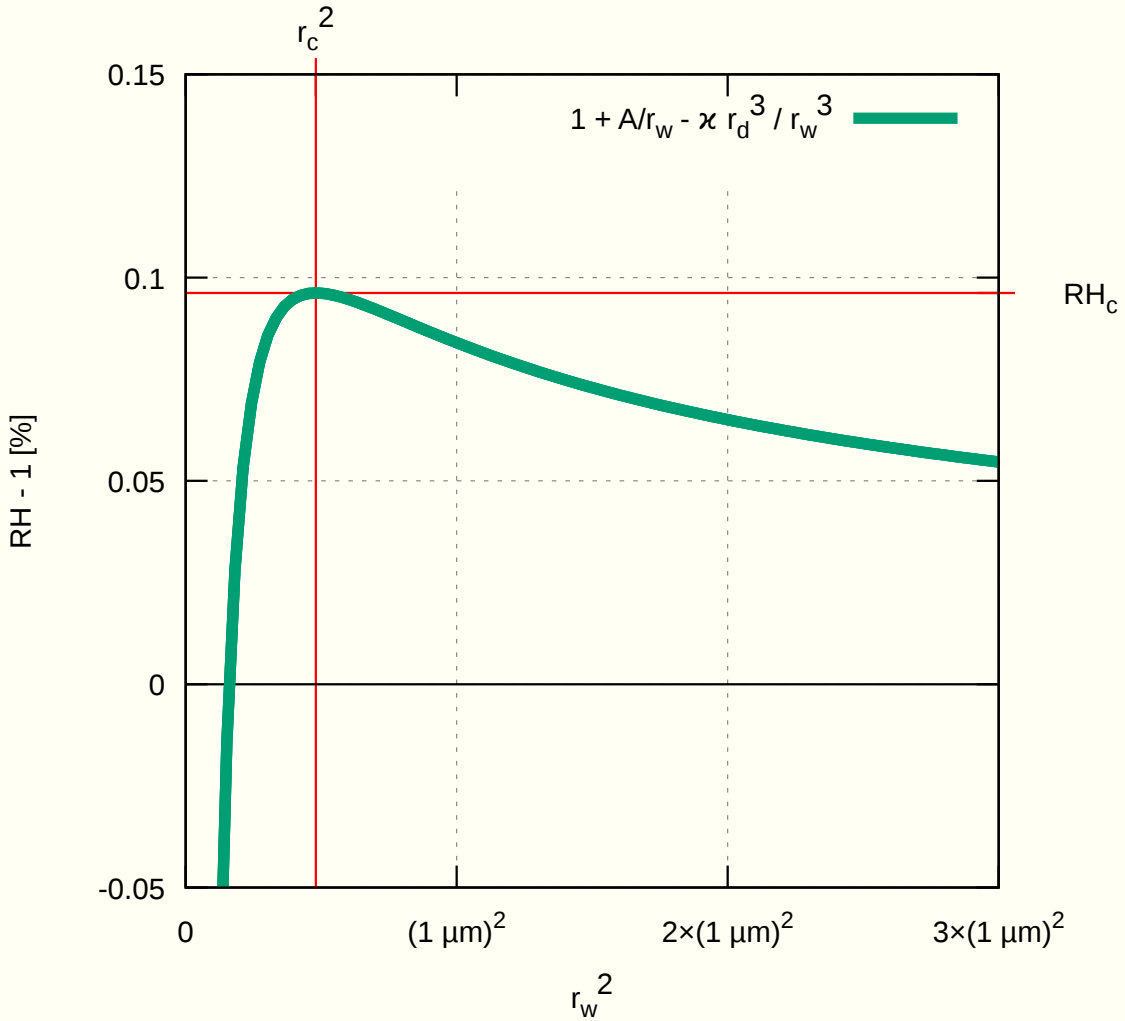


droplet growth laws in a nutshell: Köhler curve

$$\dot{r}_w = \frac{1}{r_w} D_{eff} \frac{\rho_{vs}}{\rho_w} (RH - RH_{eq})$$

$$RH_{eq} = \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1-\kappa)} \exp\left(\frac{A}{r_w}\right)$$

$$\approx 1 + \frac{A}{r_w} - \frac{\kappa r_d^3}{r_w^3}$$

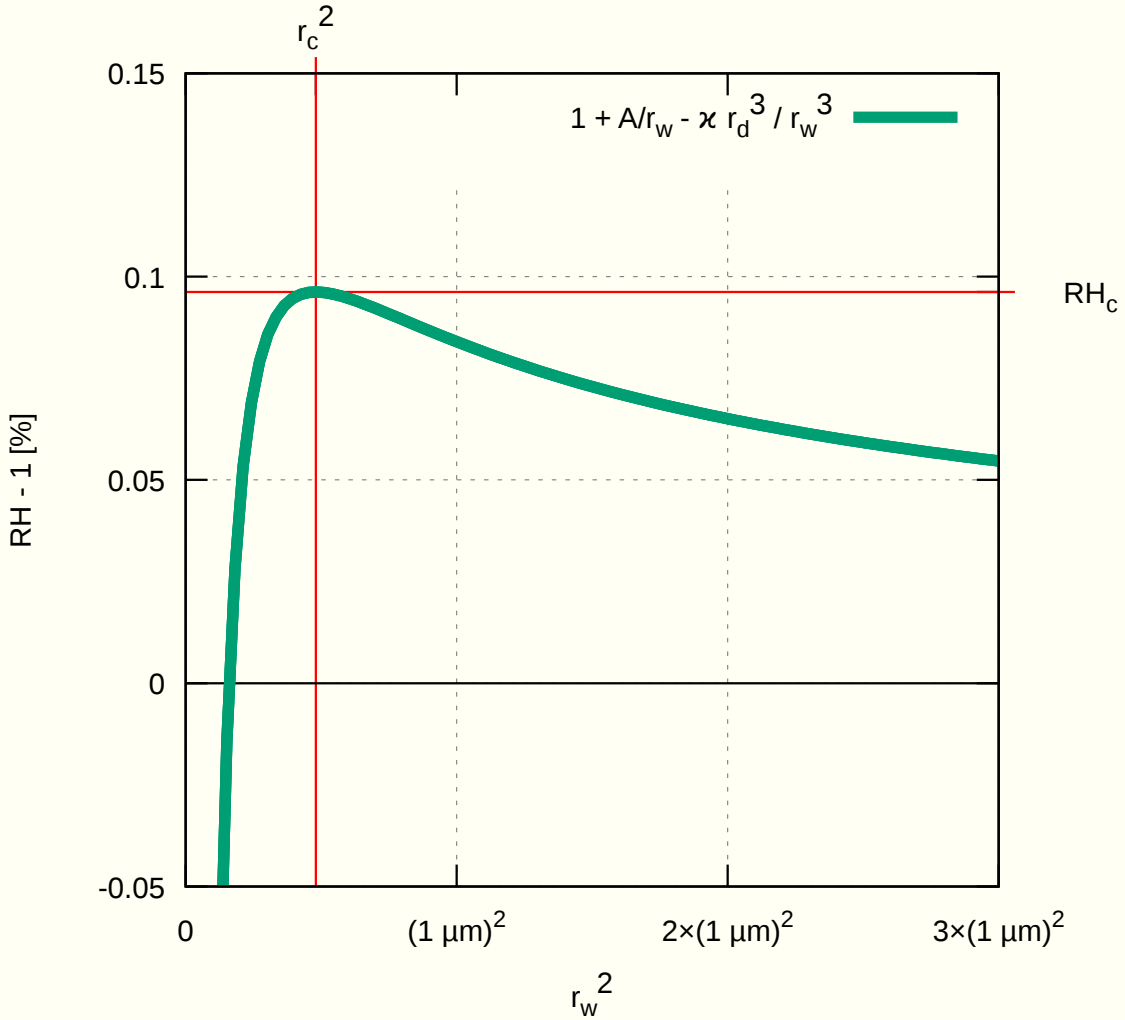


droplet growth laws in a nutshell: Köhler curve

$$\dot{r}_\mathrm{w} = \frac{1}{r_\mathrm{w}} D_\mathrm{eff} \frac{\rho_\mathrm{vs}}{\rho_\mathrm{w}} (\mathrm{RH} - \mathrm{RH_{eq}})$$

$$\mathrm{RH_{eq}} = \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1-\kappa)} \exp\left(\frac{A}{r_w}\right)$$

$$\approx 1 + \frac{A}{r_\mathrm{w}} - \frac{\kappa r_\mathrm{d}^3}{r_\mathrm{w}^3}$$

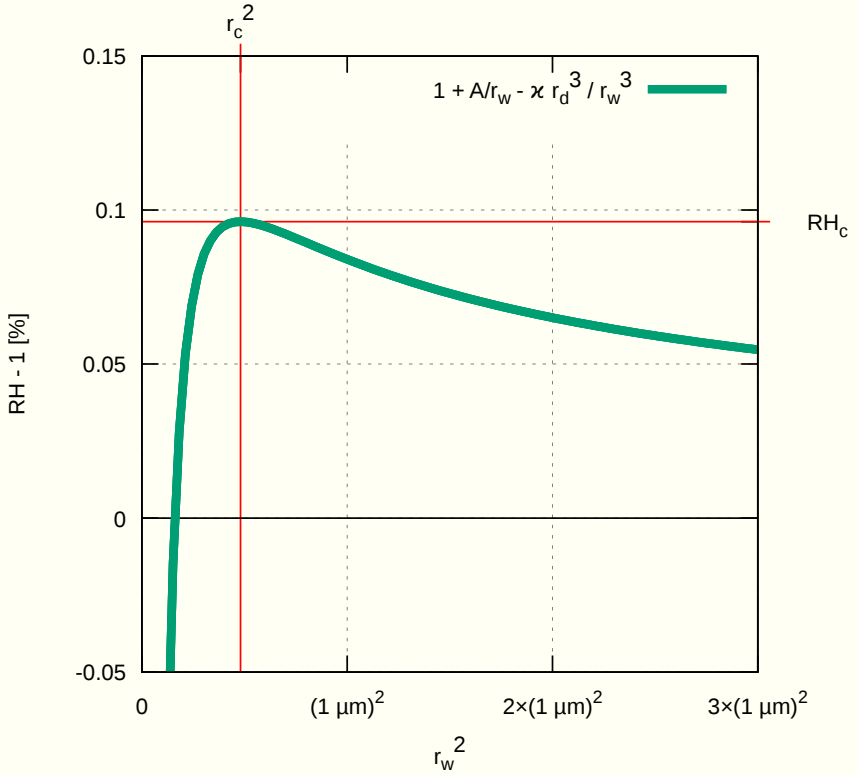


maximum at $(r_\mathrm{c}, \mathrm{RH_c})$:

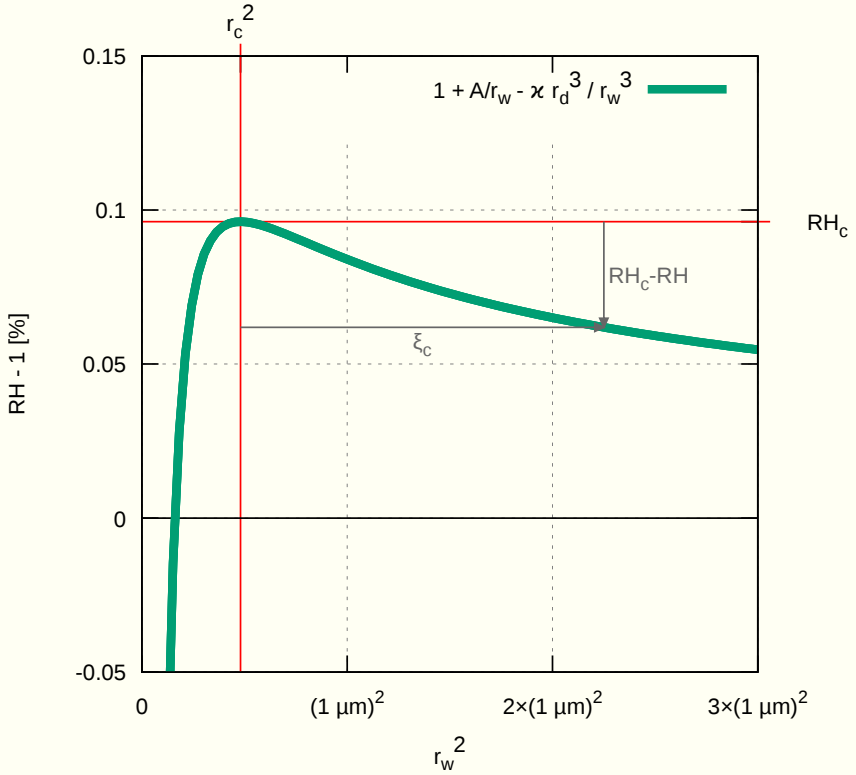
$$r_\mathrm{c} = \sqrt{3\kappa r_\mathrm{d}^3 / A}$$

$$\mathrm{RH_c} = 1 + \frac{2A}{3r_\mathrm{c}}$$

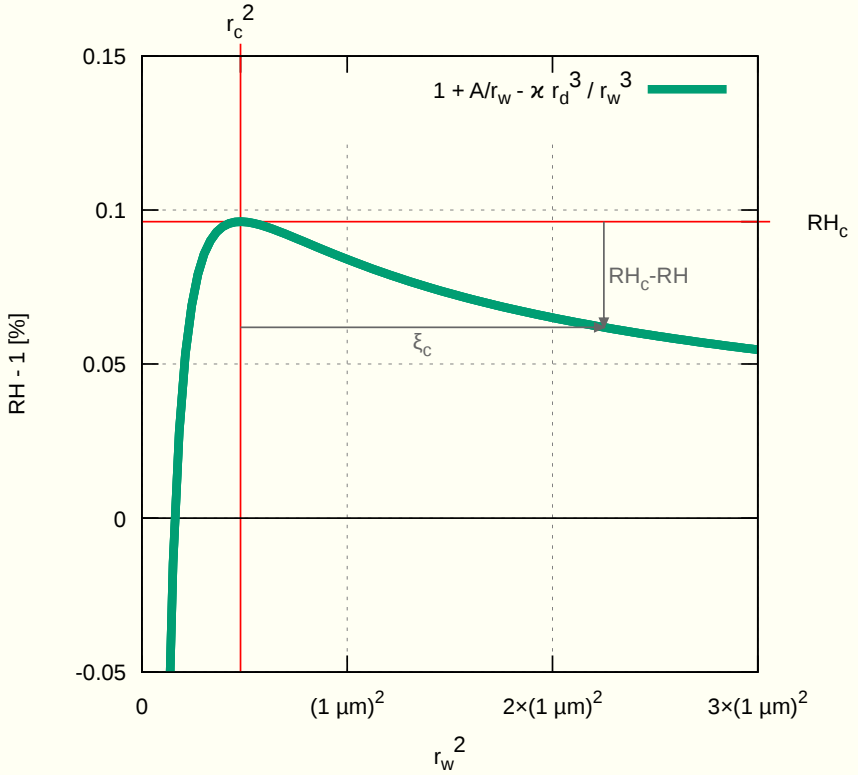
phase portrait of the system: flipped Köhler curve



phase portrait of the system: flipped Köhler curve



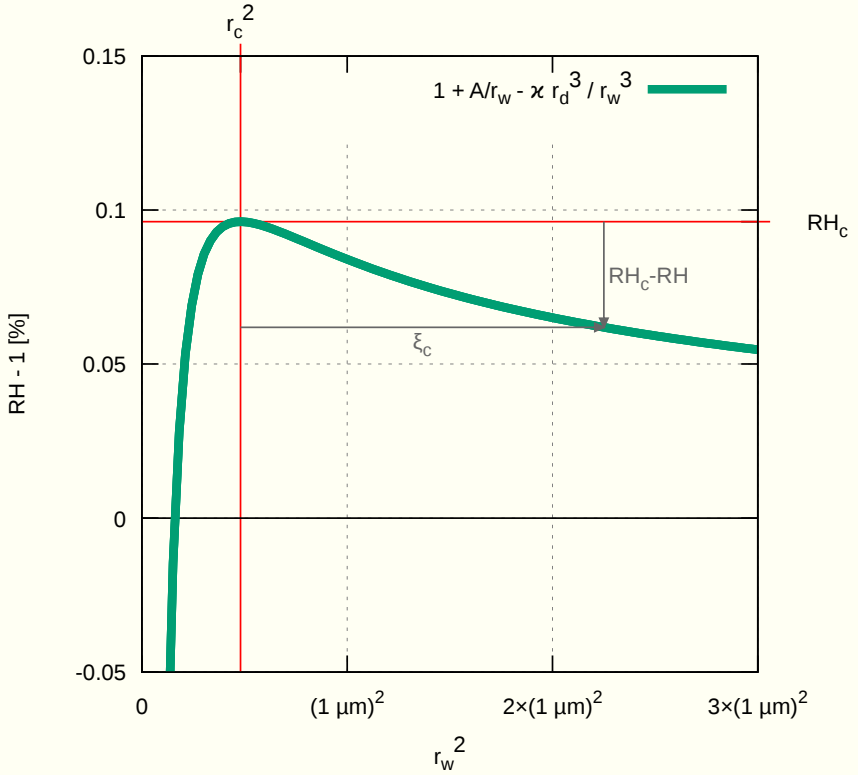
phase portrait of the system: flipped Köhler curve



$$\xi = r_\mathrm{w}^2 + C$$

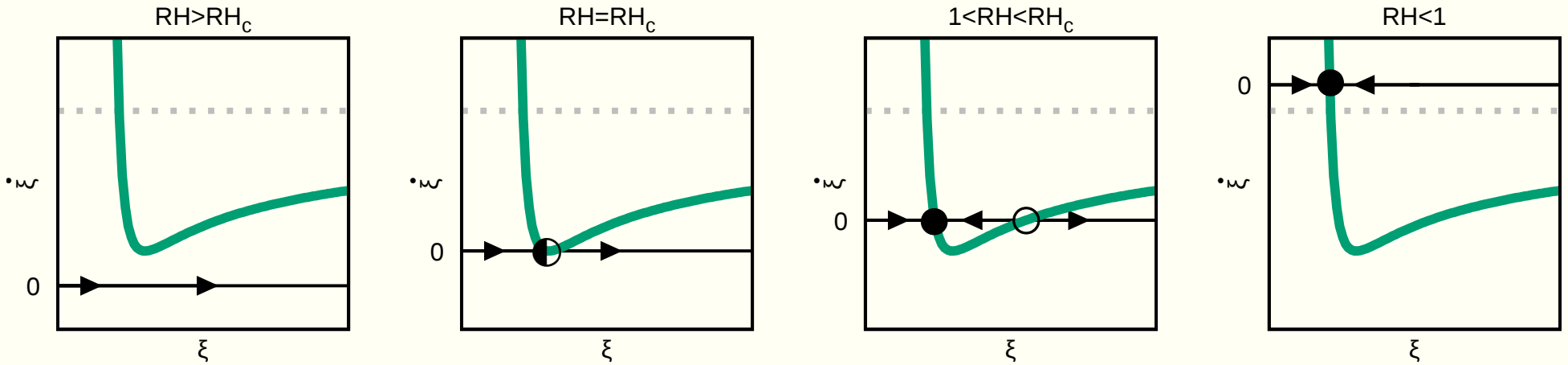
$$\dot{\xi} = 2 D_\mathrm{eff} \frac{\rho_\mathrm{vs}}{\rho_\mathrm{w}} \left(\mathrm{RH} - \mathrm{RH}_\mathrm{eq}(\xi)\right)$$

phase portrait of the system: flipped Köhler curve



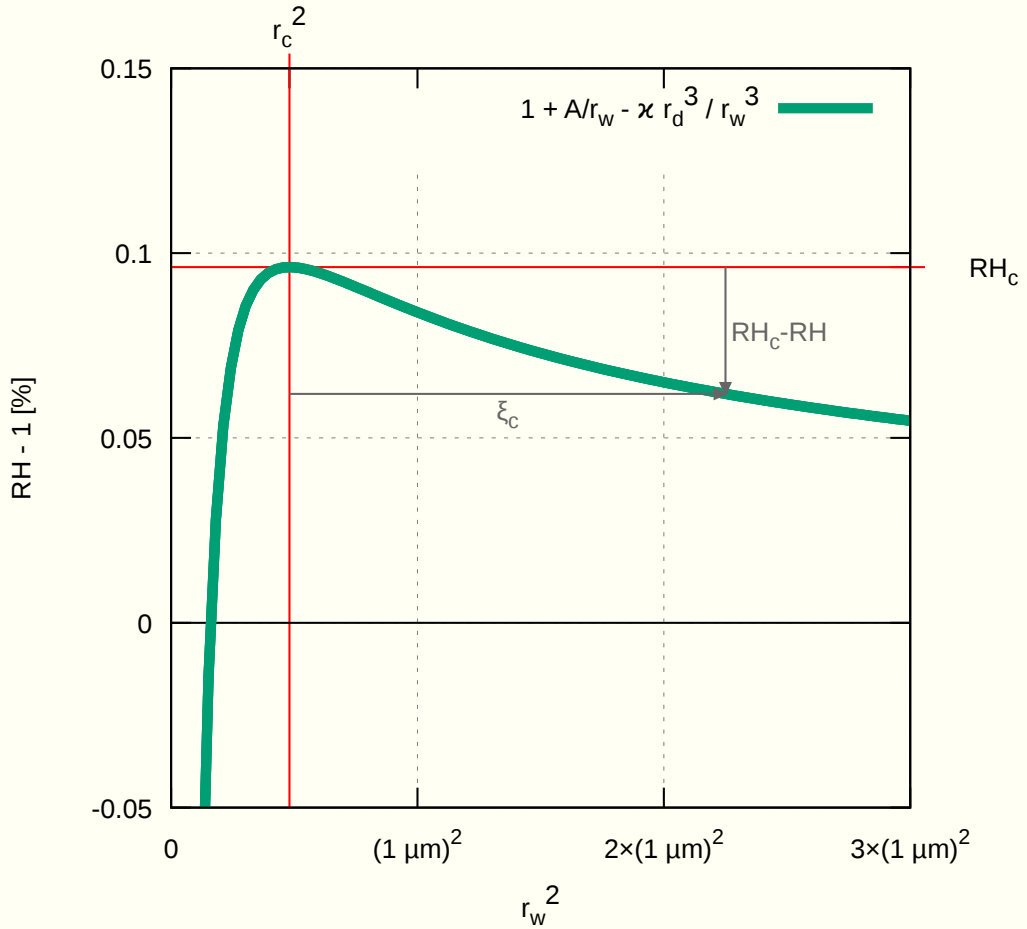
$$\xi = r_w^2 + C$$

$$\dot{\xi} = 2D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} \left(\text{RH} - \text{RH}_{\text{eq}}(\xi)\right)$$



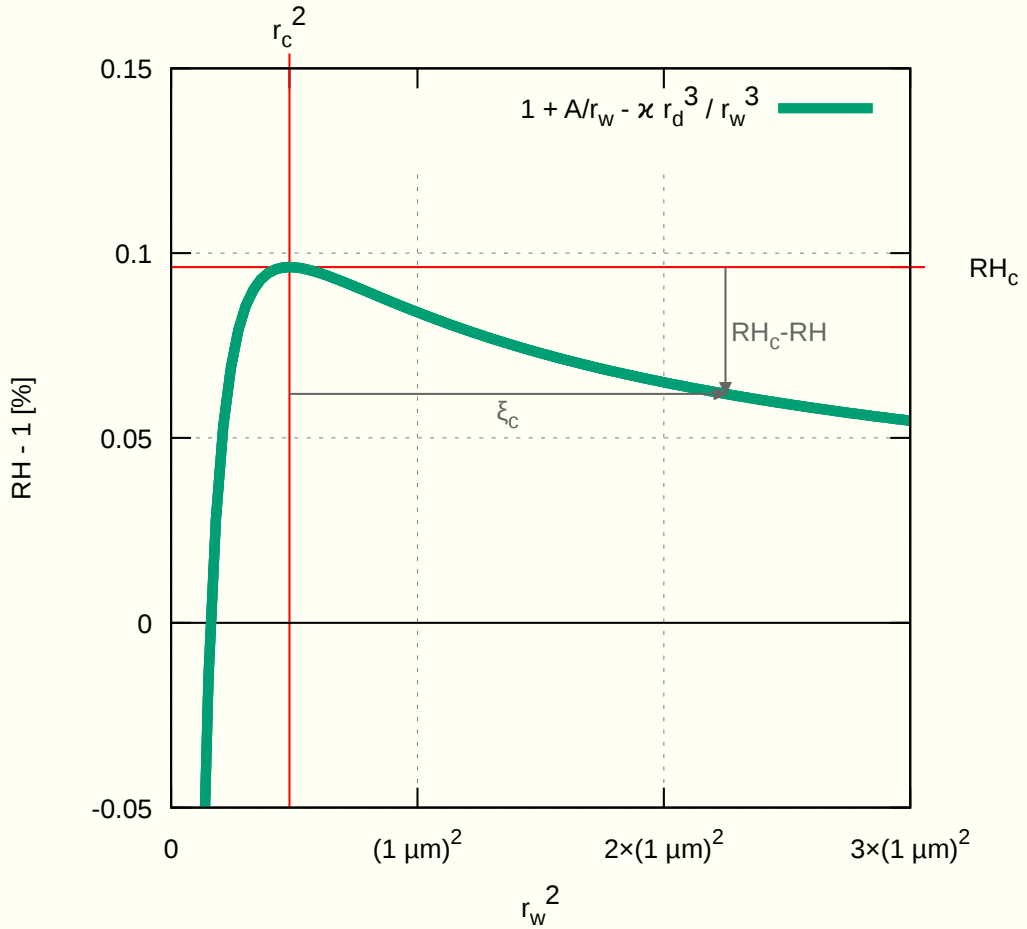
saddle-node bifurcation at Köhler curve maximum

saddle-node bifurcation at Köhler curve maximum



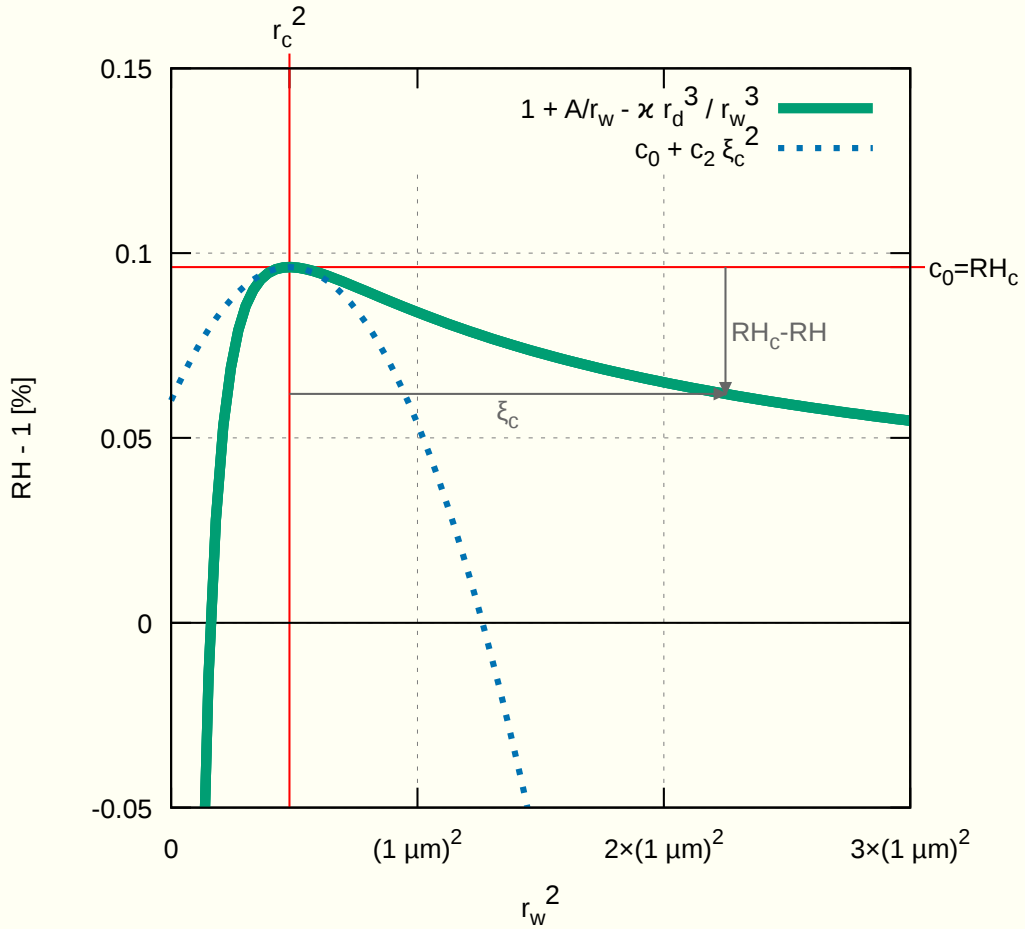
saddle-node bifurcation at Köhler curve maximum

$$\mathrm{RH}_{\mathrm{eq}}(\xi_c) = c_0 + \cancel{c_1 \xi_c} + c_2 \xi_c^2 + \ldots$$



saddle-node bifurcation at Köhler curve maximum

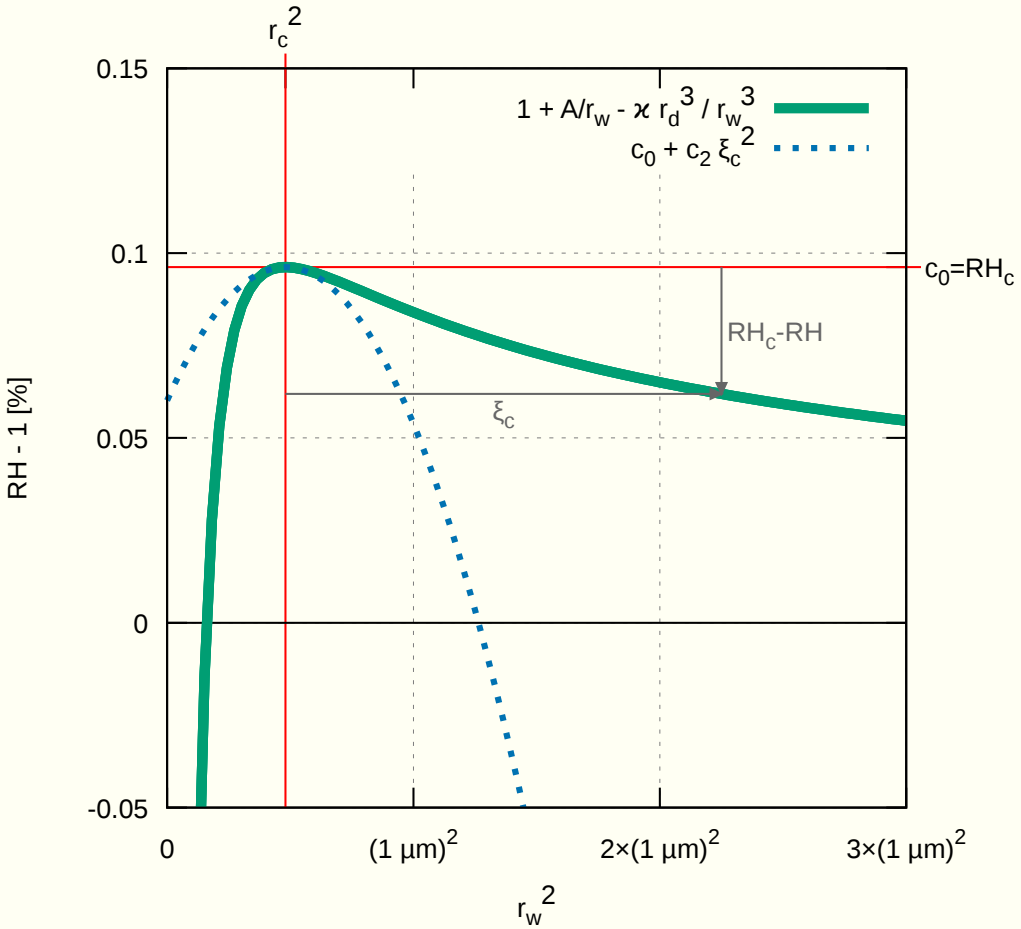
$$\mathrm{RH}_{\mathrm{eq}}(\xi_c) = c_0 + \cancel{c_1 \xi_c} + c_2 \xi_c^2 + \ldots$$



saddle-node bifurcation at Köhler curve maximum

$$\mathrm{RH}_{\mathrm{eq}}(\xi_c) = c_0 + \cancel{c_1 \xi_c} + c_2 \xi_c^2 + \ldots$$

$$\dot{\xi}_c\Big|_{\xi_c \to 0} \sim \frac{\mathrm{RH} - \mathrm{RH}_c}{A/(4r_c^5)} + \xi_c^2$$

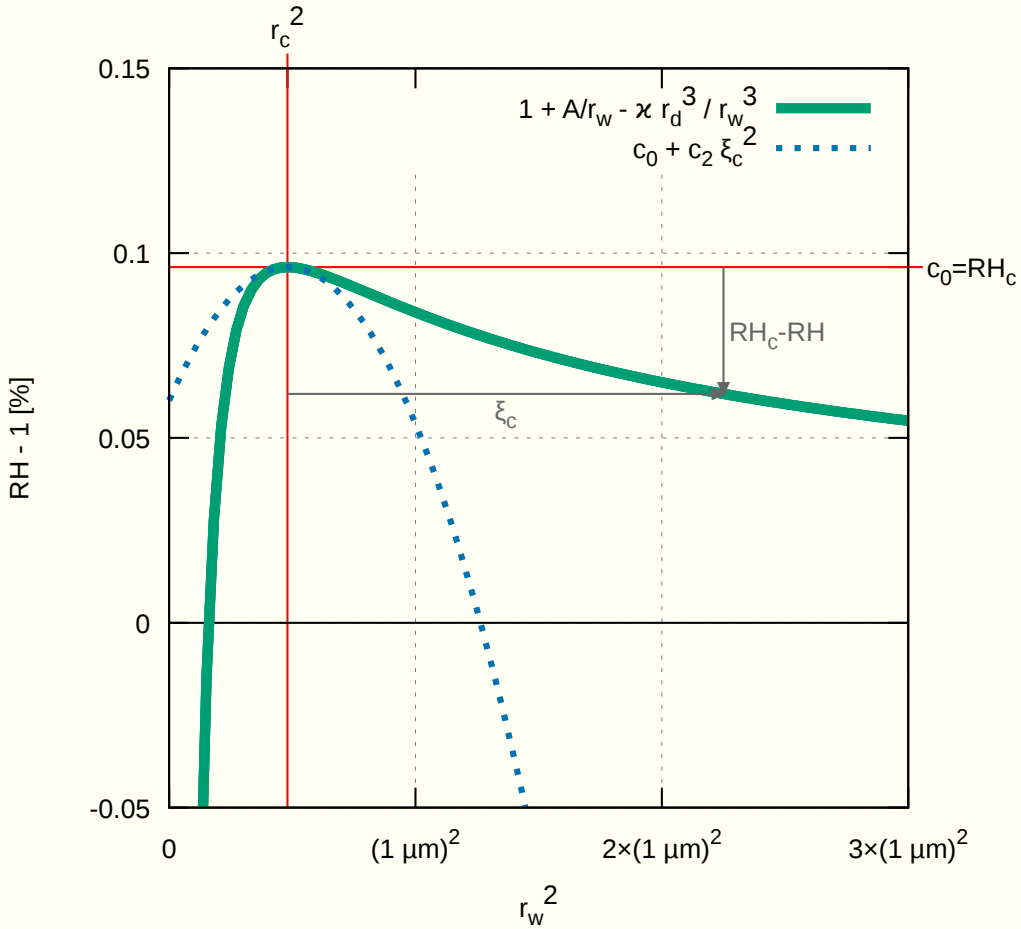


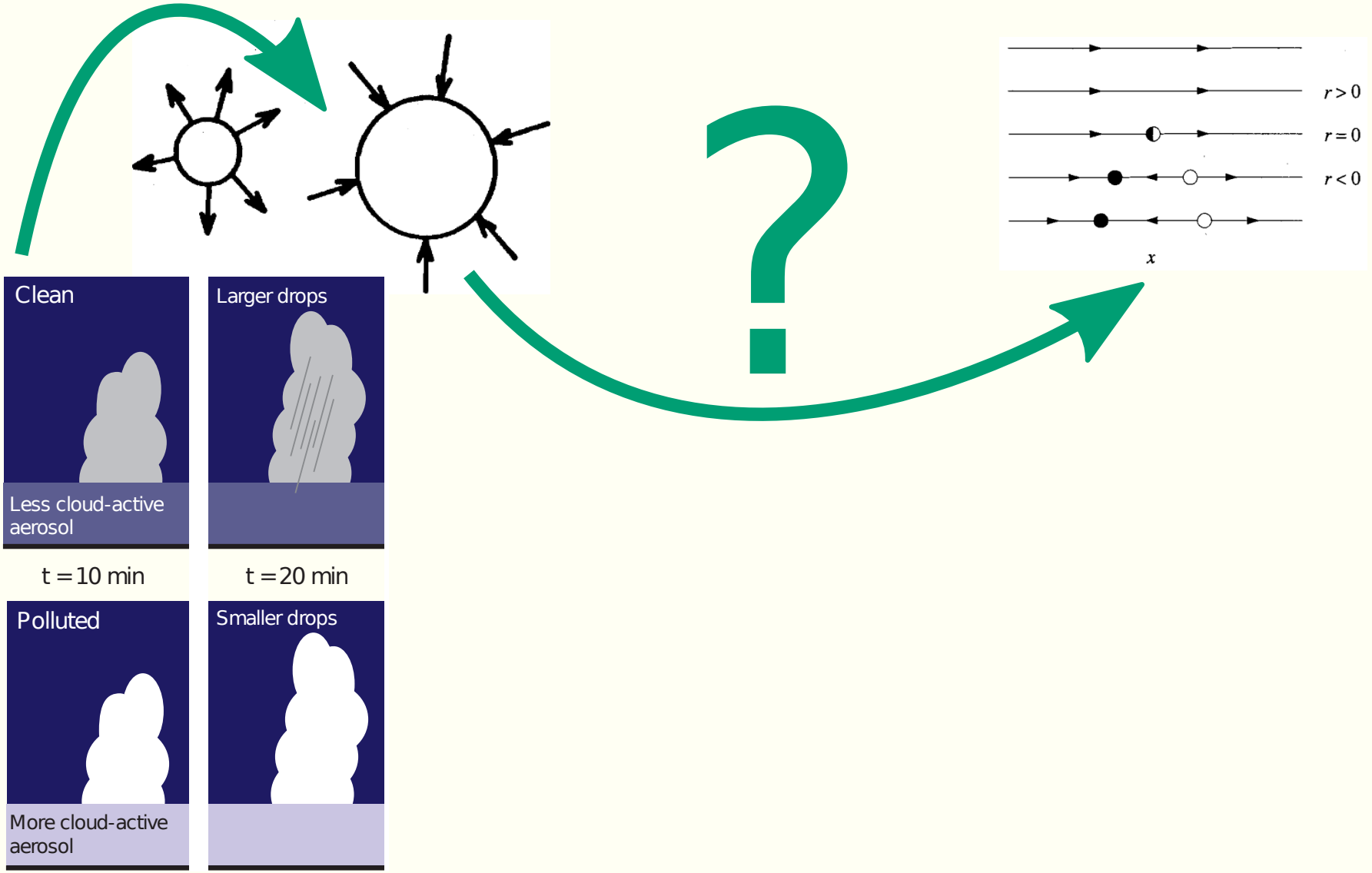
saddle-node bifurcation at Köhler curve maximum

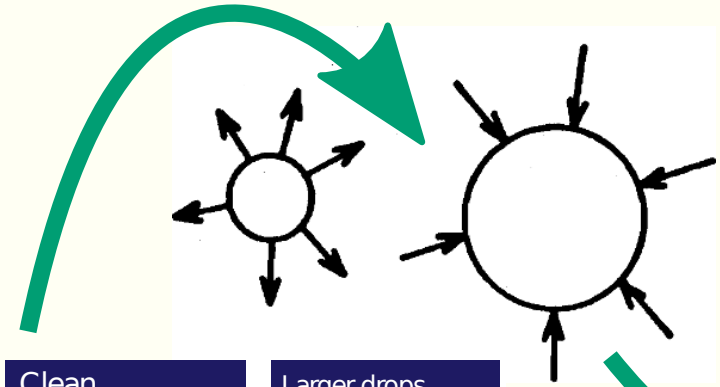
$$\mathrm{RH}_{\mathrm{eq}}(\xi_c) = c_0 + \cancel{c_1 \xi_c} + c_2 \xi_c^2 + \ldots$$

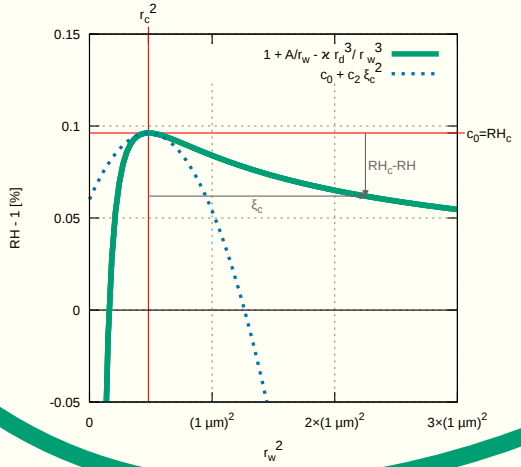
$$\dot{\xi_c}\Big|_{\xi_c \to 0} \sim \frac{\mathrm{RH} - \mathrm{RH_c}}{A/(4r_c^5)} + \xi_c^2$$

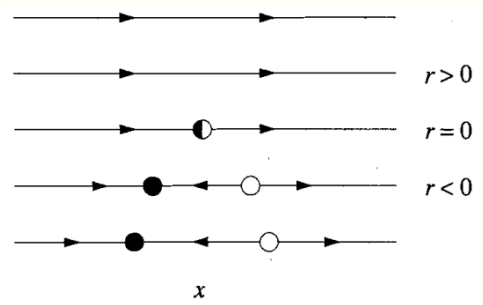
$$\dot{x} = r + x^2$$

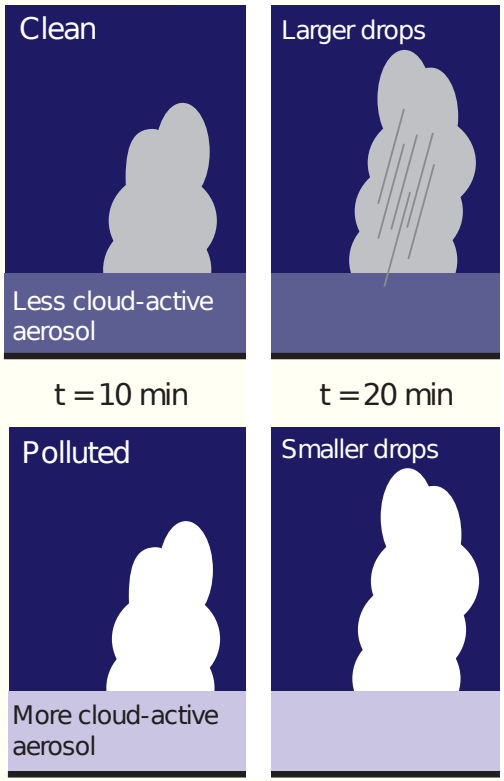


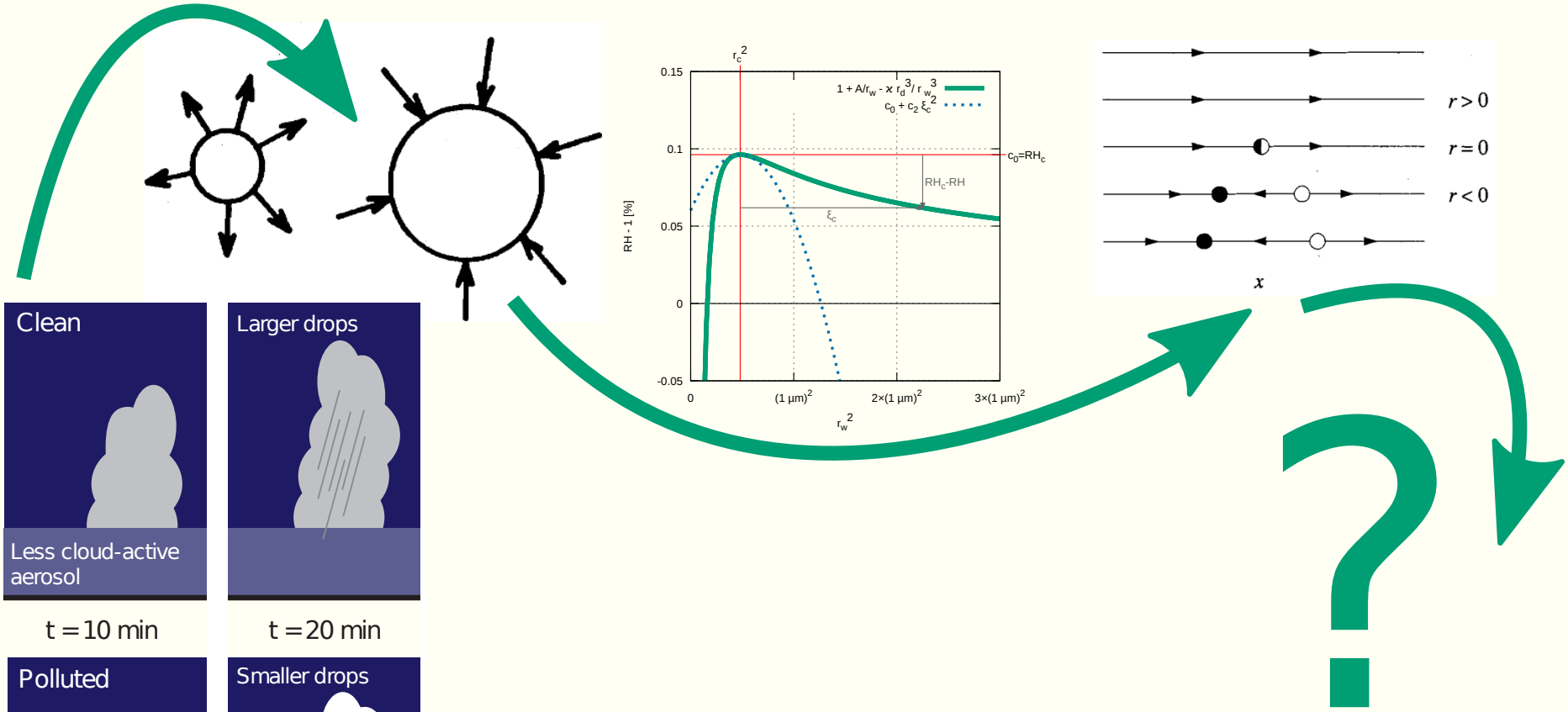

Clean
Larger drops
Less cloud-active aerosol
t = 10 min
t = 20 min
Polluted
Smaller drops
More cloud-active aerosol
r > 0
r = 0
r < 0
x




r_c^2
0.15
0.1
0.05
0
-0.05
RH - 1 [%]
$1 + A/r_w - \varkappa\, r_d^3 / r_w^3$
$c_0 + c_2\, \xi_c^2$
c_0=RH$_c$
RH$_c$-RH
ξ_c
0
(1 µm)^2
2×(1 µm)^2
3×(1 µm)^2
r_w^2


$r > 0$
$r = 0$
$r < 0$
x


Clean
Less cloud-active aerosol
Larger drops
t = 10 min
t = 20 min
Polluted
More cloud-active aerosol
Smaller drops


Clean
Less cloud-active aerosol
t = 10 min
Larger drops
t = 20 min
Polluted
More cloud-active aerosol
Smaller drops
r_c^2
0.15
0.1
0.05
0
-0.05
RH - 1 [%]
1 + A/r_w - κ r_d^3 / r_w^3
c_0 + c_2 ξ_c^2
c_0=RH_c
RH_c-RH
ξ_c
0
(1 μm)^2
2×(1 μm)^2
3×(1 μm)^2
r_w^2
r > 0
r = 0
r < 0
x

coalescence in the saddle-node bottleneck (sic!)

coalescence in the saddle-node bottleneck (sic!)

Strogatz 2014 (sect. 4.3): *coalescence* of the fixed points
is associated with a passage through a *bottleneck*,

key observation: time of passage through the parabolic *bottleneck*
dominates all other timescales

coalescence in the saddle-node bottleneck (sic!)

Strogatz 2014 (sect. 4.3): *coalescence* of the fixed points
is associated with a passage through a *bottleneck*,

key observation: time of passage through the parabolic *bottleneck*
dominates all other timescales

coalescence in the saddle-node bottleneck (sic!)

Strogatz 2014 (sect. 4.3): *coalescence* of the fixed points is associated with a passage through a *bottleneck*,

key observation: time of passage through the parabolic *bottleneck* dominates all other timescales

$$\tau_{act} \approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c}$$

coalescence in the saddle-node bottleneck (sic!)

Strogatz 2014 (sect. 4.3): *coalescence* of the fixed points is associated with a passage through a *bottleneck*,

key observation: time of passage through the parabolic *bottleneck* dominates all other timescales

$$\tau_{act} \approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c}$$

$$= \frac{r_c^{5/2}}{\sqrt{A}} \frac{\rho_w/\rho_{vs}}{D_{eff}} \frac{\pi}{\sqrt{RH - RH_c}}$$

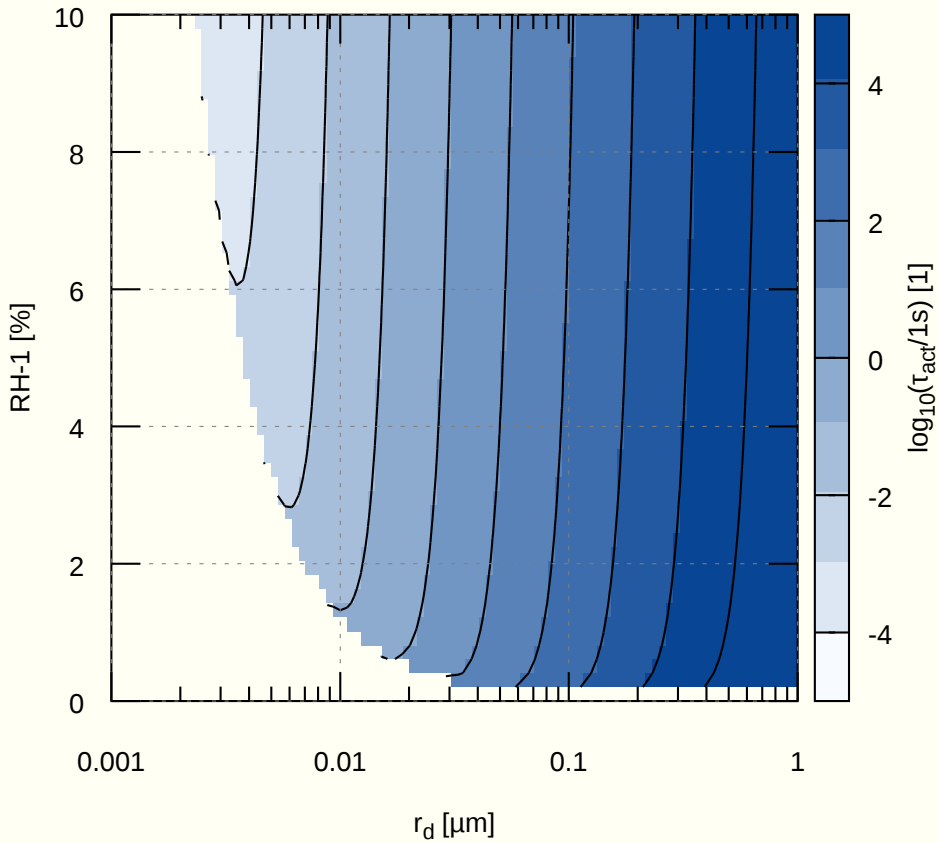
coalescence in the saddle-node bottleneck (sic!)

Strogatz 2014 (sect. 4.3): *coalescence* of the fixed points is associated with a passage through a *bottleneck*,

key observation: time of passage through the parabolic *bottleneck* dominates all other timescales

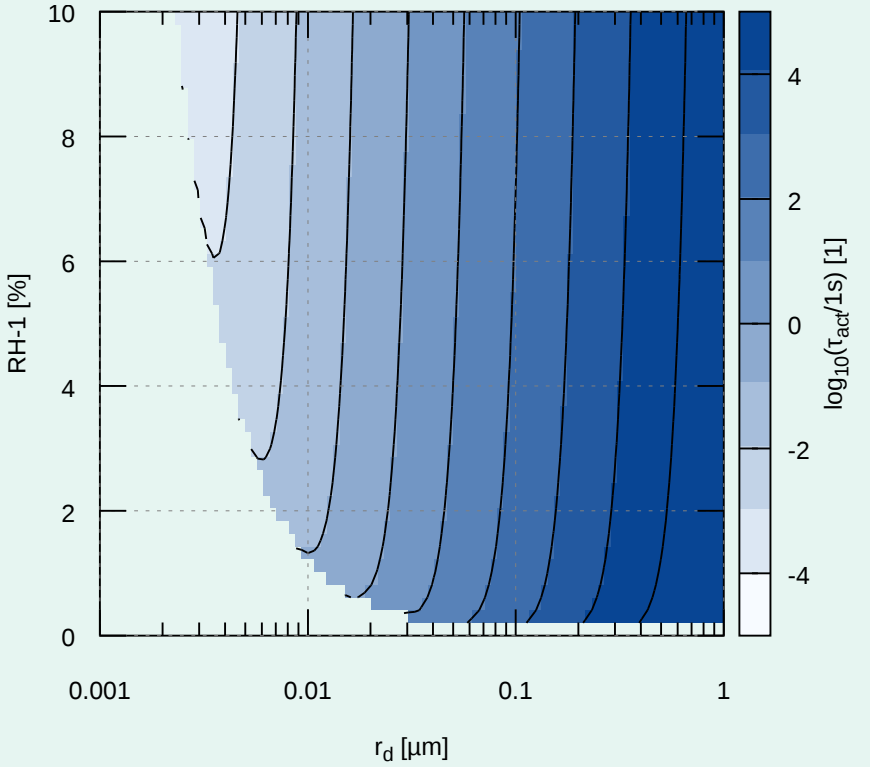
$$\tau_{act} \approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c}$$

$$= \frac{r_c^{5/2}}{\sqrt{A}} \frac{\rho_w/\rho_{vs}}{D_{eff}} \frac{\pi}{\sqrt{RH - RH_c}}$$



activation timescale: analytic vs. numerical

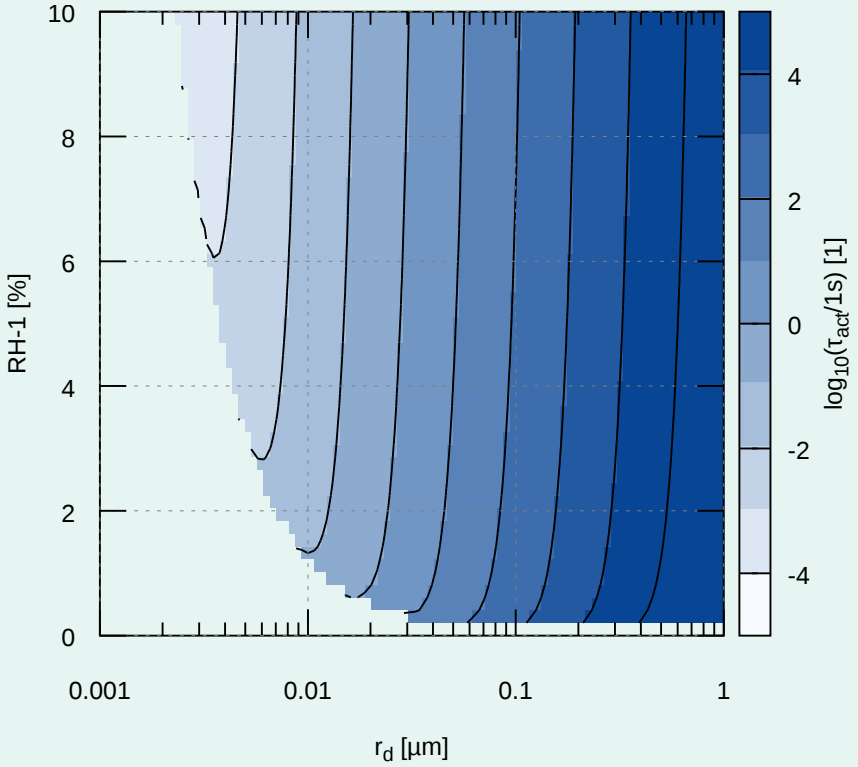
Arabas & Shima 2017



note: axes ranges vs. close-to-equilibrium assumption

activation timescale: analytic vs. numerical

Arabas & Shima 2017



note: axes ranges vs. close-to-equilibrium assumption

Hoffmann, 2016 (MWR)

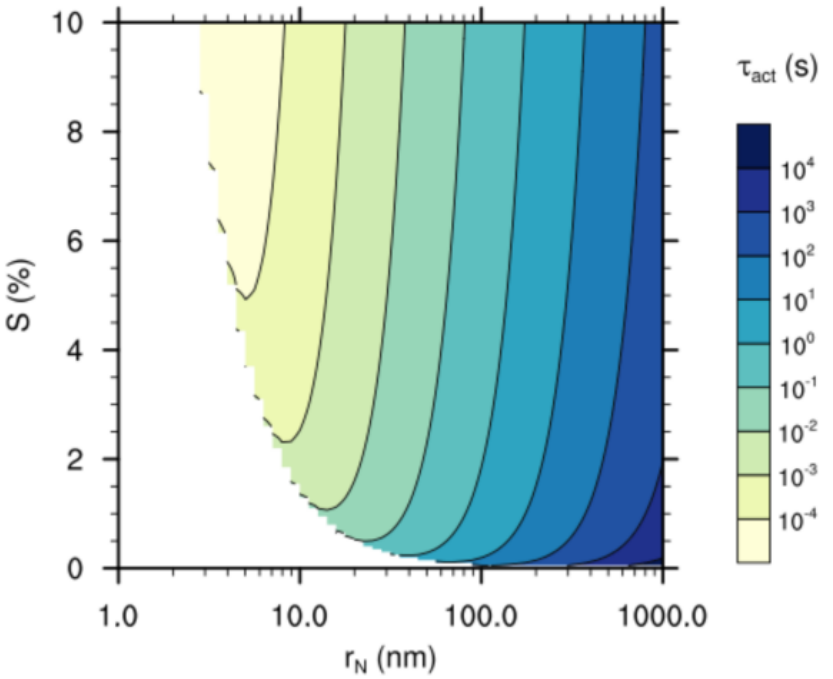
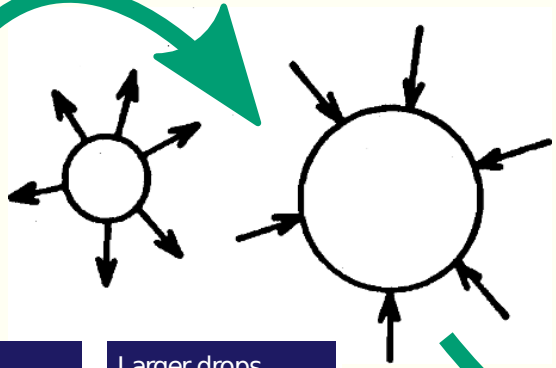
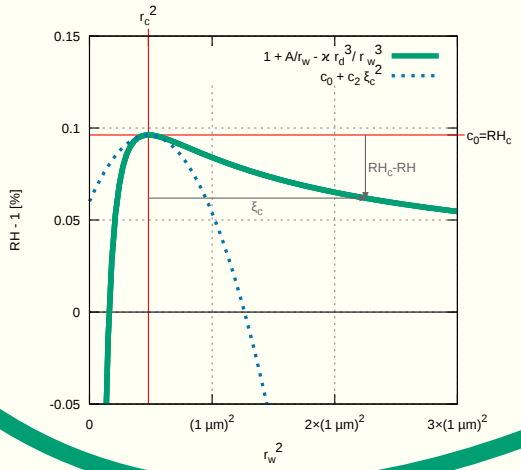


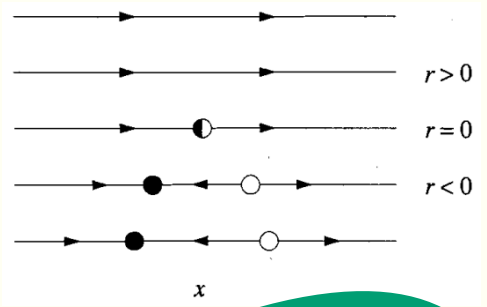
FIG. 2. The activation time scale τ_{act} as a function of dry aerosol radius r_N and supersaturation S. For values of $S < S_{crit}$ (white areas), τ_{act} does not exist.

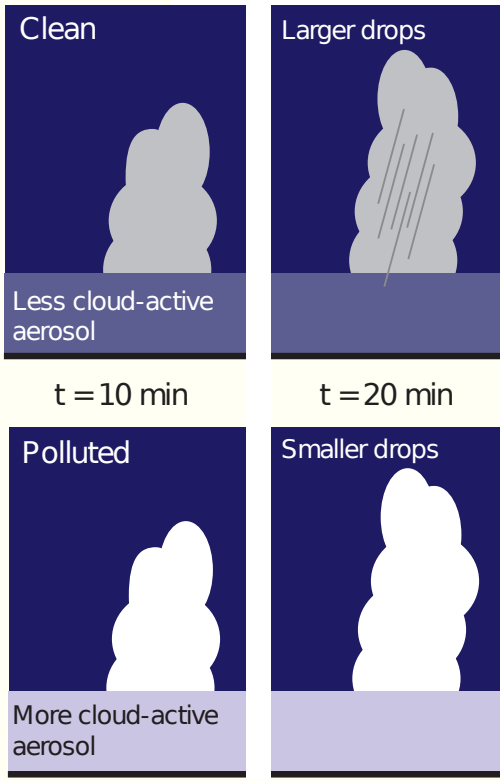
$$r\frac{dr}{dt} = \left(S - \frac{A}{r} + \frac{Br_N^3}{r^3}\right) \Big/ (F_k + F_D), \qquad (10)$$

The second time scale is associated with the activation of particles, for which Köhler theory is essential. This makes an analytic solution for (10) impossible. Numerically calculated values of τ_{act} measuring the time needed for a wetted aerosol to grow beyond its critical radius $r_{crit} = \sqrt{3Br_N^3/A}$ are given in Fig. 2 as a function of




r_c^2
0.15
0.1
0.05
0
-0.05
RH - 1 [%]
1 + A/r_w - κ r_d^3 / r_w^3
c_0 + c_2 ξ_c^2
c_0=RH_c
RH_c-RH
ξ_c
0
(1 μm)^2
2×(1 μm)^2
3×(1 μm)^2
r_w^2


r > 0
r = 0
r < 0
x


Clean
Larger drops
Less cloud-active aerosol
t = 10 min
t = 20 min
Polluted
Smaller drops
More cloud-active aerosol

RH-coupled system & particle concentration as parameter

RH-coupled system & particle concentration as parameter

simple moisture budget (const T,p):

$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_\mathrm{v}}{\rho_\mathrm{vs}} = -N \underbrace{\frac{4\pi\rho_\mathrm{w}}{3\rho_\mathrm{vs}}}_{\alpha} 3 r_\mathrm{w}^2 \dot{r}_\mathrm{w}$$

RH-coupled system & particle concentration as parameter

simple moisture budget (const T,p):

$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_\mathrm{v}}{\rho_\mathrm{vs}} = -N \underbrace{\frac{4\pi\rho_\mathrm{w}}{3\rho_\mathrm{vs}}}_{\alpha} 3 r_\mathrm{w}^2 \dot{r}_\mathrm{w}$$

integrating in time:

$$\mathrm{RH} = \mathrm{RH}_0 - \alpha N r_\mathrm{w}^3$$

RH-coupled system & particle concentration as parameter

simple moisture budget (const T,p):

$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_\mathrm{v}}{\rho_\mathrm{vs}} = -N \underbrace{\frac{4\pi\rho_\mathrm{w}}{3\rho_\mathrm{vs}}}_{\alpha} 3 r_\mathrm{w}^2 \dot{r}_\mathrm{w}$$

integrating in time:

$$\mathrm{RH} = \mathrm{RH}_0 - \alpha N r_\mathrm{w}^3$$

new phase portrait:

$$\dot{\xi} \sim (\mathrm{RH}_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_\mathrm{d}^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_{f}$$

RH-coupled system & particle concentration as parameter

simple moisture budget (const T,p):

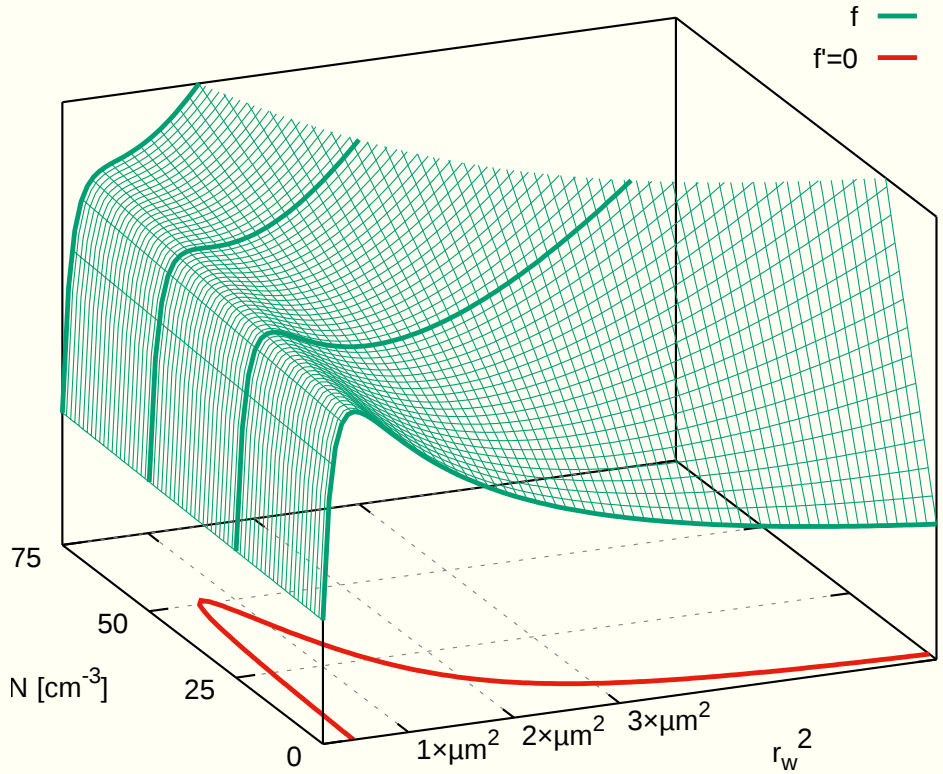
$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_\mathrm{v}}{\rho_\mathrm{vs}} = -N \underbrace{\frac{4\pi\rho_\mathrm{w}}{3\rho_\mathrm{vs}}}_{\alpha} 3 r_\mathrm{w}^2 \dot{r}_\mathrm{w}$$

integrating in time:

$$\mathrm{RH} = \mathrm{RH}_0 - \alpha N r_\mathrm{w}^3$$

new phase portrait:

$$\dot{\xi} \sim (\mathrm{RH}_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_\mathrm{d}^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_{f}$$



RH-coupled system & particle concentration as parameter

simple moisture budget (const T,p):

$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_\mathrm{v}}{\rho_\mathrm{vs}} = -N \underbrace{\frac{4\pi\rho_\mathrm{w}}{3\rho_\mathrm{vs}}}_{\alpha} 3 r_\mathrm{w}^2 \dot{r}_\mathrm{w}$$

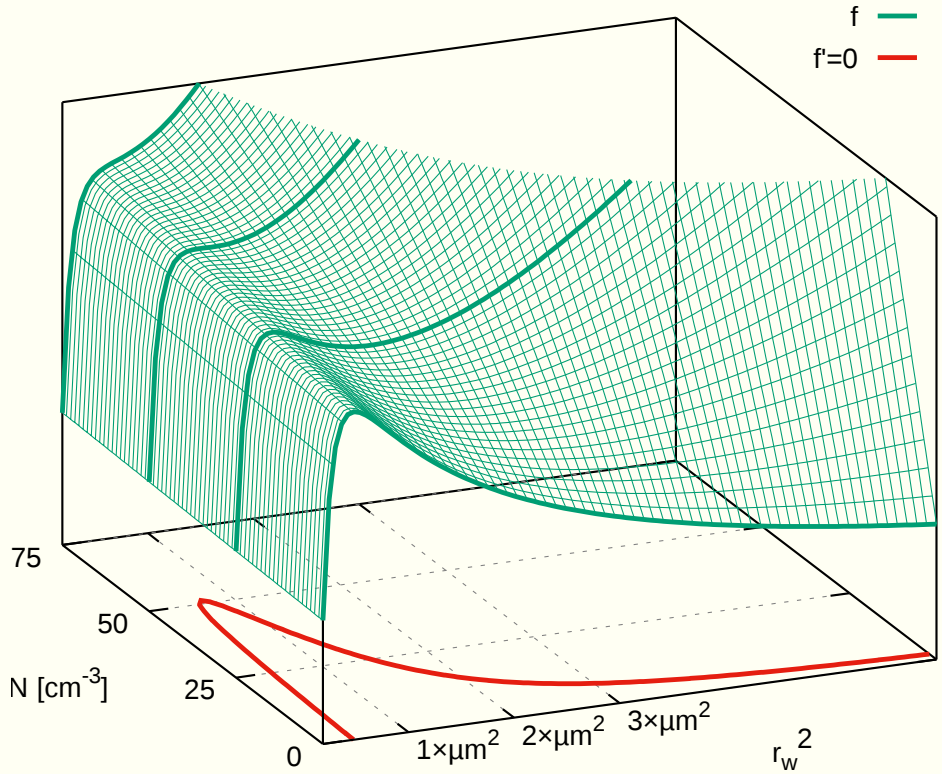
integrating in time:

$$\mathrm{RH} = \mathrm{RH}_0 - \alpha N r_\mathrm{w}^3$$

new phase portrait:

$$\dot{\xi} \sim (\mathrm{RH}_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_\mathrm{d}^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_{f}$$

regime-controlling params: RH, N



RH-coupled system & particle concentration as parameter

simple moisture budget (const T,p):

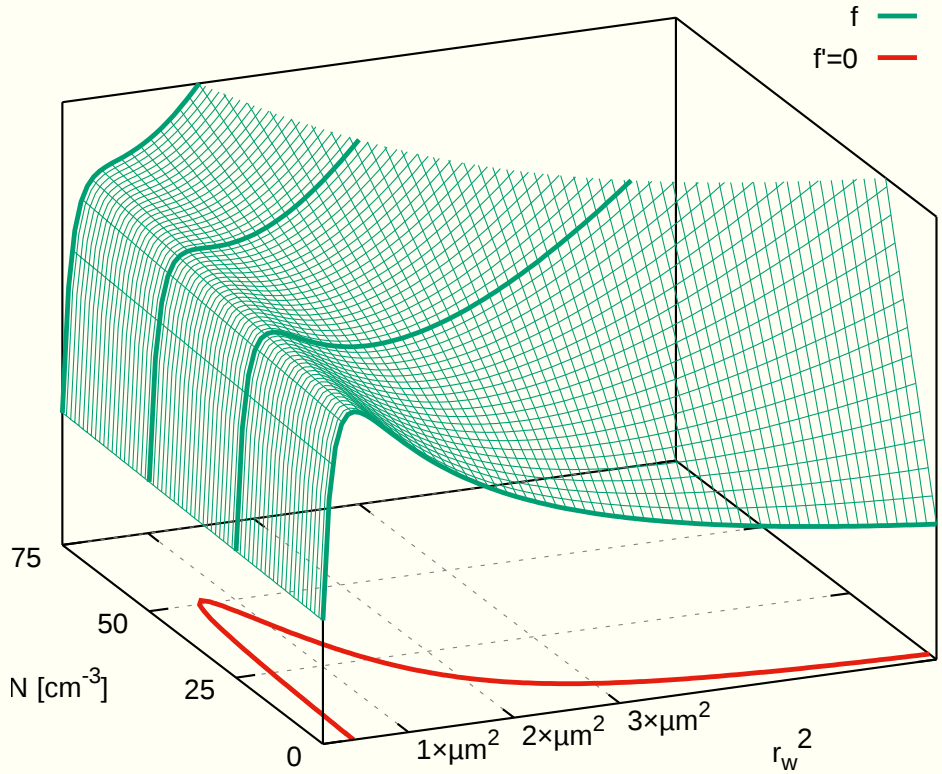
$$\dot{\mathrm{RH}} \approx \frac{\dot{\rho}_\mathrm{v}}{\rho_\mathrm{vs}} = -N \underbrace{\frac{4\pi\rho_\mathrm{w}}{3\rho_\mathrm{vs}}}_{\alpha} 3 r_\mathrm{w}^2 \dot{r}_\mathrm{w}$$

integrating in time:

$$\mathrm{RH} = \mathrm{RH}_0 - \alpha N r_\mathrm{w}^3$$

new phase portrait:

$$\dot{\xi} \sim (\mathrm{RH}_0 - 1) - \underbrace{\left(\frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_\mathrm{d}^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_{f}$$



$$\mathrm{sgn}(f') = \mathrm{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3 \right)$$

regime-controlling params: RH, N

bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

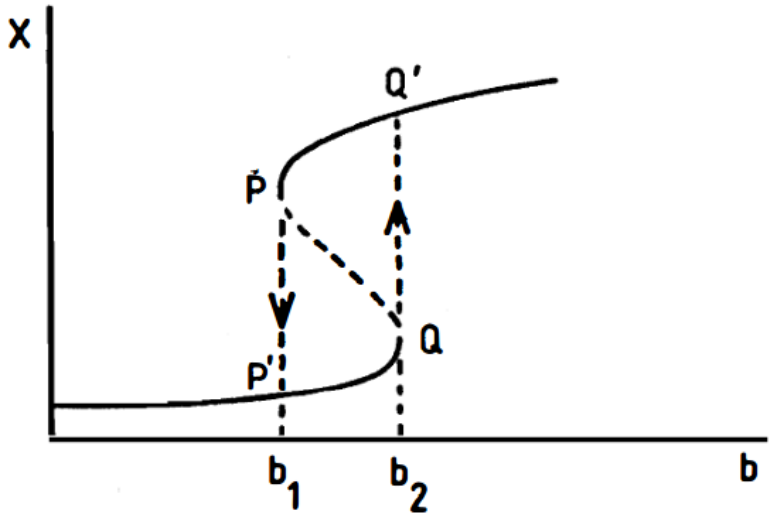


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter b first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while b grows. But at $b=b_2$, there will be a discontinuity: The system jumps from Q to Q', on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till $b=b_1$, when it will jump down to P'. Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

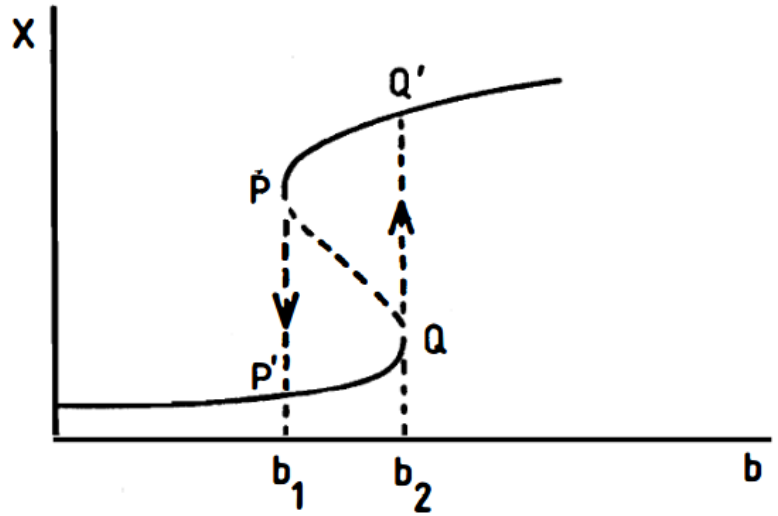
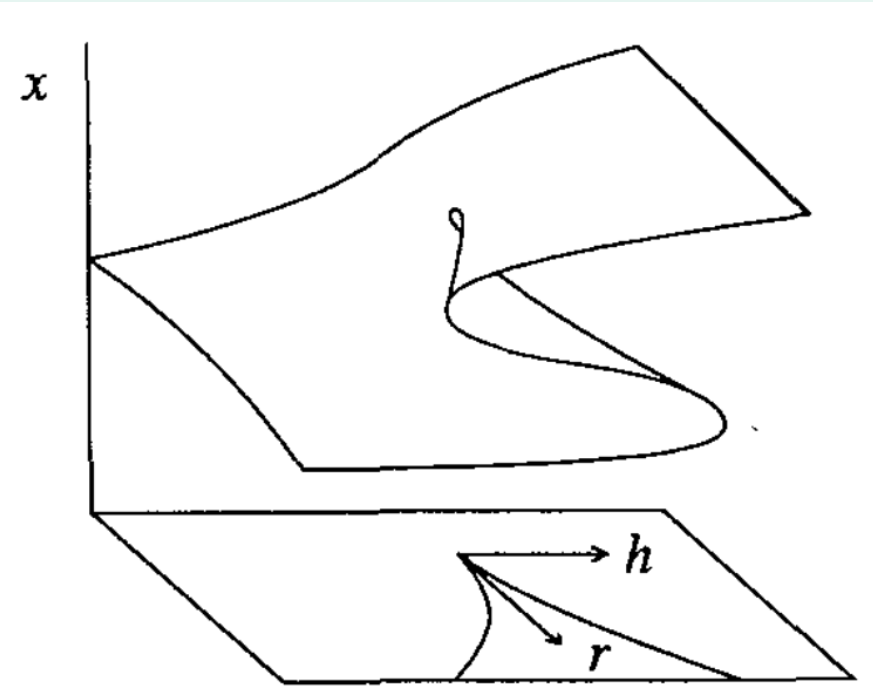


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter b first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while b grows. But at $b=b_2$, there will be a discontinuity: The system jumps from Q to Q', on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till $b=b_1$, when it will jump down to P'. Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

Strogatz 2014



"cusp catastrophe"

bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

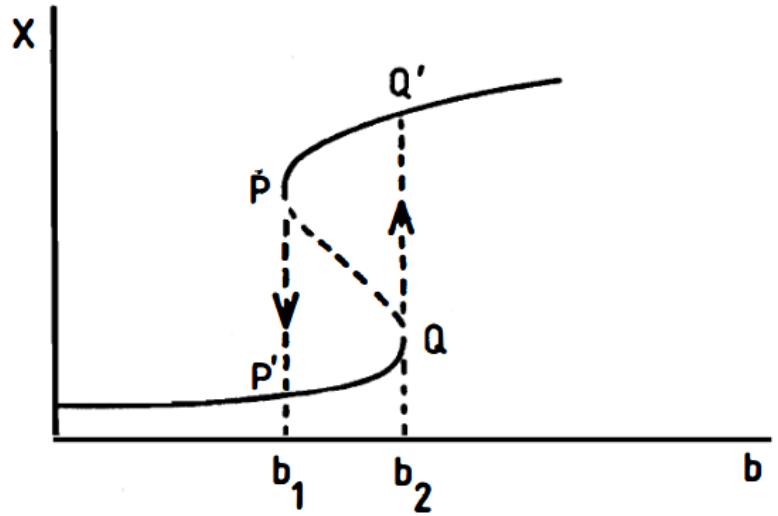
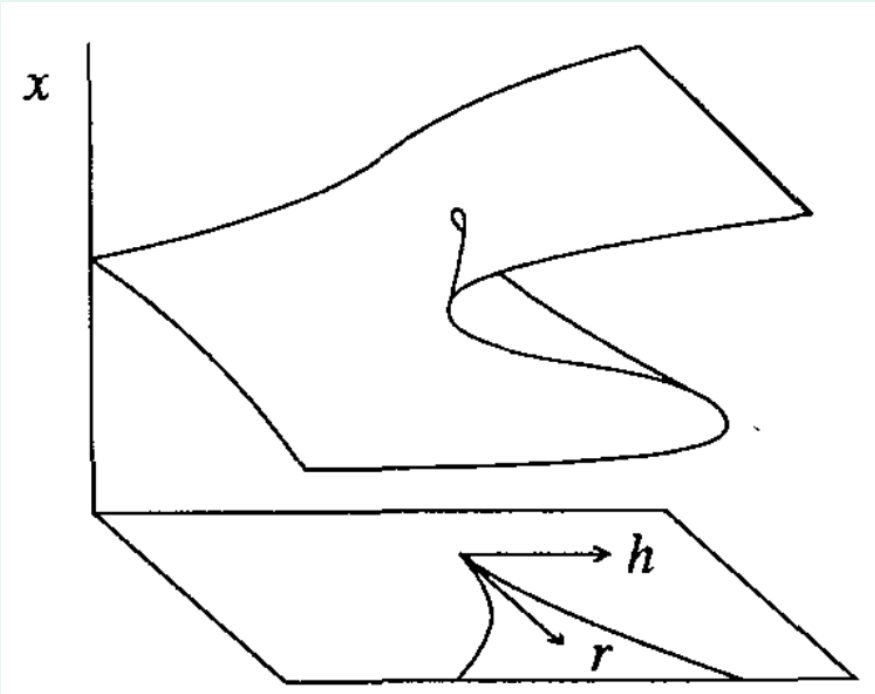


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter b first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while b grows. But at $b=b_2$, there will be a discontinuity: The system jumps from Q to Q', on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till $b=b_1$, when it will jump down to P'. Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

Strogatz 2014



"cusp catastrophe"

$\rightsquigarrow$ "jumps", hysteretic behaviour (r_w, RH) for small enough N, close to equilibrium (slow process)

more in the paper...

Arabas & Shima 2017

Nonlin. Processes Geophys., 24, 535–542, 2017
https://doi.org/10.5194/npg-24-535-2017

Nonlinear Processes in Geophysics
Open Access
EGU

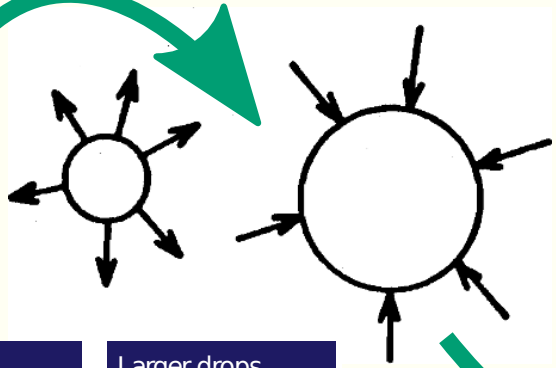
On the CCN (de)activation nonlinearities

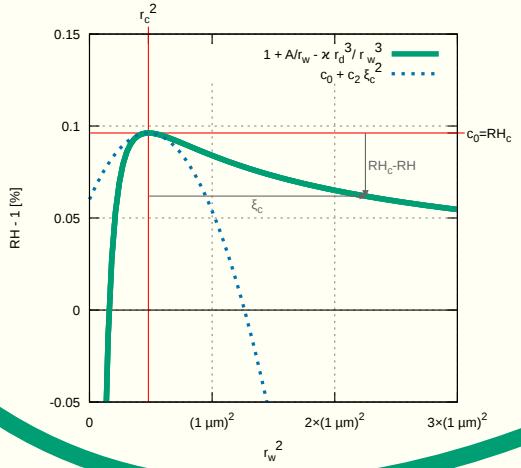
Sylwester Arabas[1,2] **and Shin-ichiro Shima**[3]

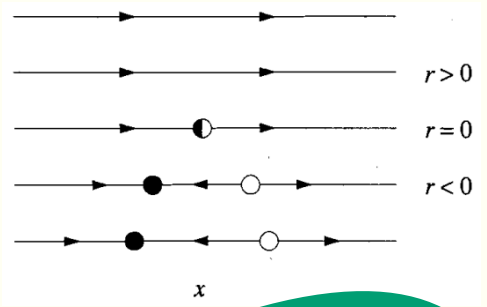
[1]Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland
[2]Chatham Financial Corporation Europe, Cracow, Poland
[3]Graduate School of Simulation Studies, University of Hyogo, Kobe, Japan

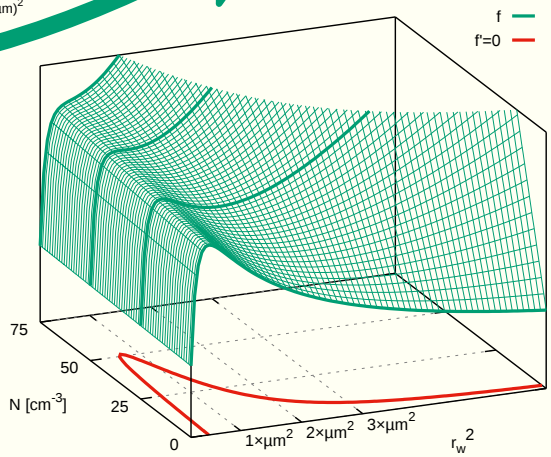
Correspondence to: Sylwester Arabas (sarabas@chathamfinancial.eu) and Shin-ichiro Shima (s_shima@sim.u-hyogo.ac.jp)

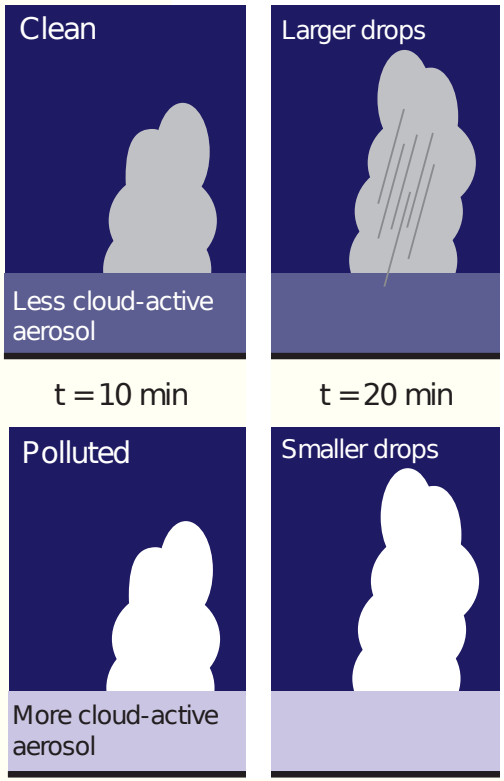
Received: 9 September 2016 – Discussion started: 4 October 2016
Revised: 23 May 2017 – Accepted: 24 July 2017 – Published: 5 September 2017

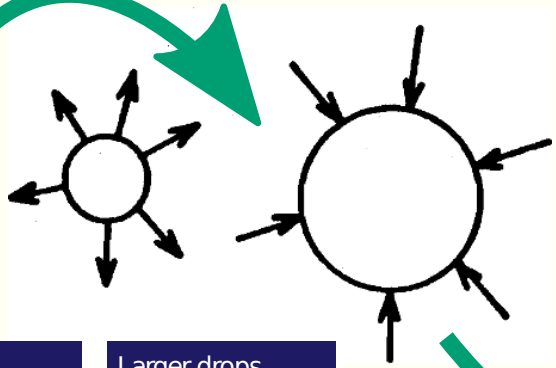


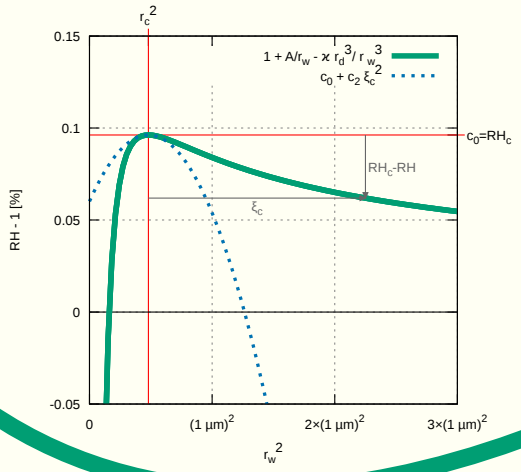

r_c^2
0.15
0.1
0.05
0
-0.05
RH - 1 [%]
$1 + A/r_w - \varkappa\, r_d^3 / r_w^3$
$c_0 + c_2\, \xi_c^2$
c_0=RH$_c$
RH$_c$-RH
ξ_c
0
(1 µm)^2
2×(1 µm)^2
3×(1 µm)^2
r_w^2

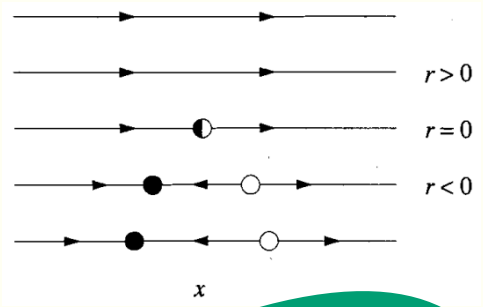

$r > 0$
$r = 0$
$r < 0$
x

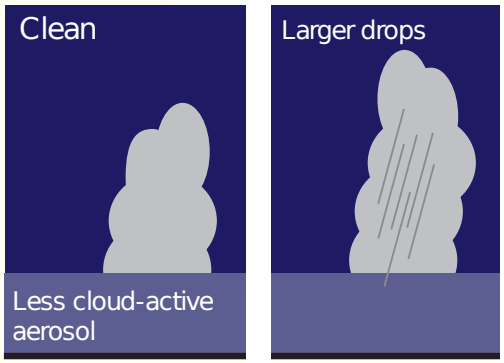

f
f'=0
75
50
25
0
N [cm^{-3}]
1×µm^2
2×µm^2
3×µm^2
r_w^2


Clean
Larger drops
Less cloud-active aerosol
t = 10 min
t = 20 min
Polluted
Smaller drops
More cloud-active aerosol



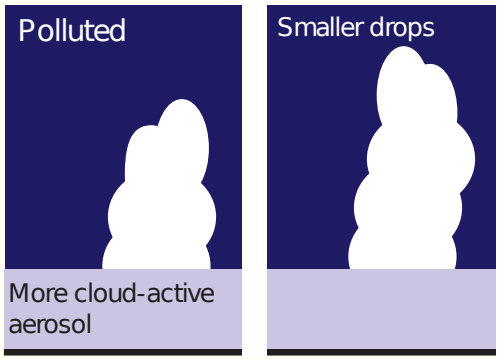




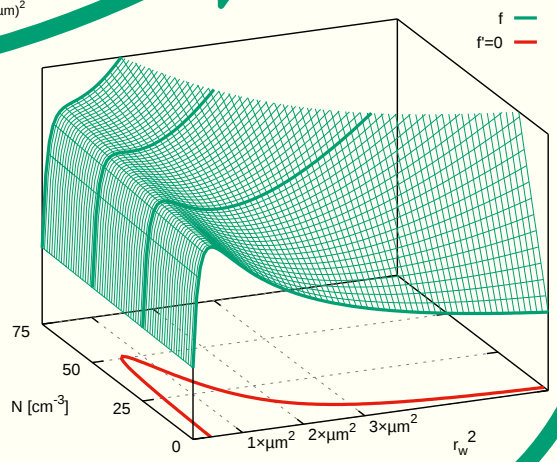


t = 10 min

t = 20 min



?



limitations:

limitations:

- no spectral width representation
  (key for modelling precipitation onset)

limitations:

- no spectral width representation
  (key for modelling precipitation onset)
- no activated/unactivated partitioning
  (documented excitable behaviour)

limitations:

- no spectral width representation
  (key for modelling precipitation onset)
- no activated/unactivated partitioning
  (documented excitable behaviour)
- no droplet-to-droplet vapour transfer
  (Ostwald ripening = Ouzo effect)

limitations:

- no spectral width representation
  (key for modelling precipitation onset)
- no activated/unactivated partitioning
  (documented excitable behaviour)
- no droplet-to-droplet vapour transfer
  (Ostwald ripening = Ouzo effect)

... applicability?

limitations:

- no spectral width representation
  (key for modelling precipitation onset)
- no activated/unactivated partitioning
  (documented excitable behaviour)
- no droplet-to-droplet vapour transfer
  (Ostwald ripening = Ouzo effect)

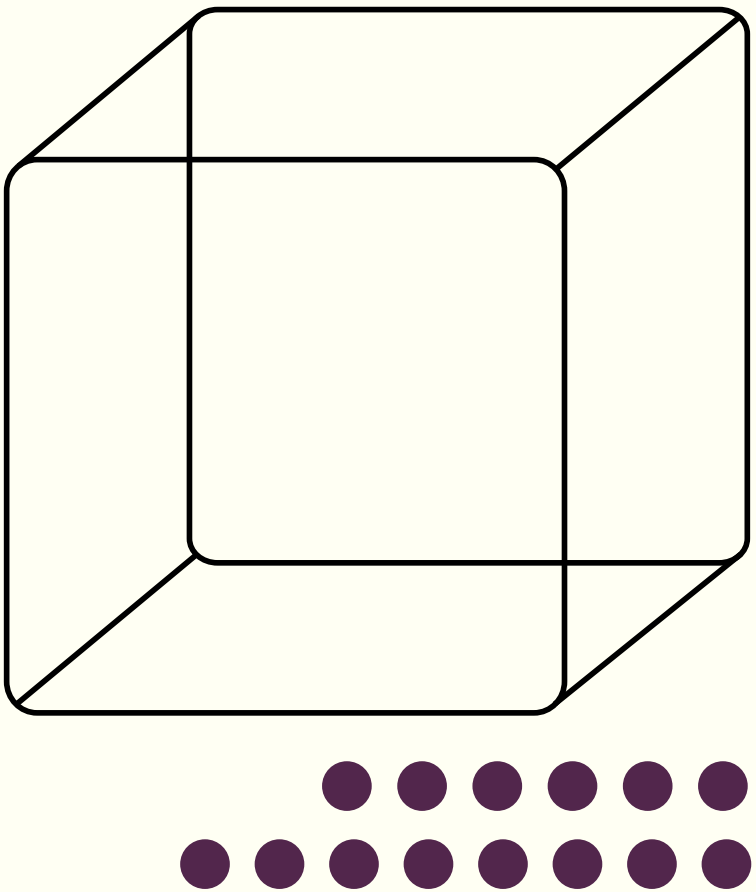
... applicability?

particle-based μ-physics schemes for LES!
(Lagrangian Cloud Models / Super-Droplet Models)

particle-based μ-physics for LES

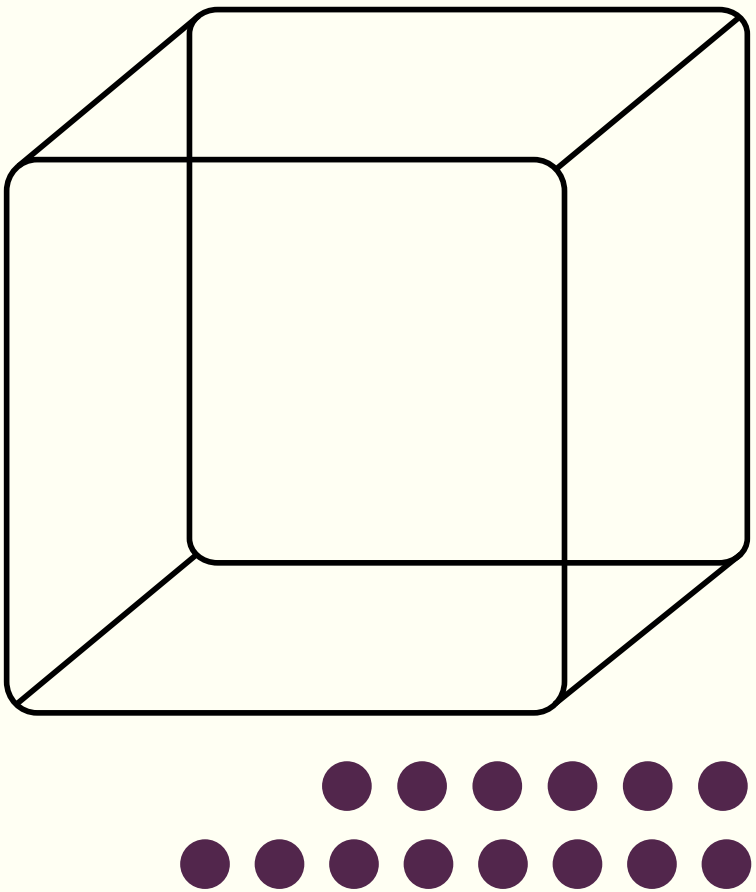
- "information carriers" in LES domain

particle-based μ-physics for LES



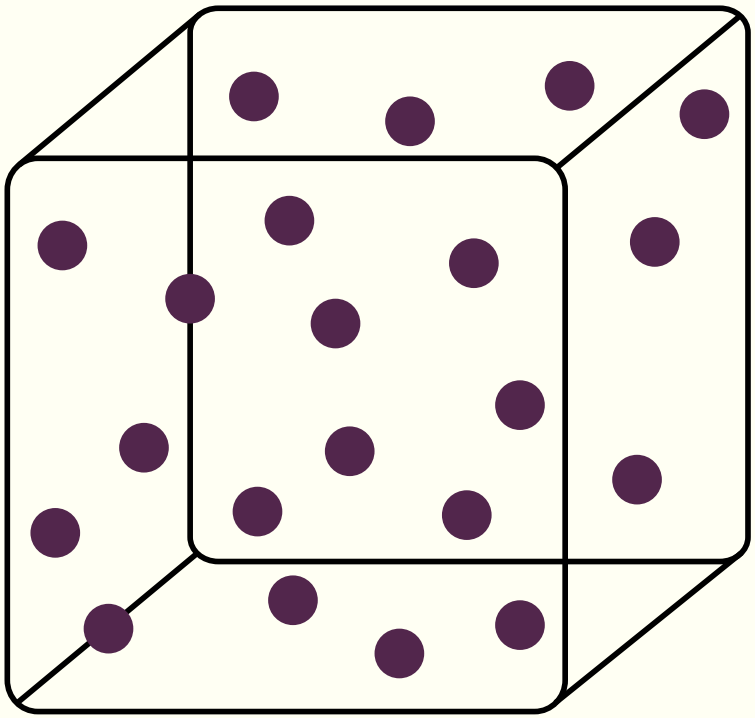
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain

particle-based μ-physics for LES



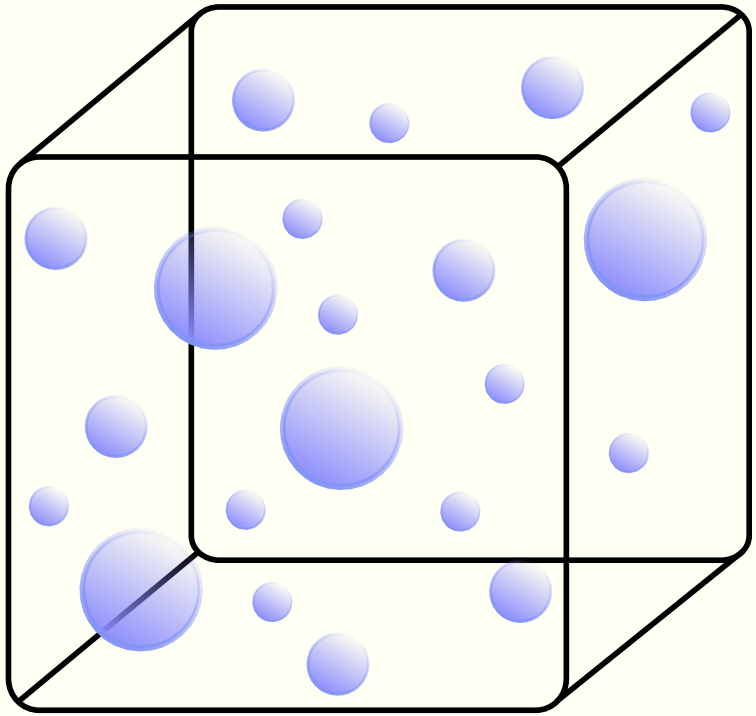
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:

particle-based μ-physics for LES



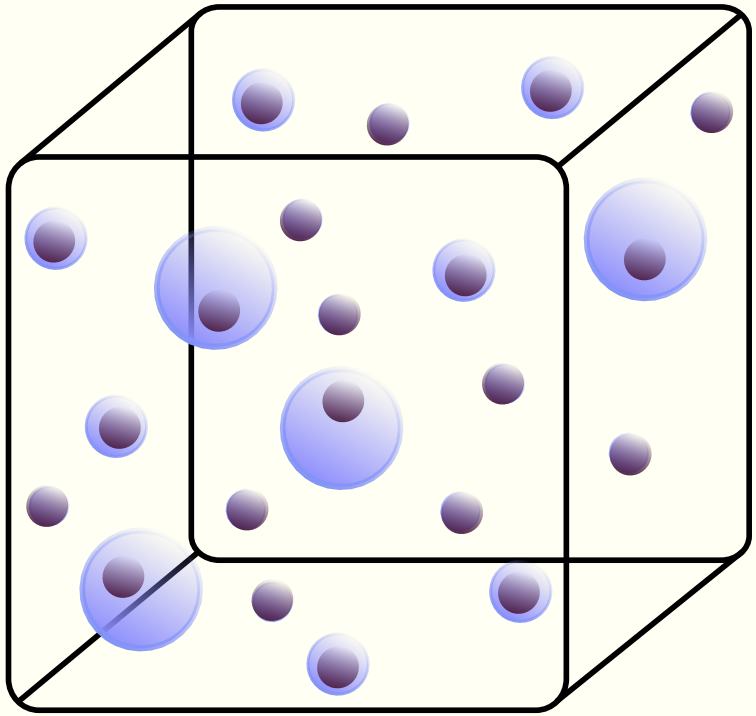
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates

particle-based μ-physics for LES



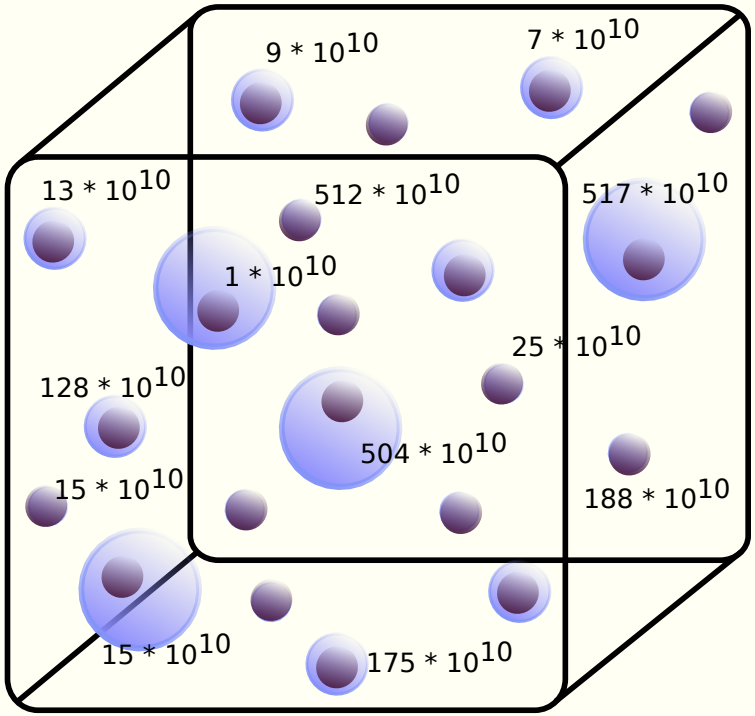
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius

particle-based μ-physics for LES



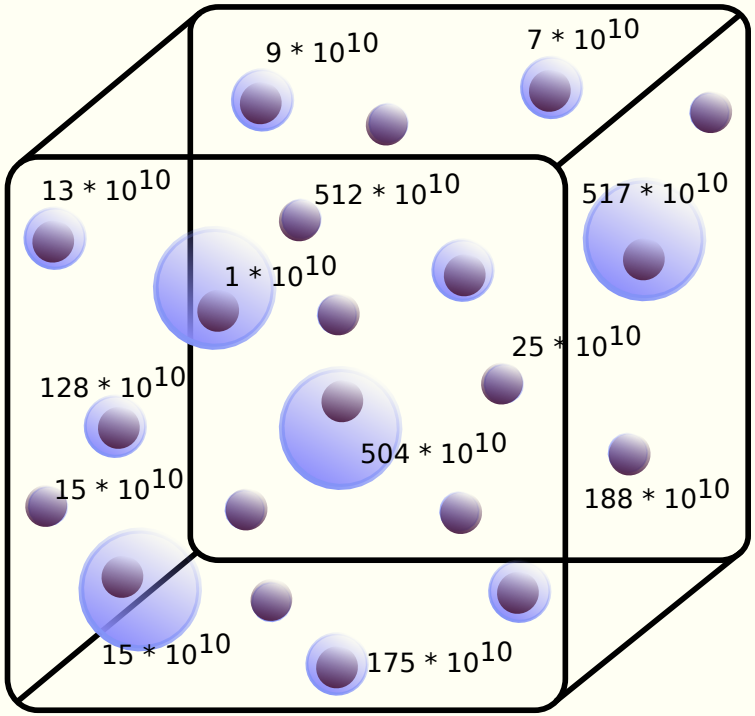
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius
  - dry radius

particle-based μ-physics for LES



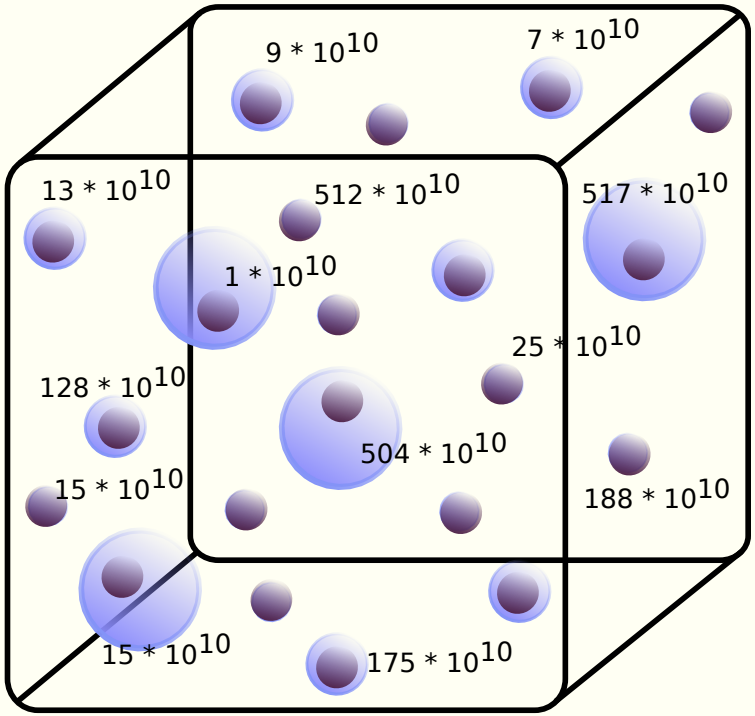
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius
  - dry radius
  - multiplicity

particle-based μ-physics for LES



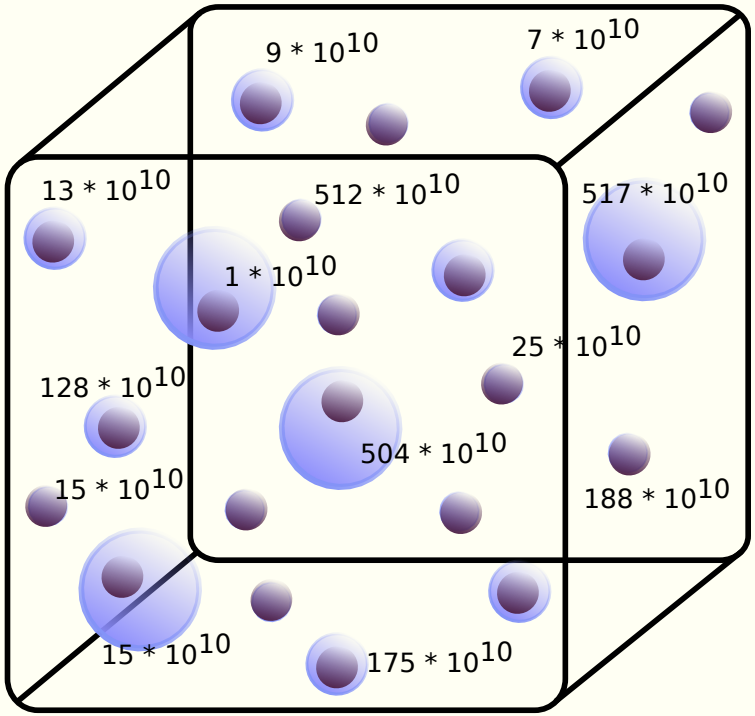
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius
  - dry radius
  - multiplicity
  - ...

particle-based μ-physics for LES



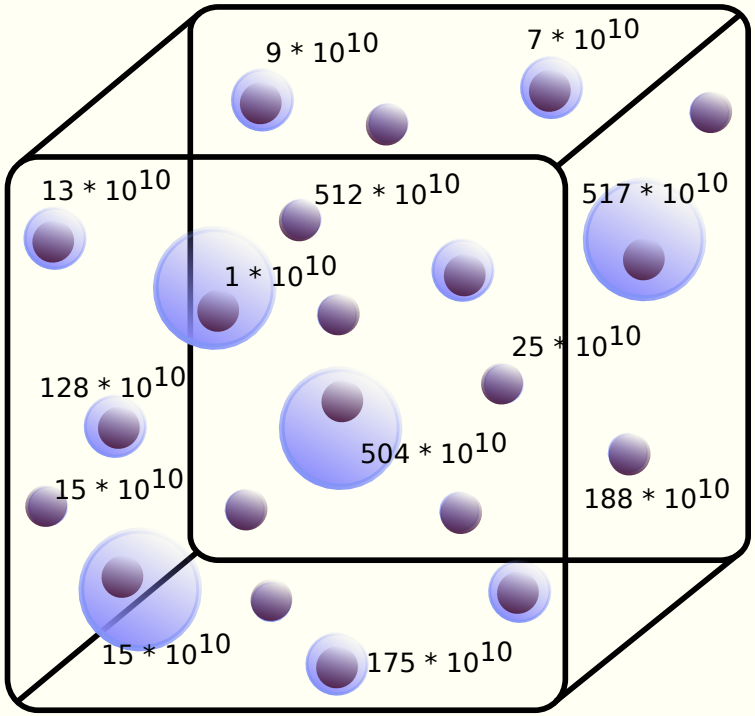
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius
  - dry radius
  - multiplicity
  - ...
  - chemistry, charge, isotopic composition, ...

particle-based μ-physics for LES

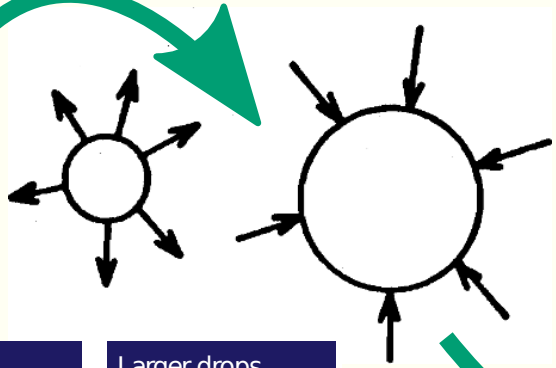


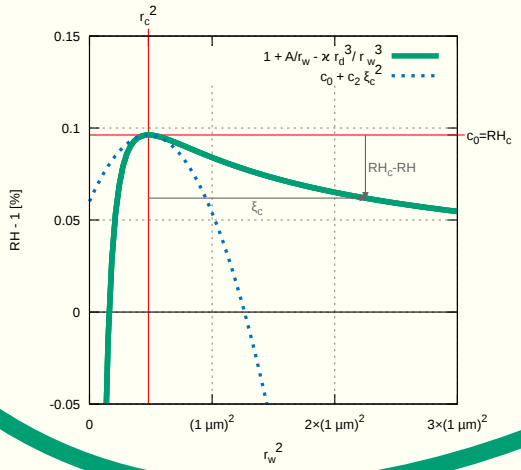
- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius
  - dry radius
  - multiplicity
  - ...
  - chemistry, charge, isotopic composition, ...
- each particle: **monodisperse**!

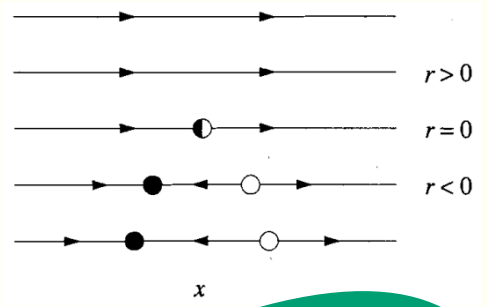
particle-based μ-physics for LES

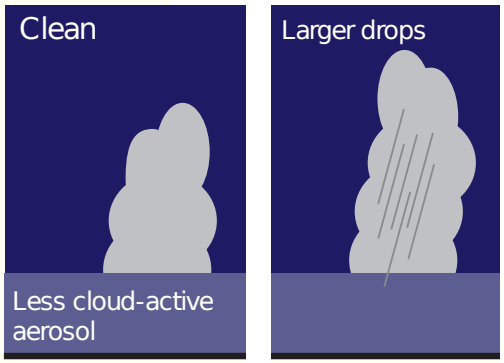


- "information carriers" in LES domain
- ab-initio approach:
  particle=aerosol/cloud/rain
- attributes:
  - spatial coordinates
  - wet radius
  - dry radius
  - multiplicity
  - ...
  - chemistry, charge, isotopic composition, ...
- each particle: **monodisperse**!
- each timestep: **constant RH**!



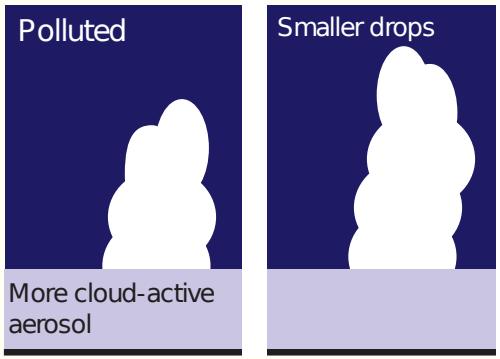


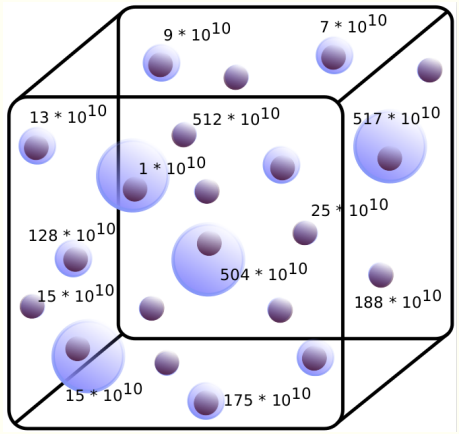


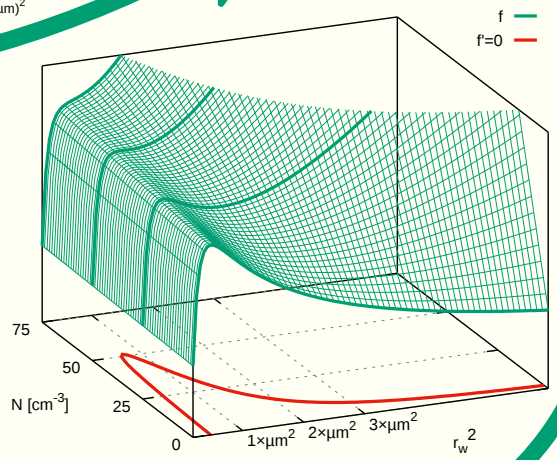


t = 10 min

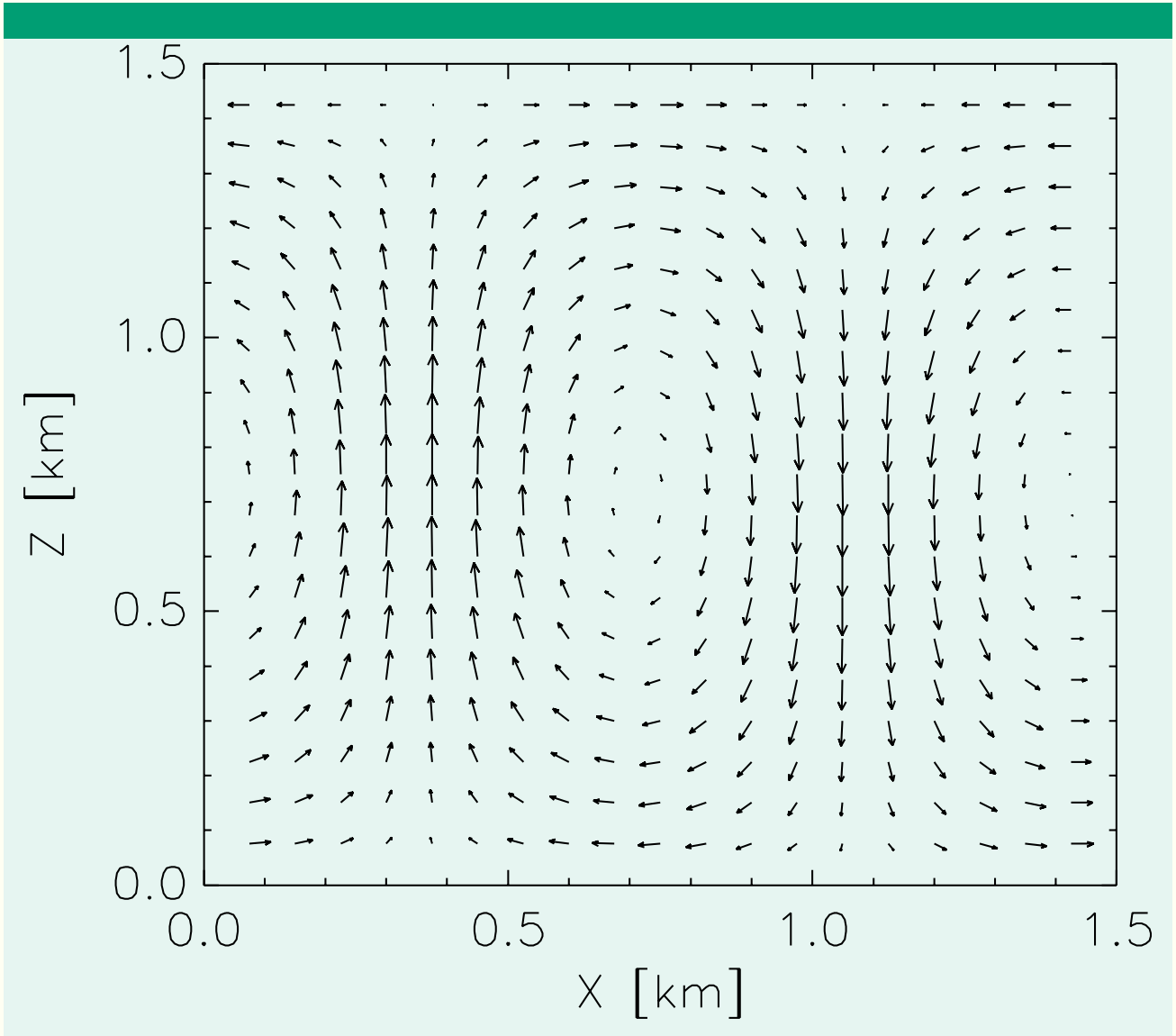
t = 20 min



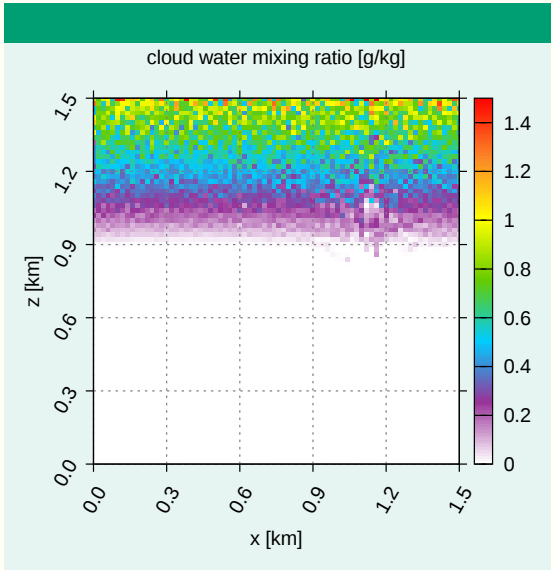


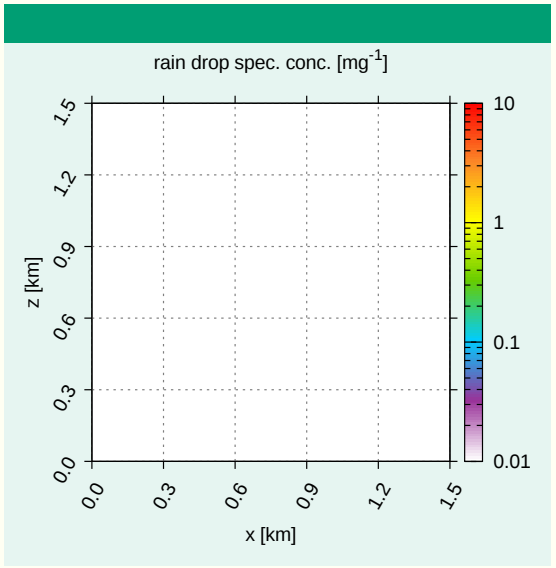


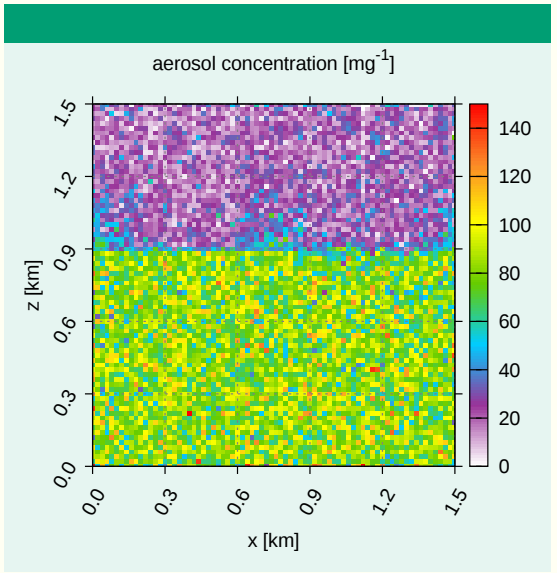
example simulation (2D, prescribed flow)

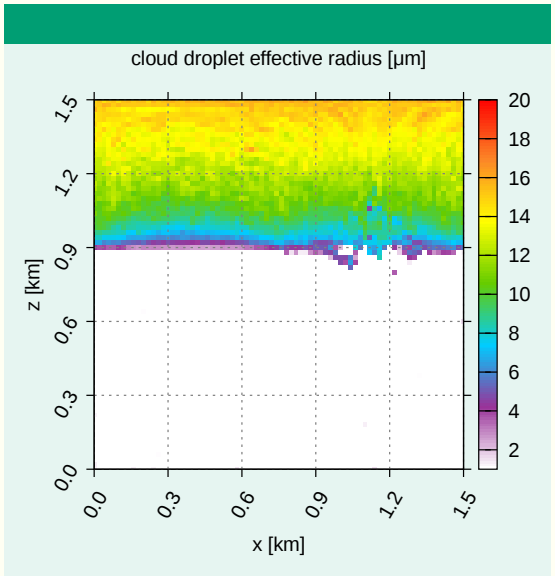


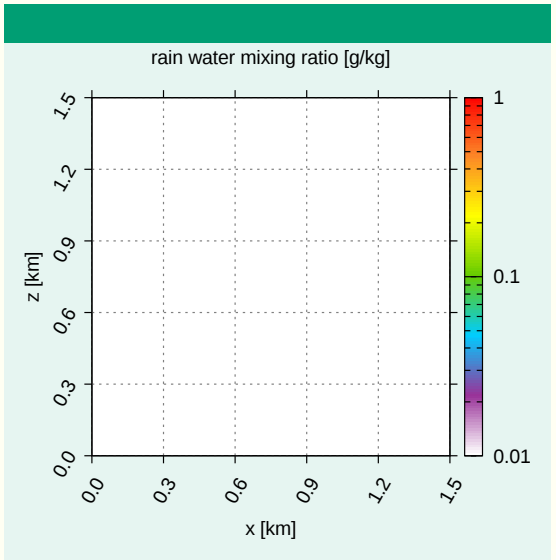
example simulation (2D, prescribed flow)



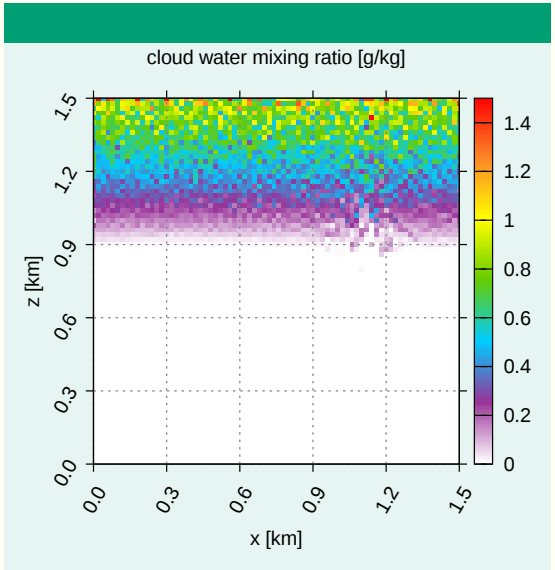


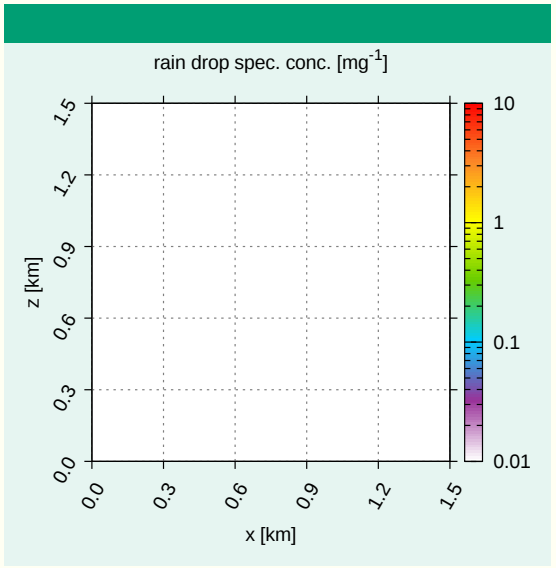


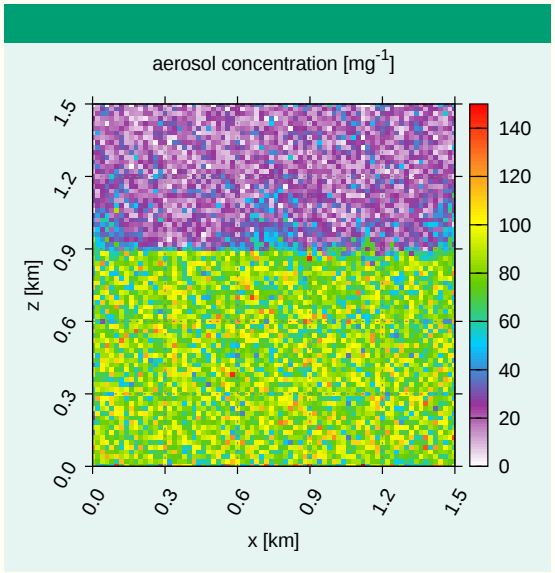


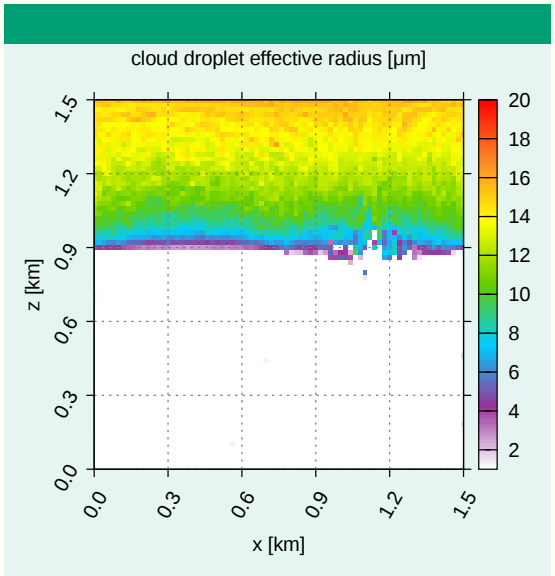


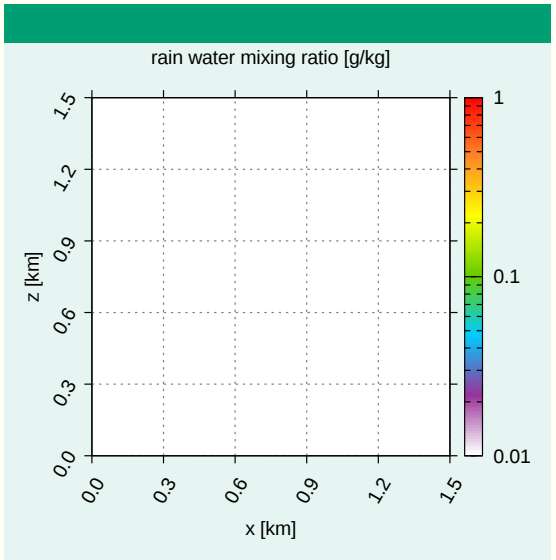
example simulation (2D, prescribed flow)



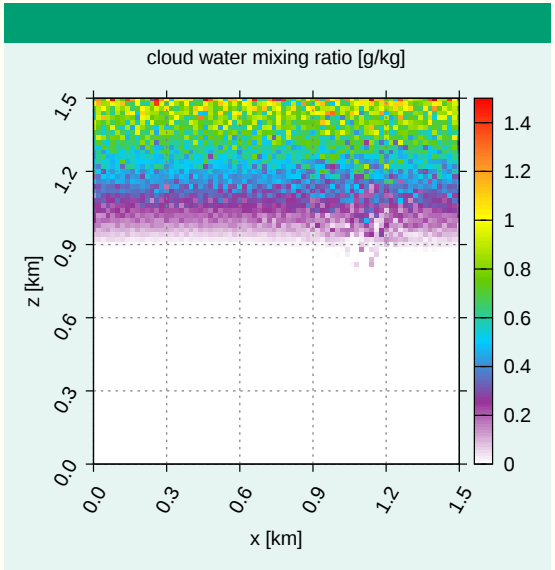


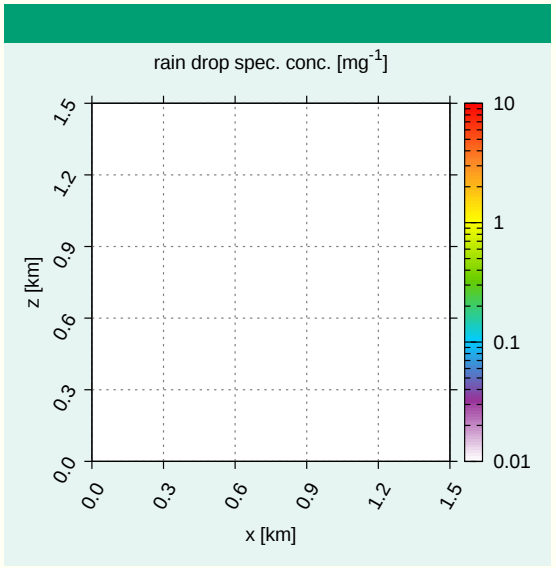


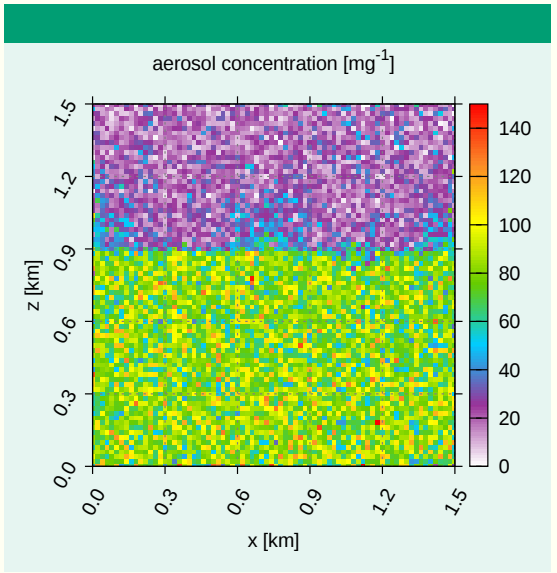


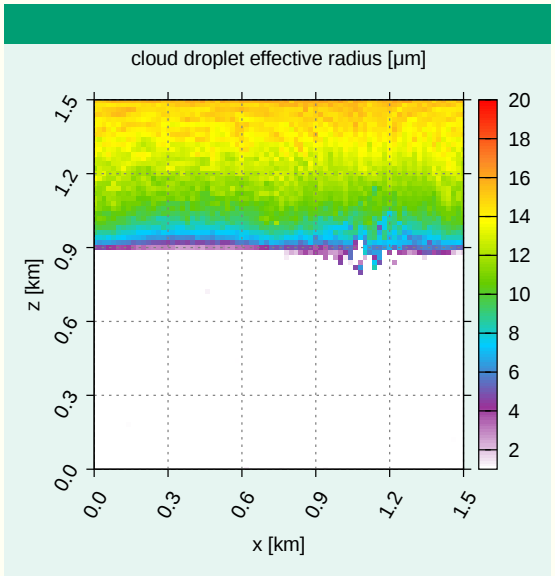


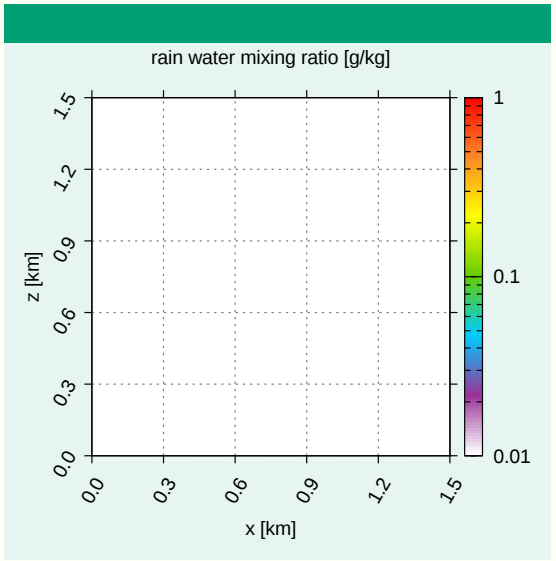
example simulation (2D, prescribed flow)



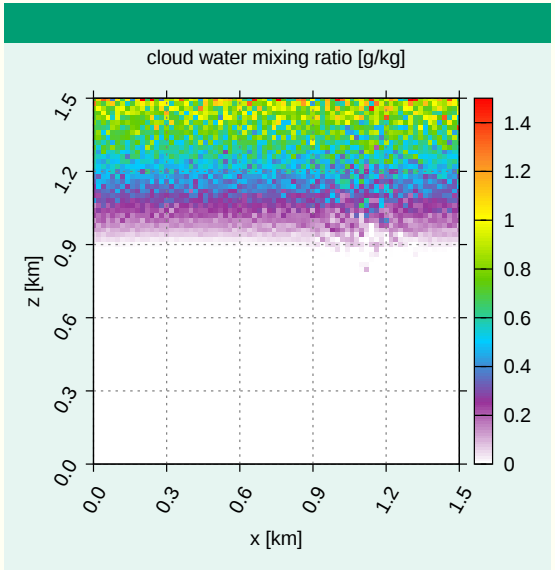


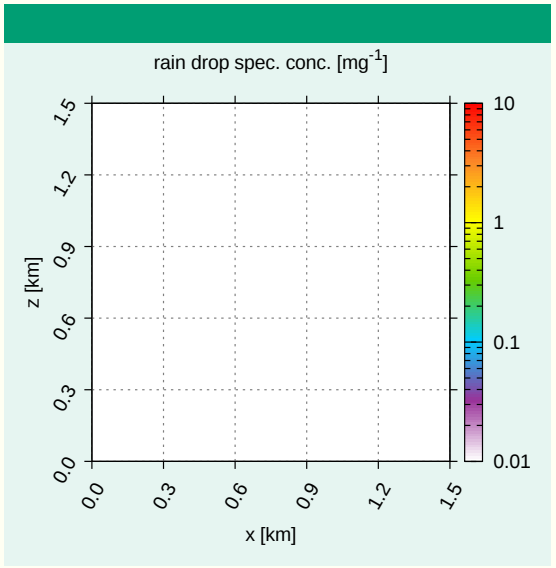


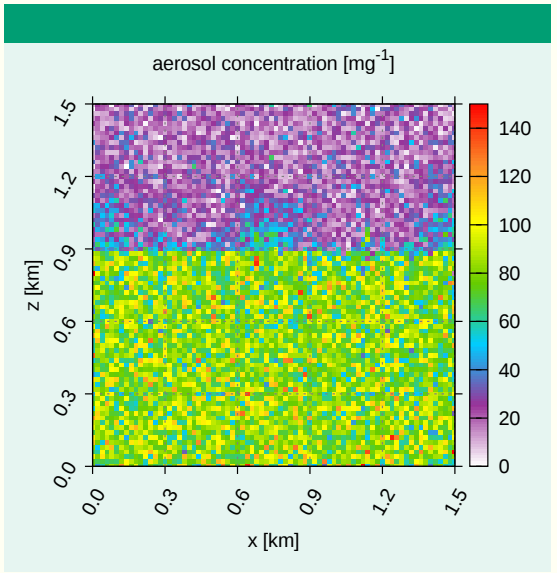


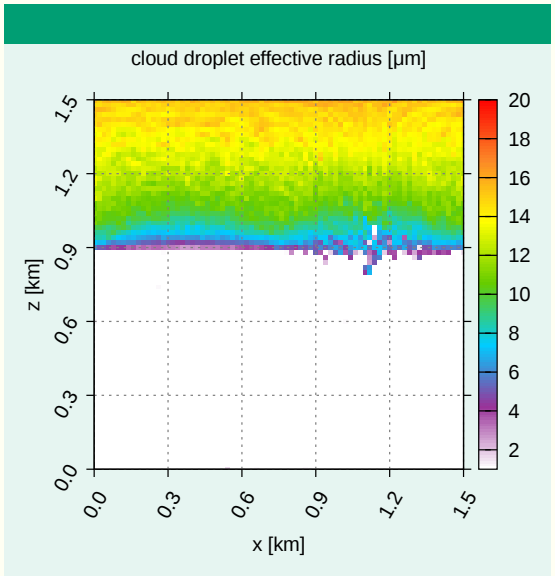


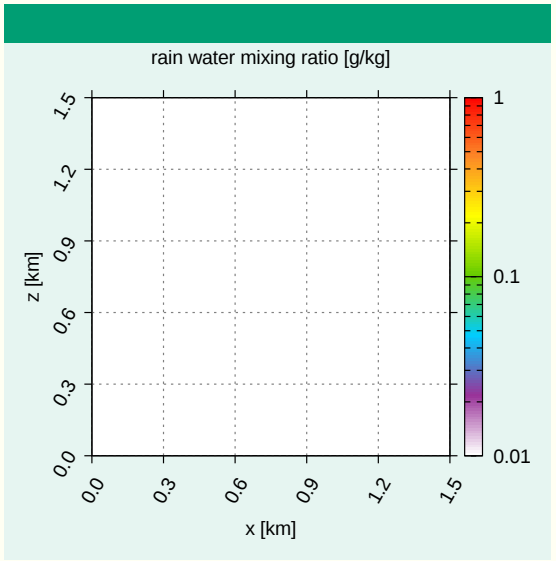
example simulation (2D, prescribed flow)



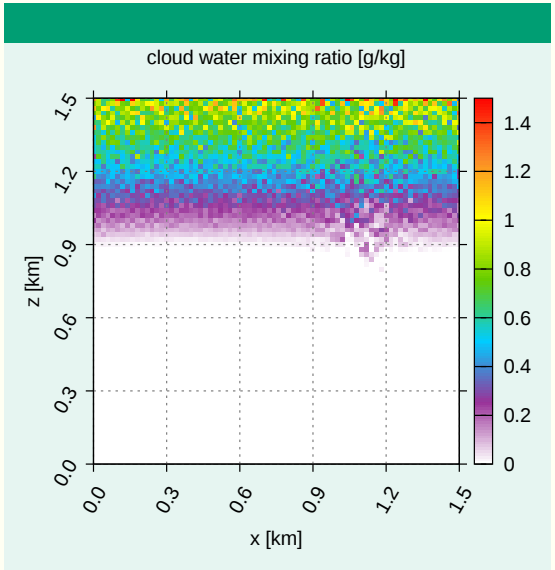


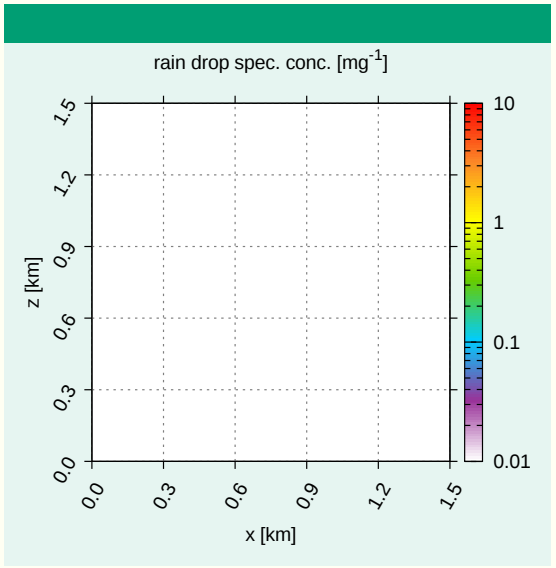


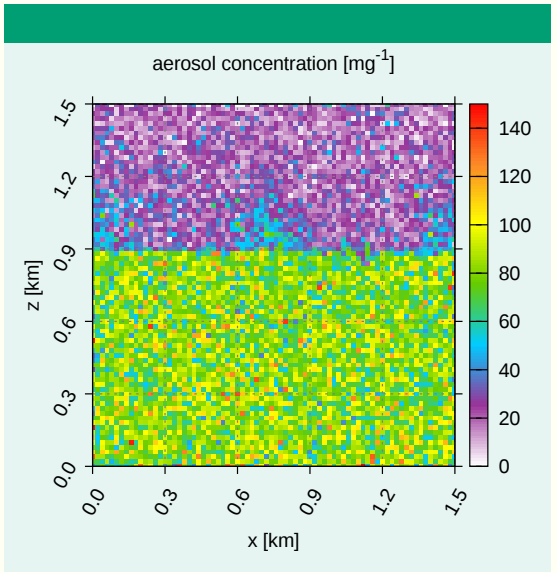


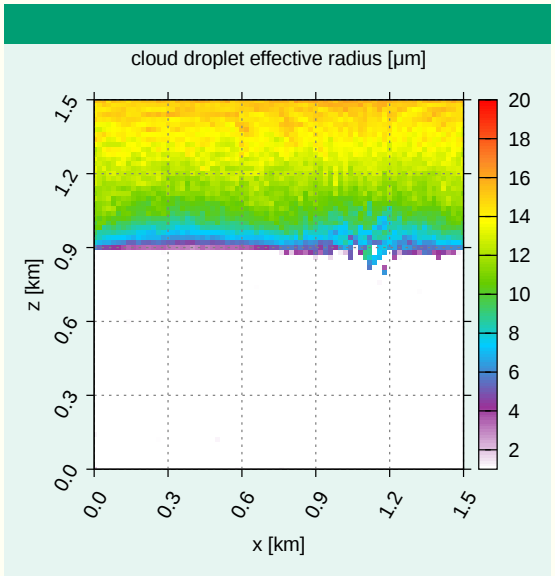


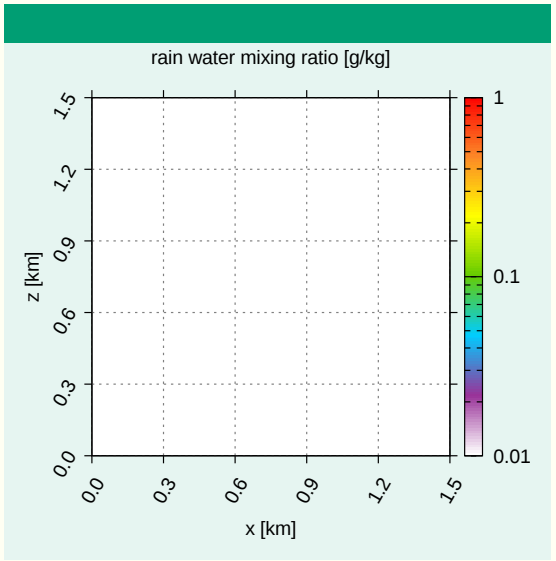
example simulation (2D, prescribed flow)



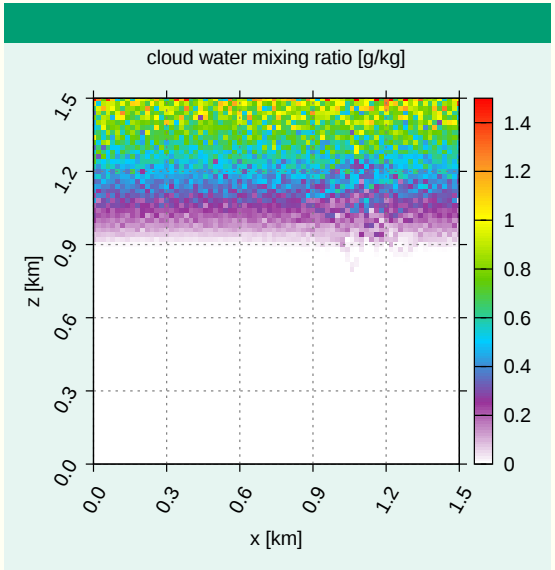


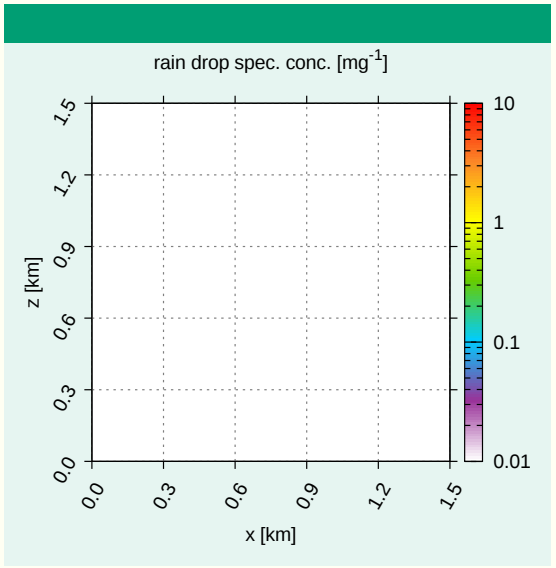


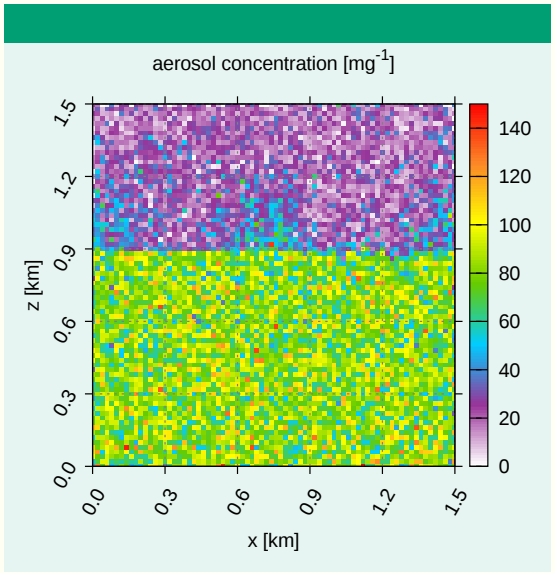


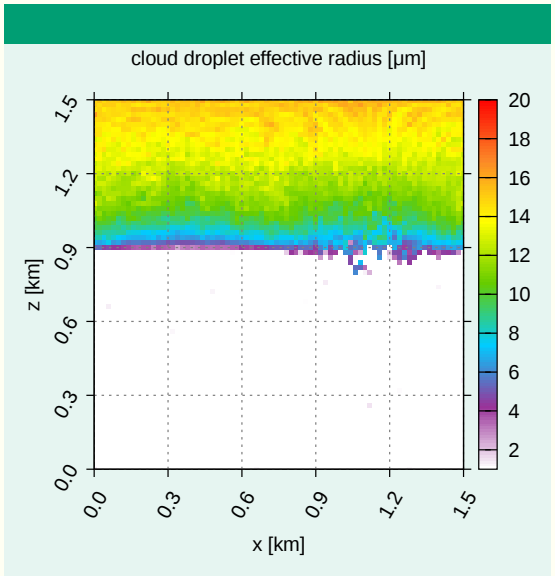


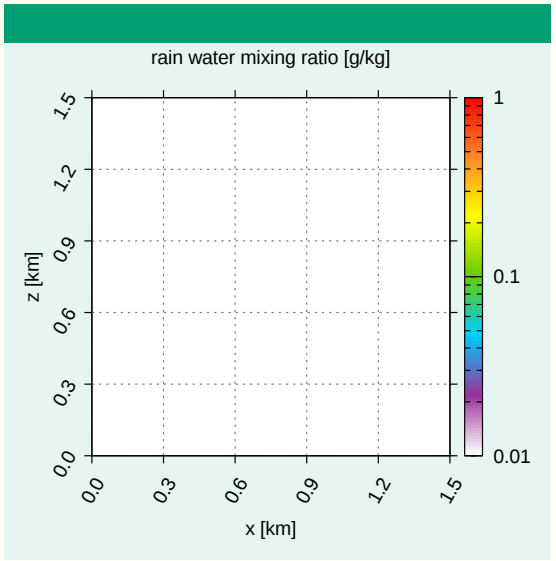
example simulation (2D, prescribed flow)



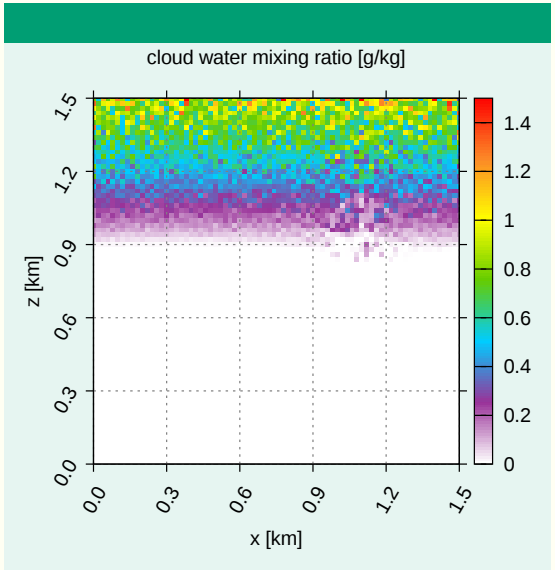


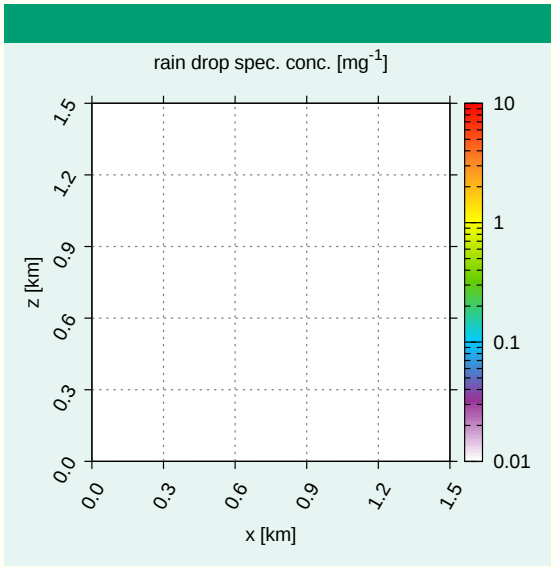


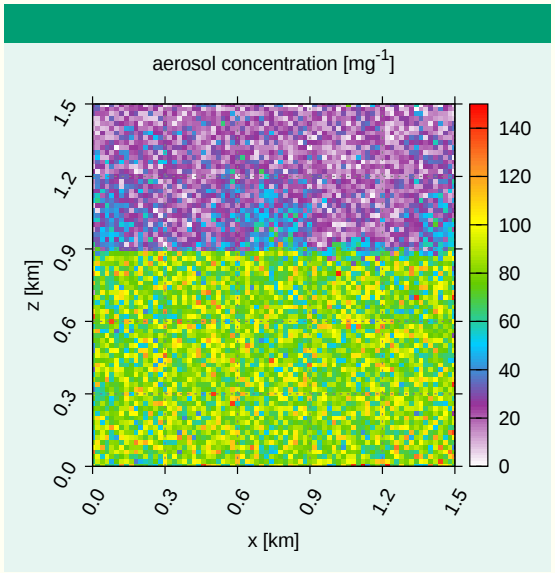


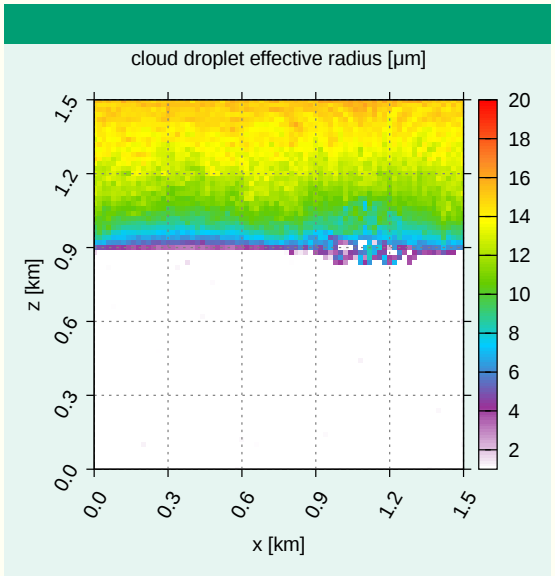


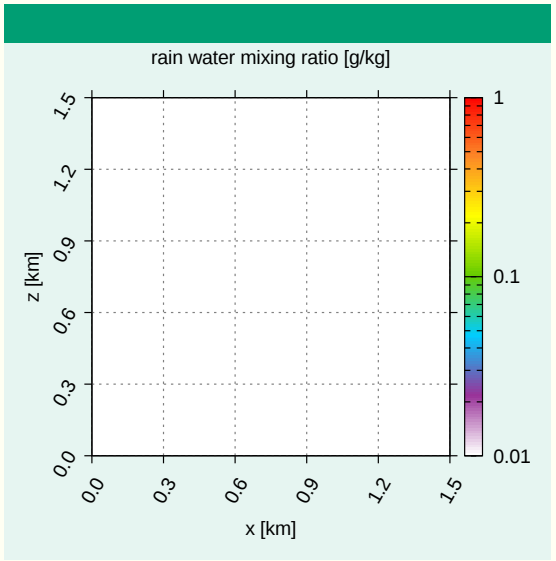
example simulation (2D, prescribed flow)



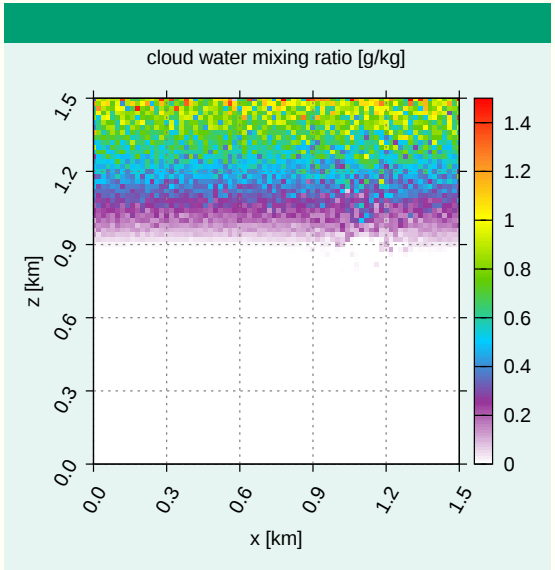


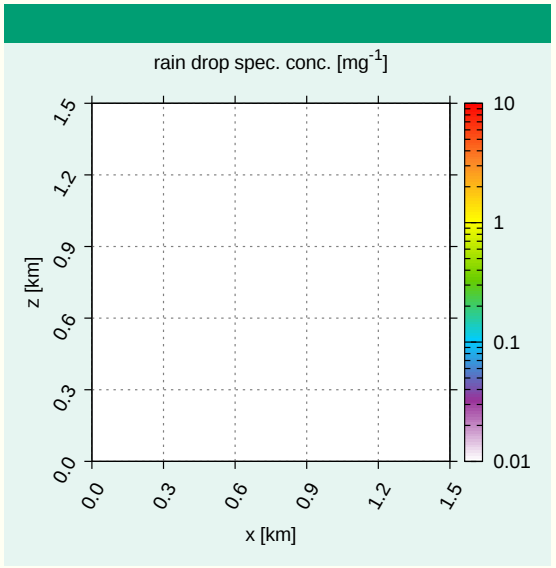


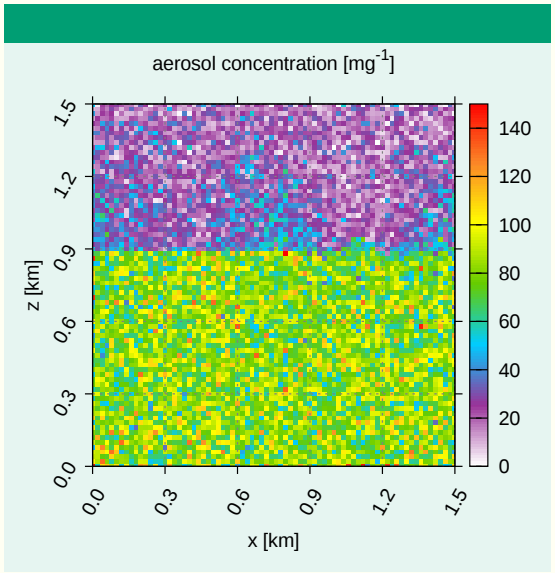


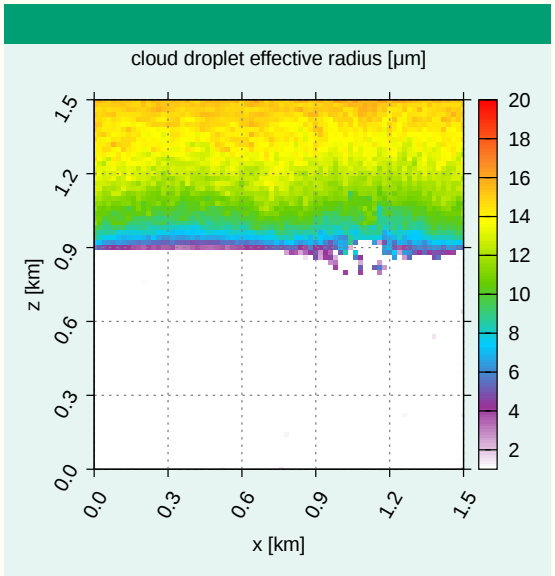


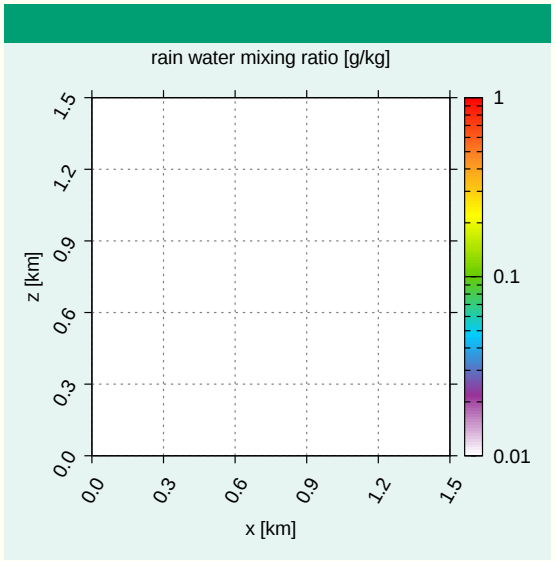
example simulation (2D, prescribed flow)



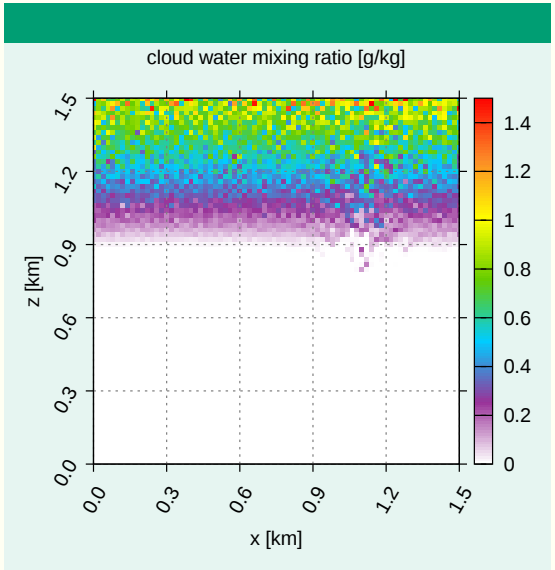


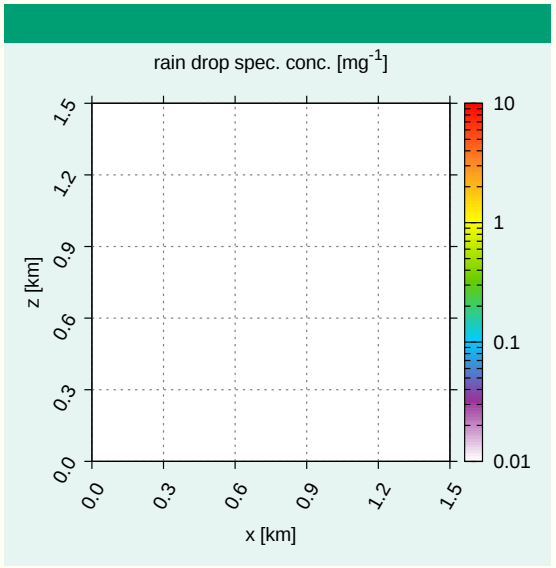


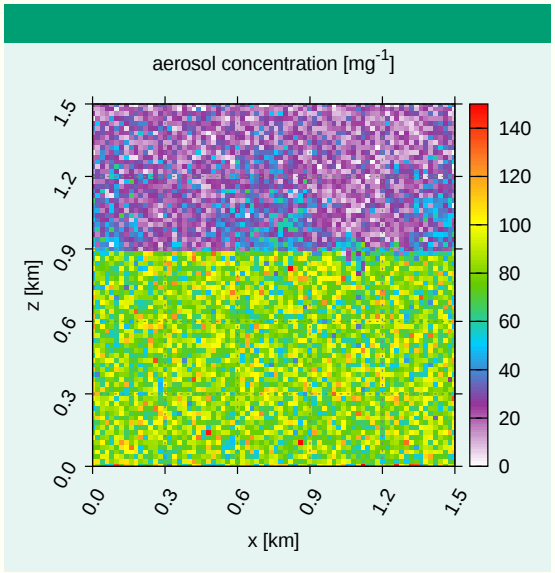


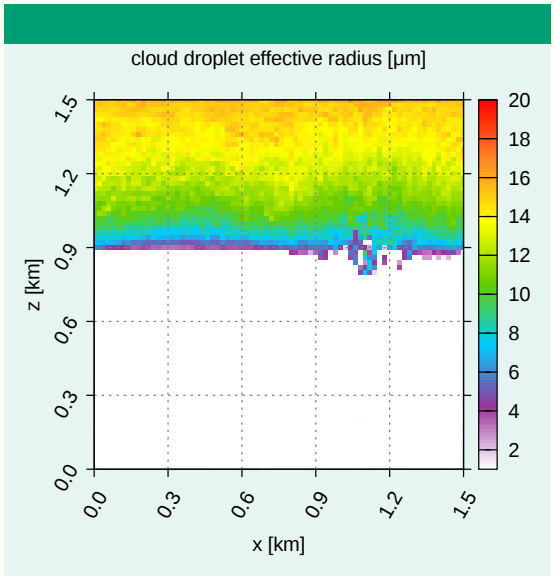


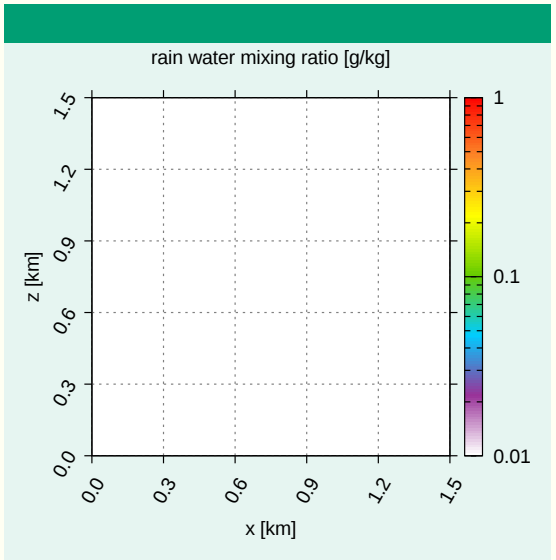
example simulation (2D, prescribed flow)



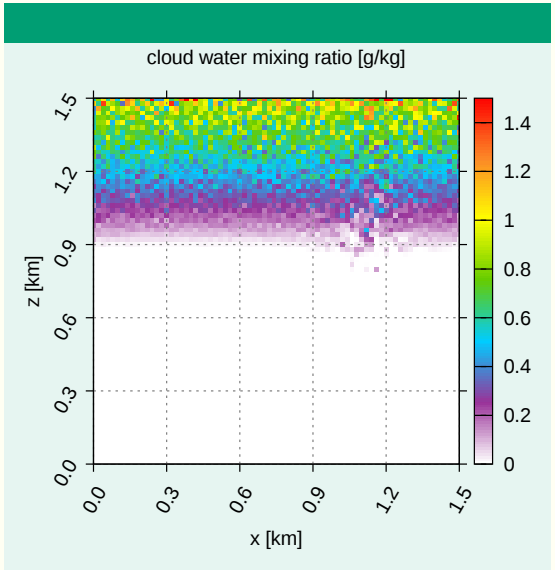


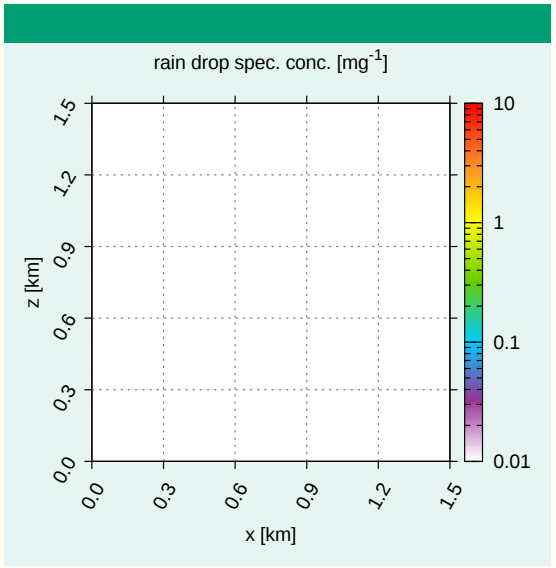


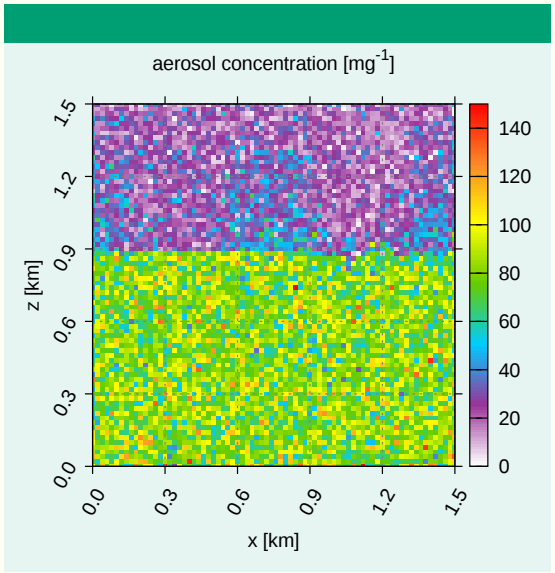


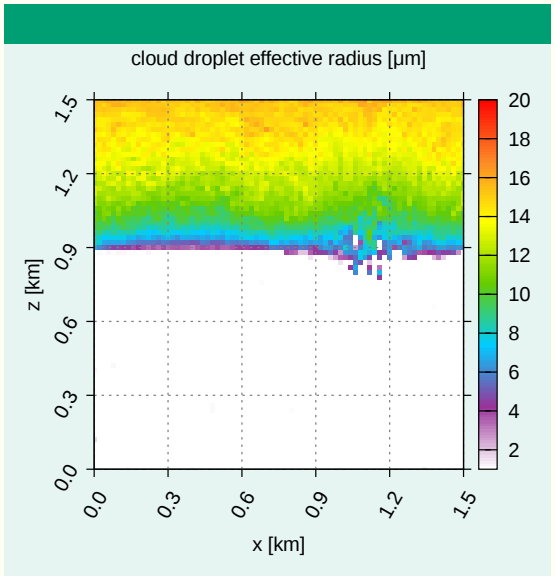


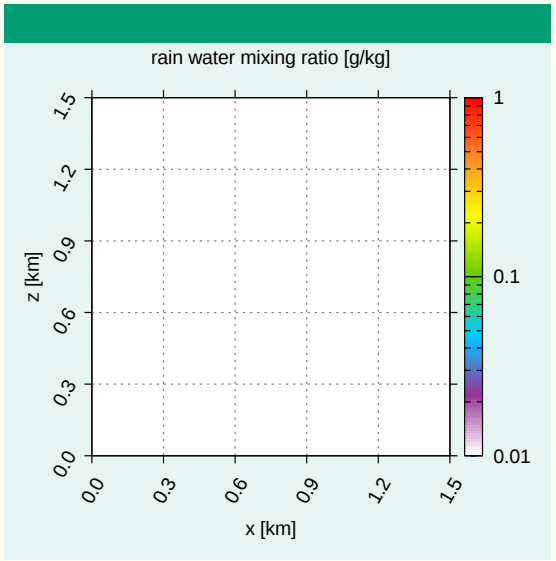
example simulation (2D, prescribed flow)



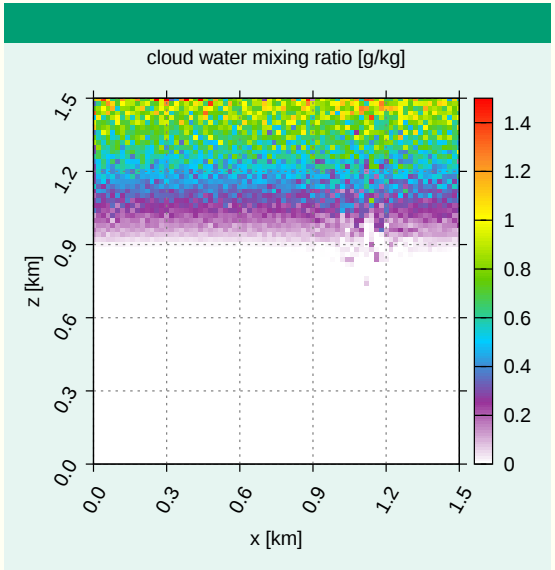


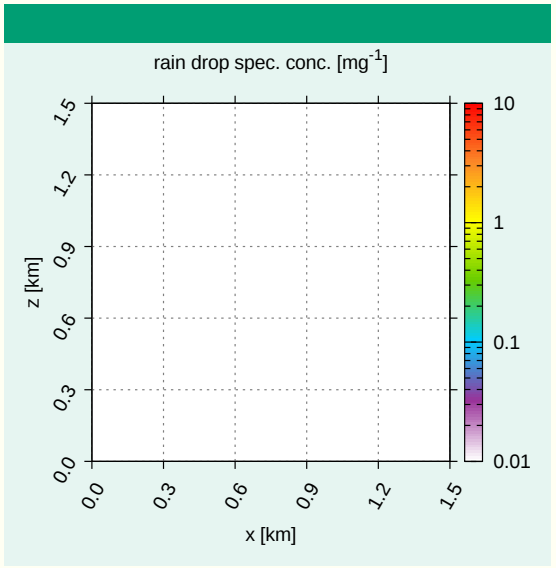


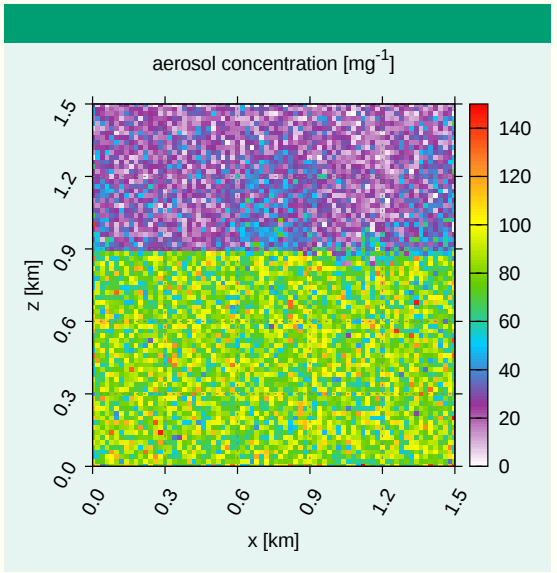


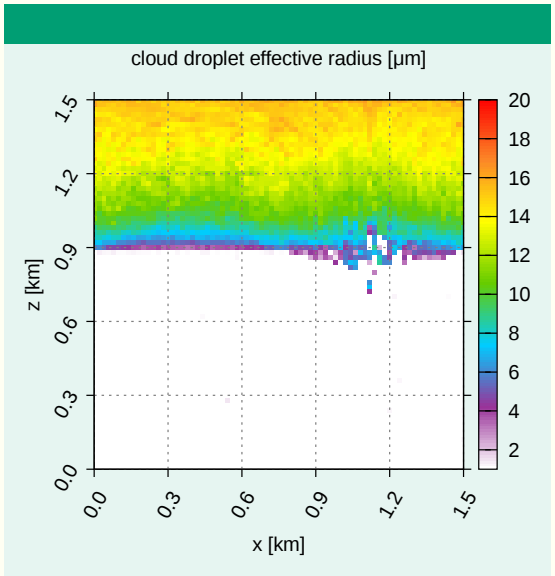


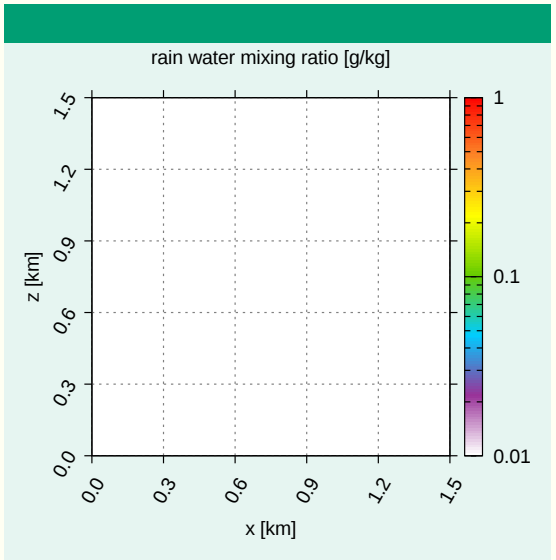
example simulation (2D, prescribed flow)



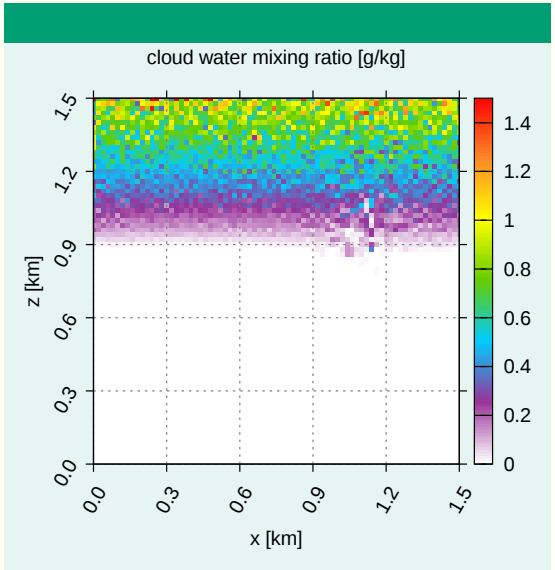


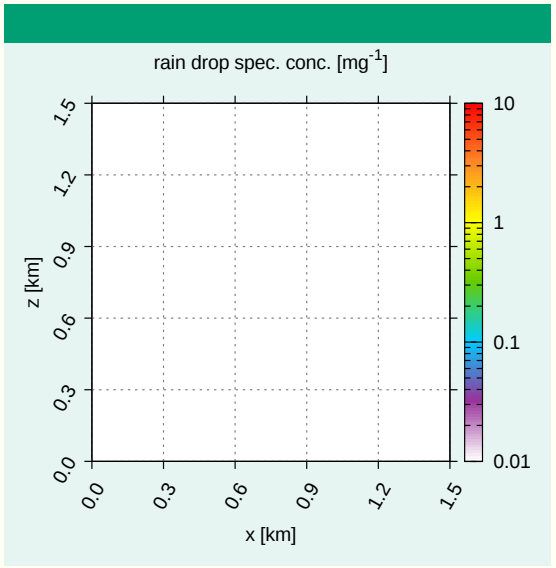


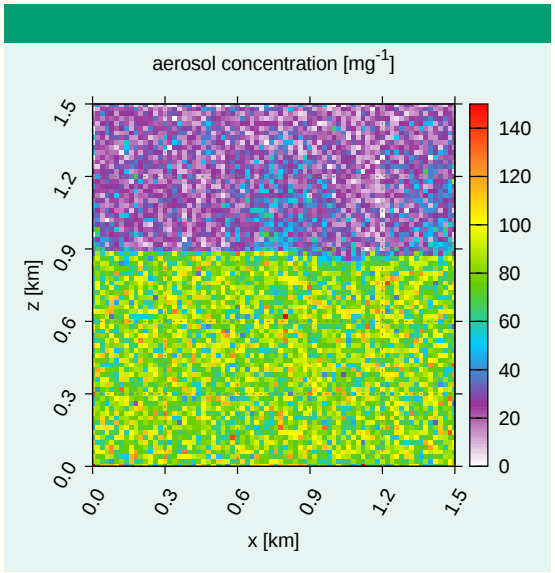


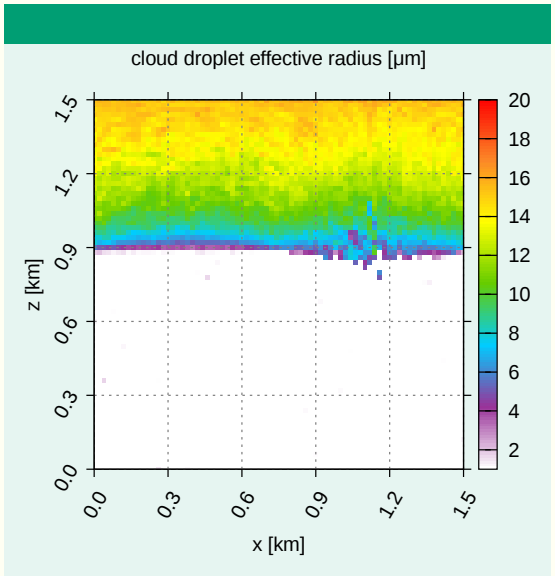


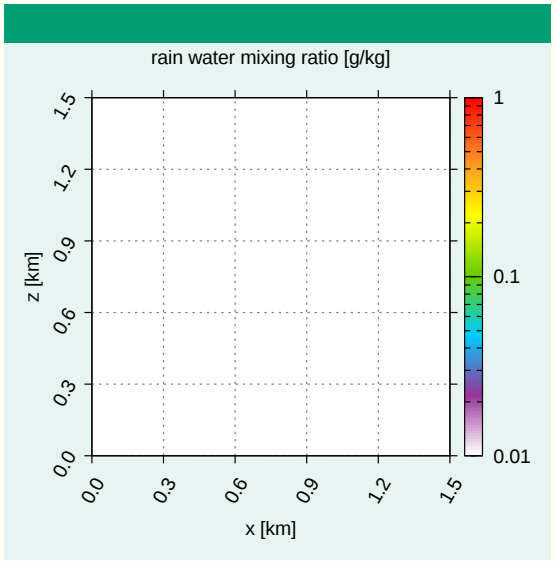
example simulation (2D, prescribed flow)



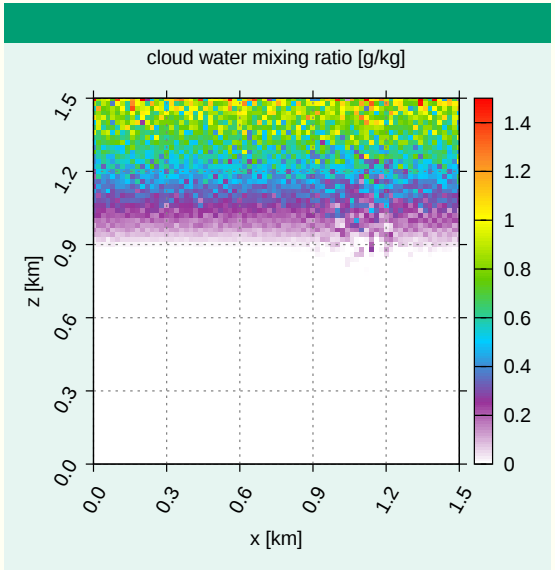


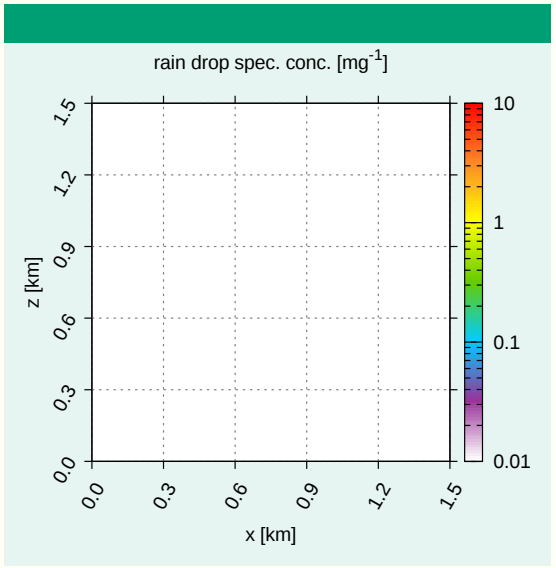


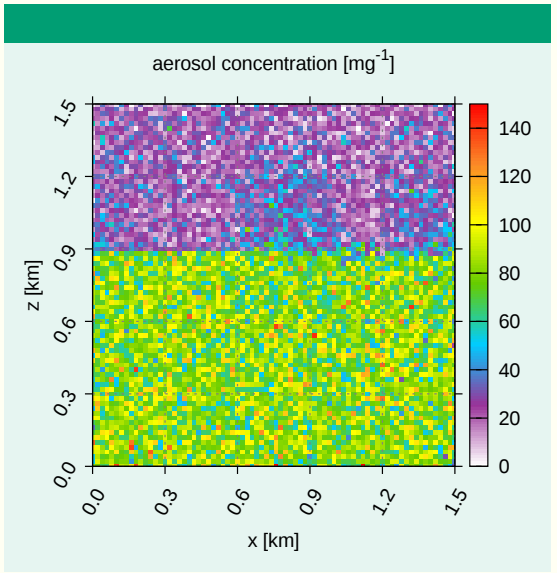


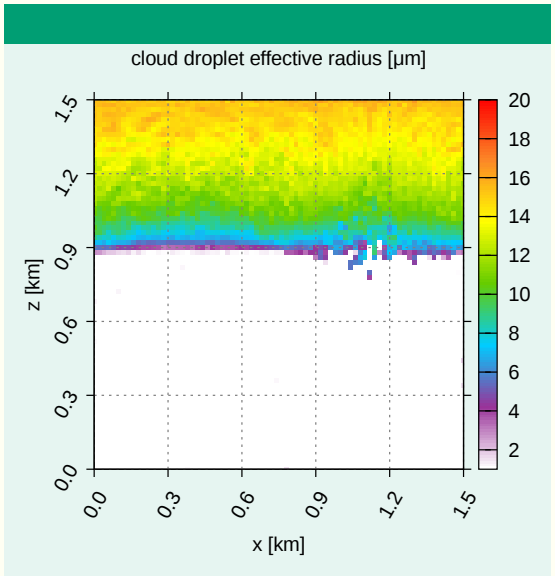


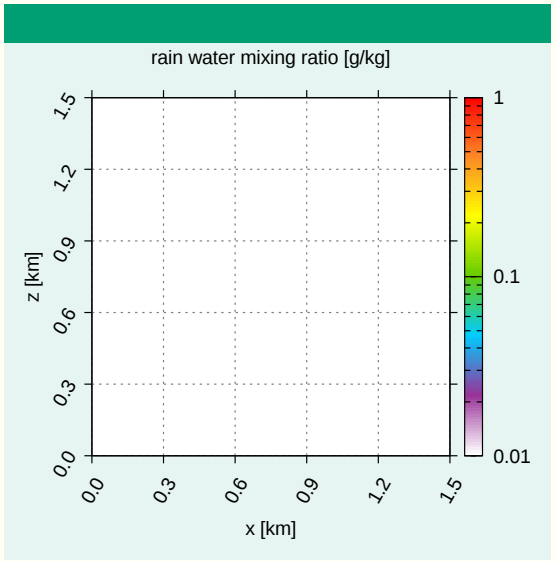
example simulation (2D, prescribed flow)



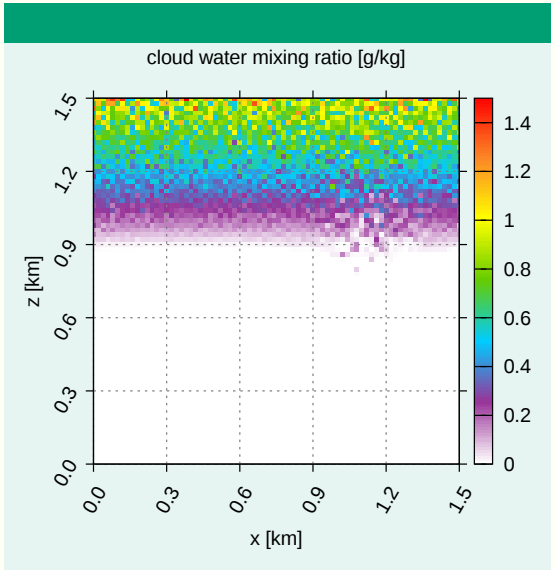


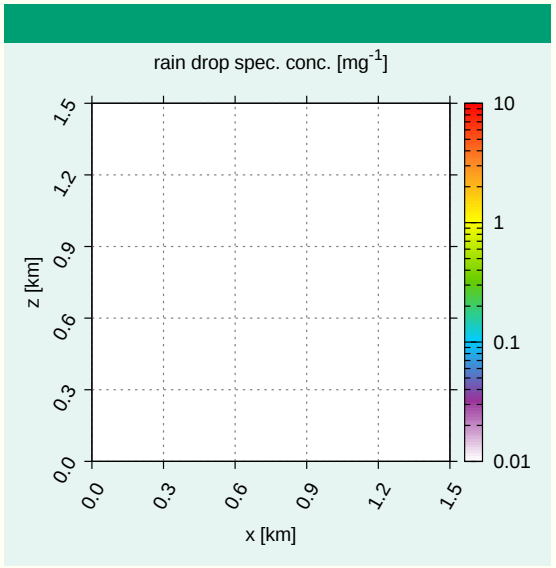


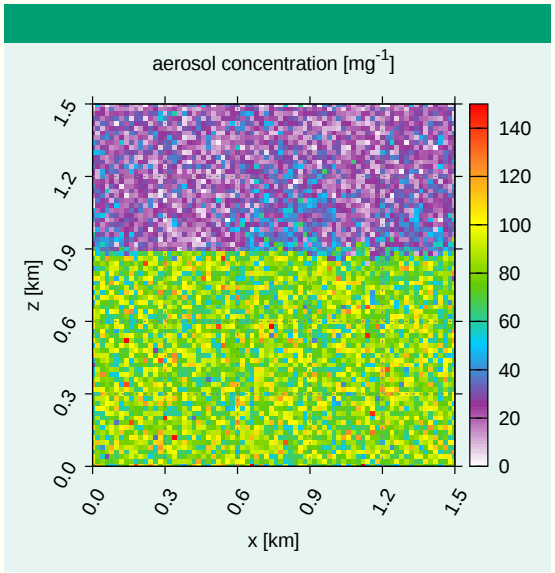


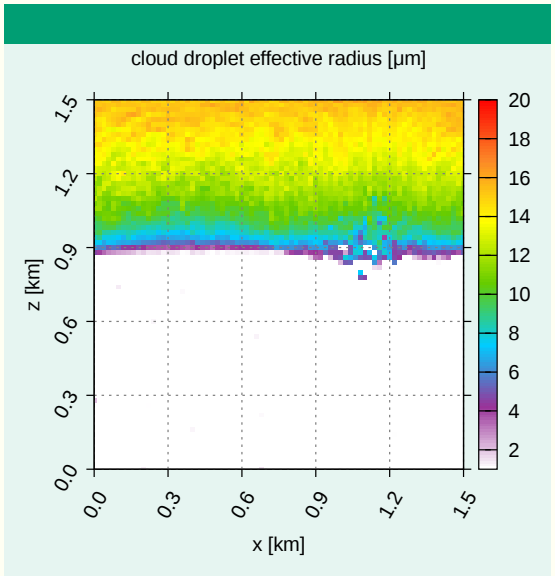


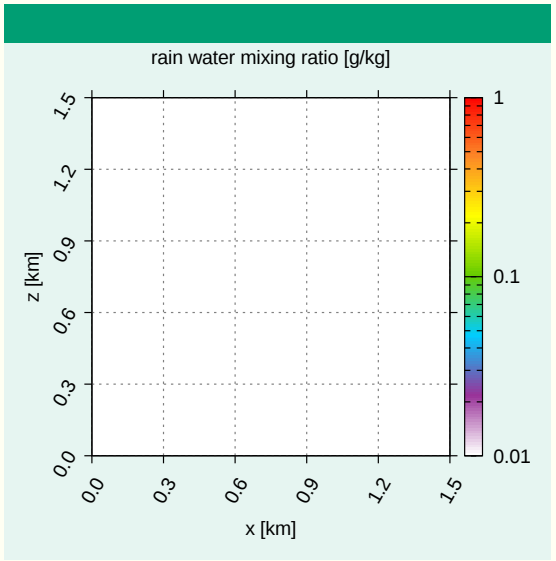
example simulation (2D, prescribed flow)



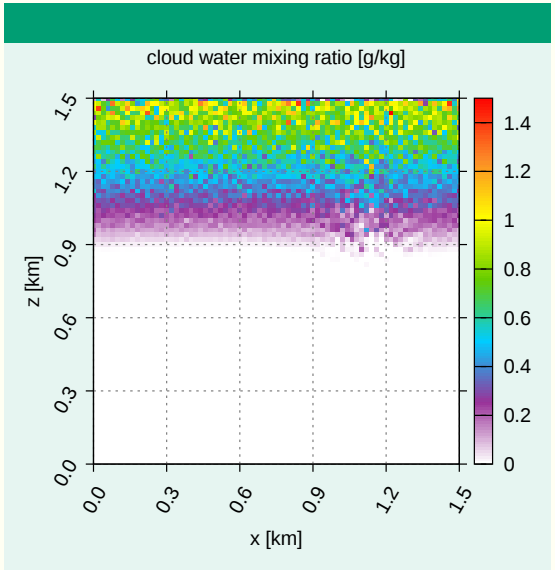


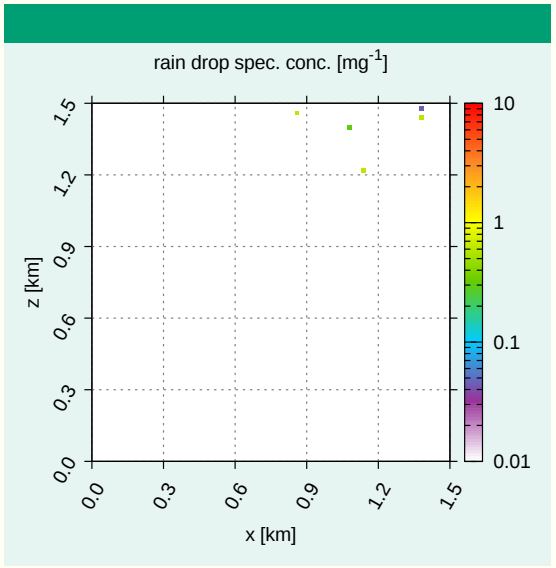


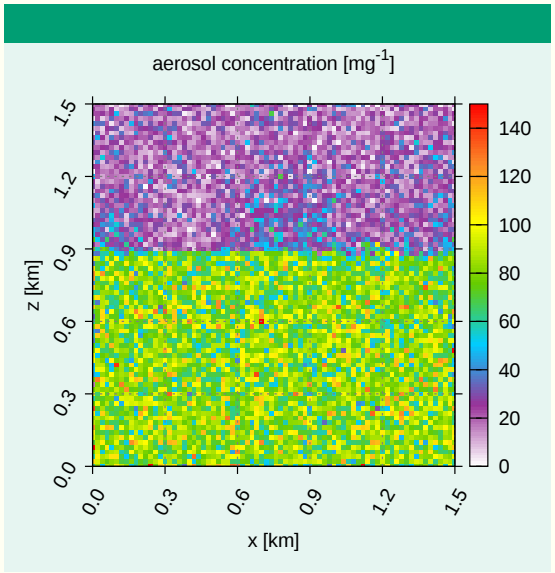


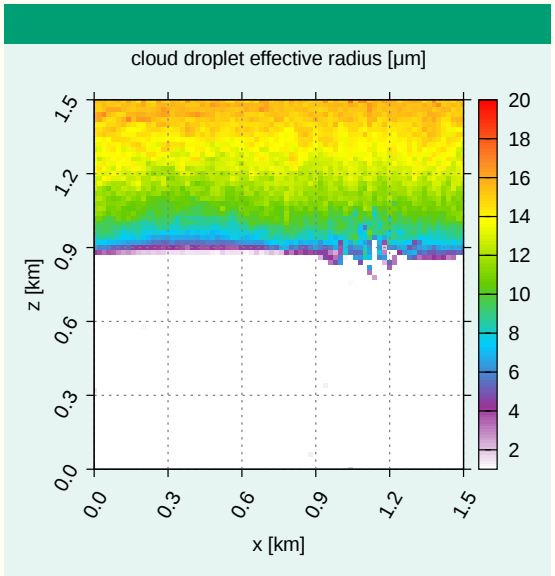


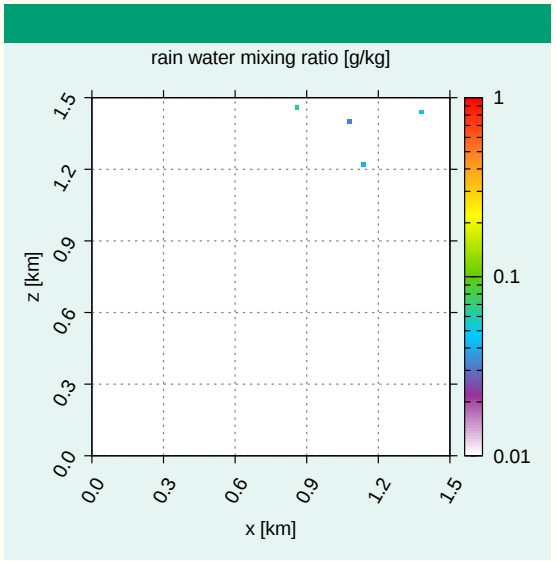
example simulation (2D, prescribed flow)



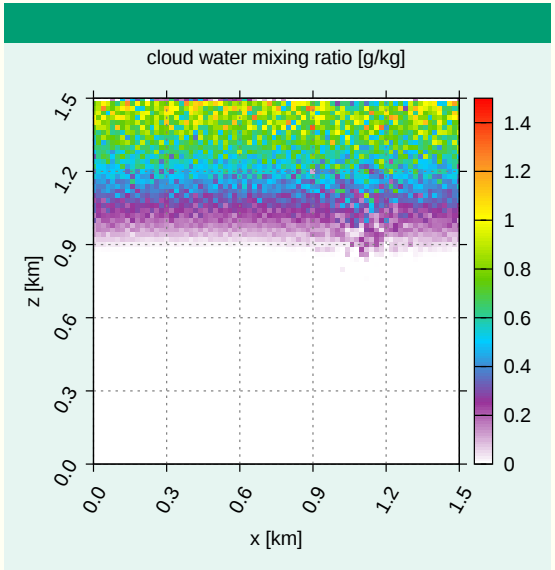


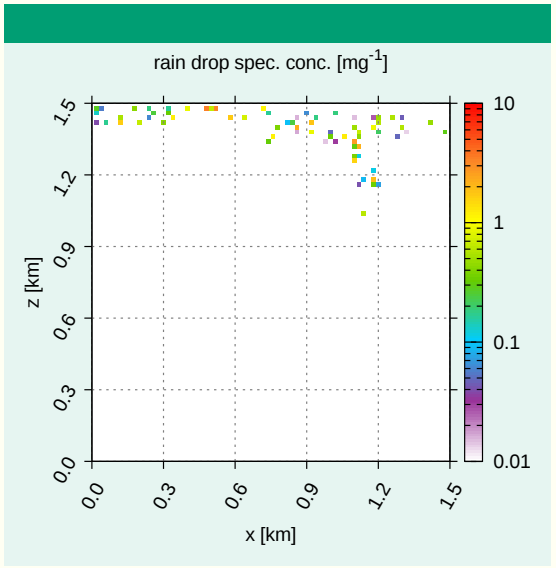


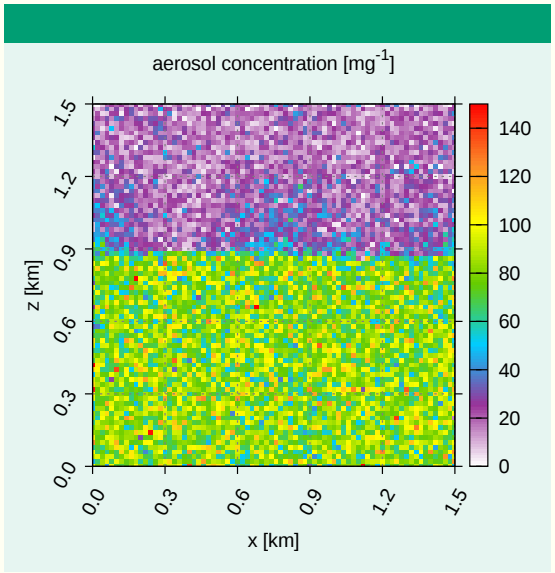


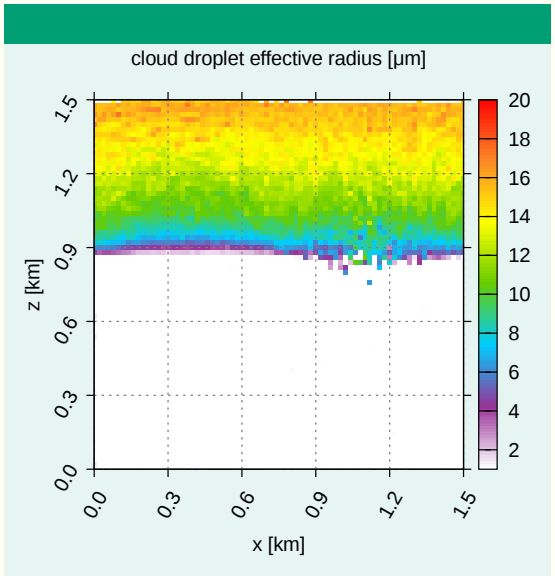


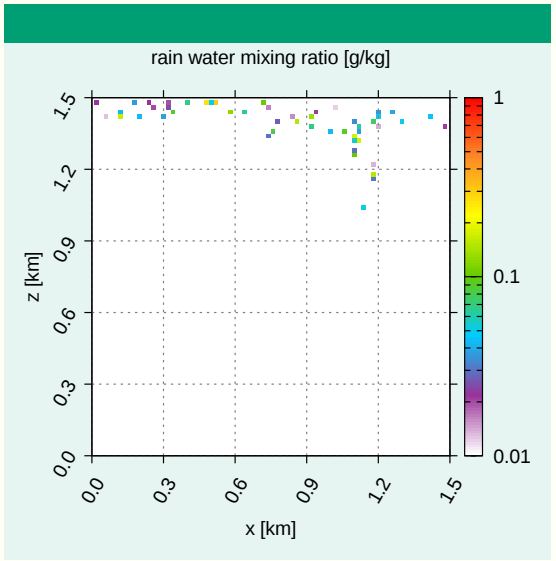
example simulation (2D, prescribed flow)



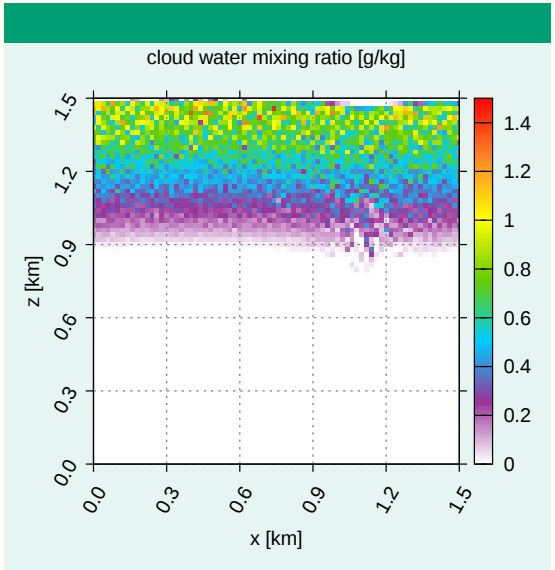


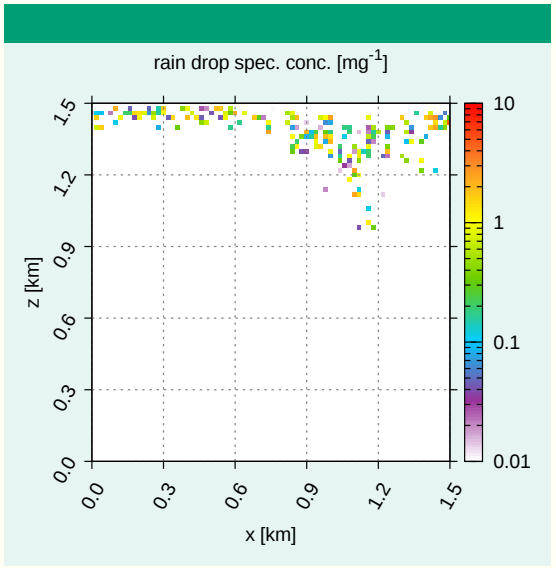


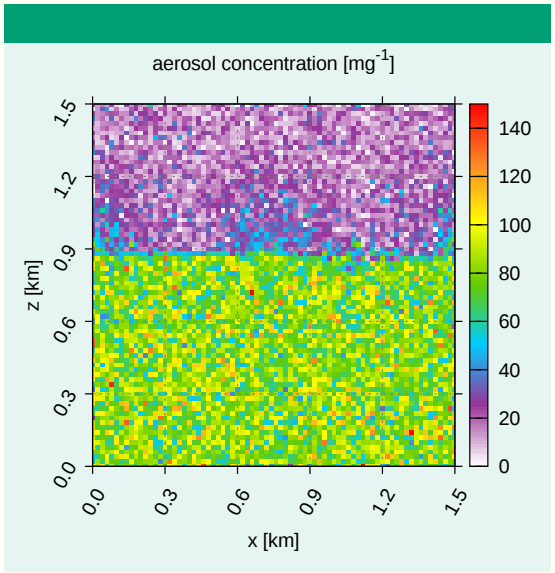


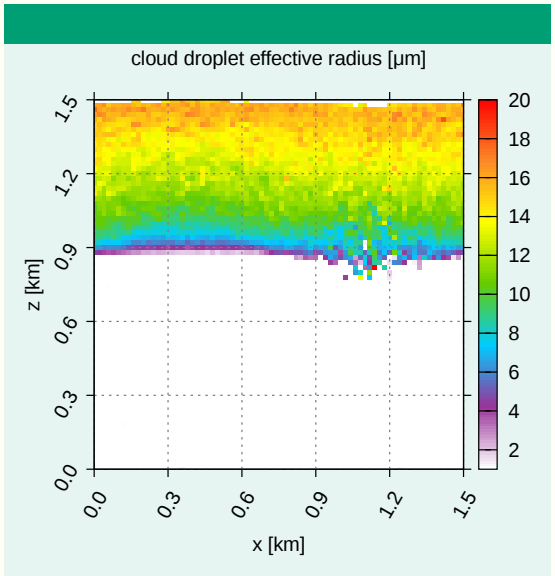


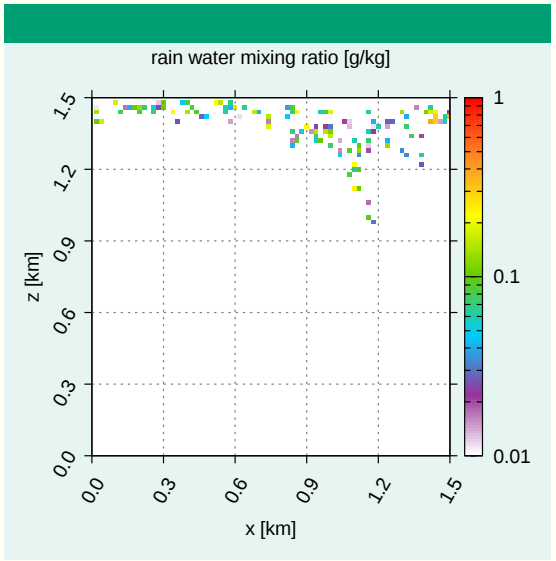
example simulation (2D, prescribed flow)



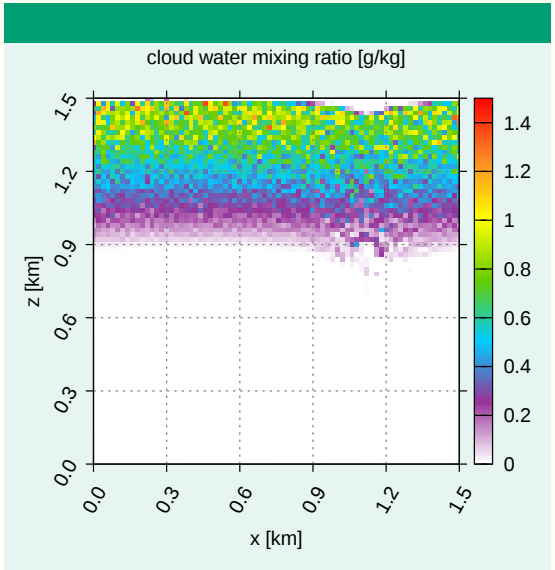


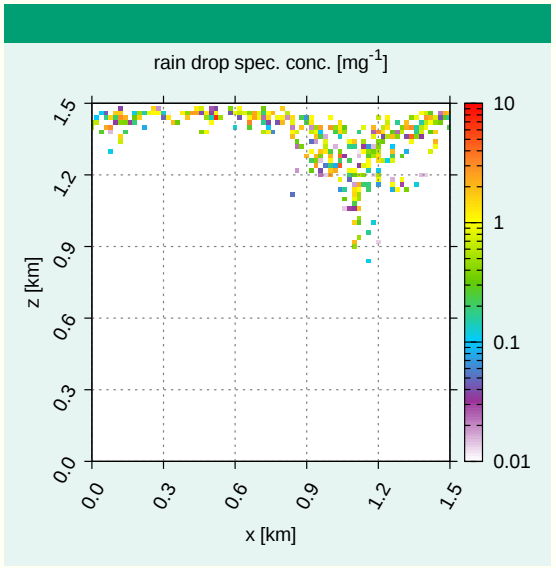


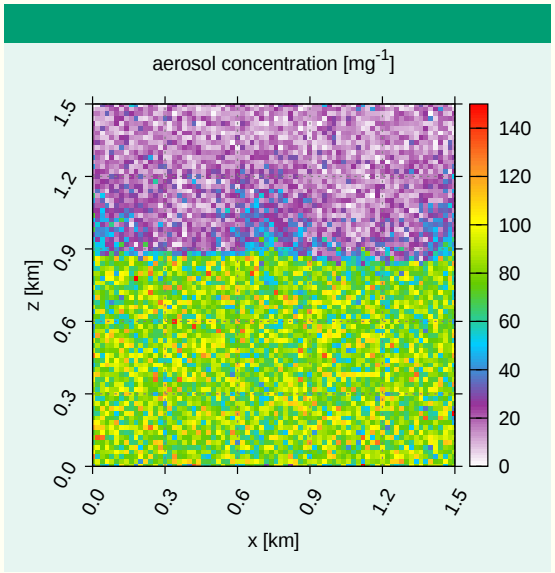


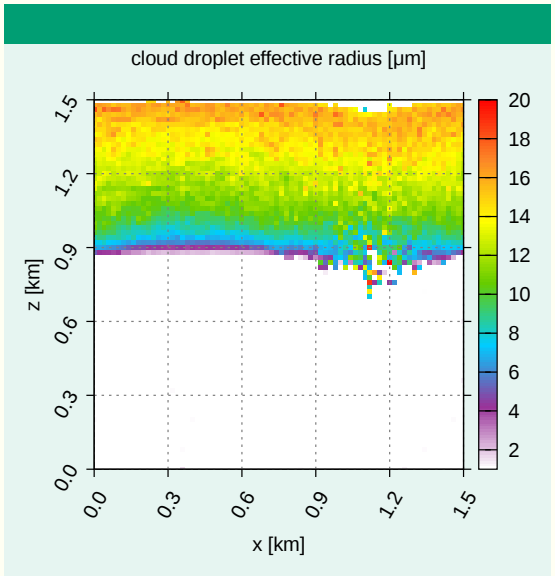


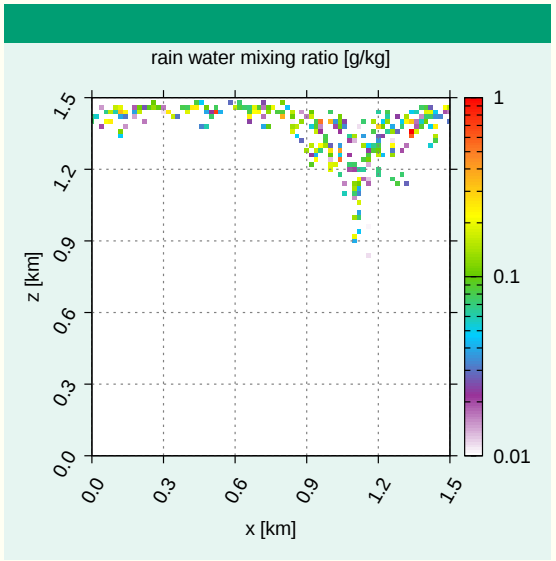
example simulation (2D, prescribed flow)



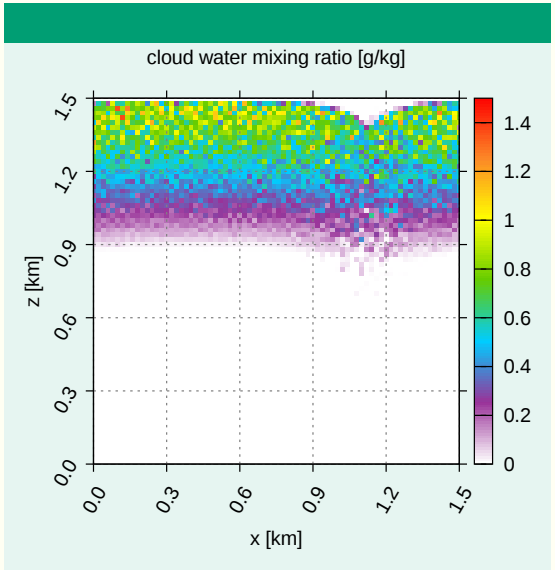


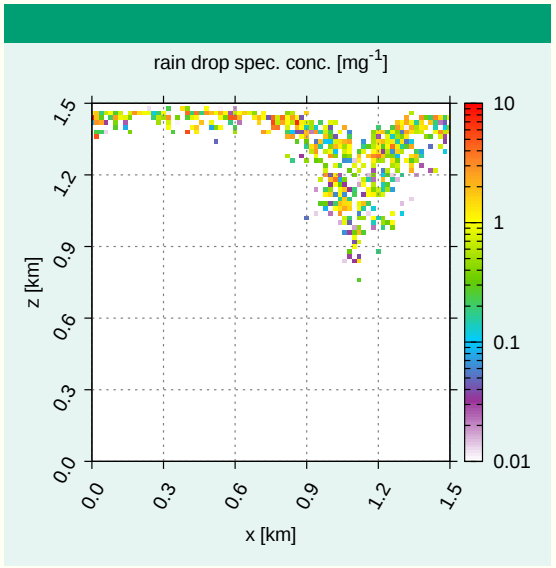


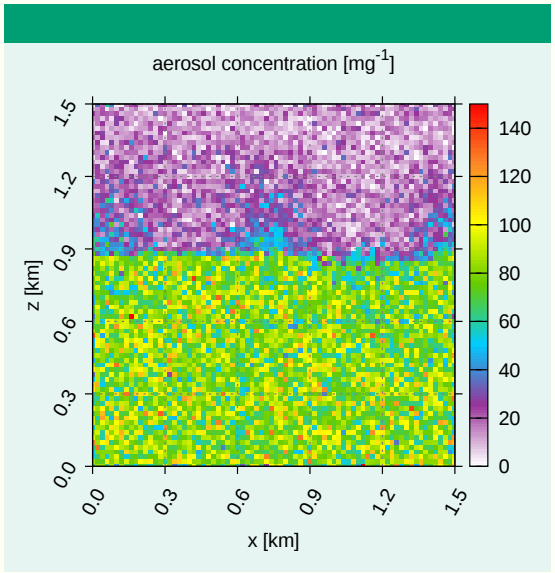


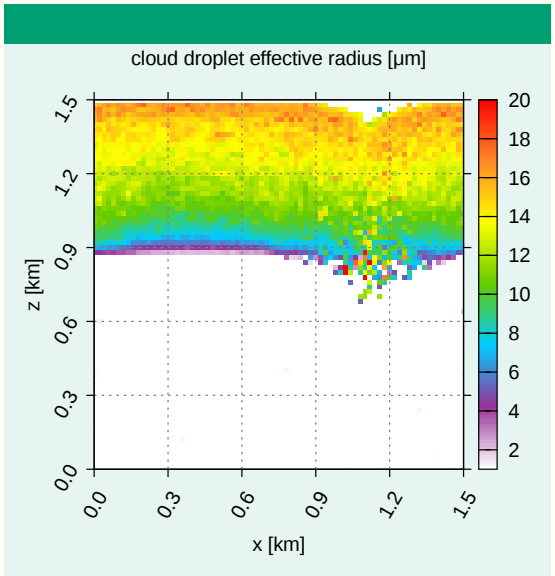


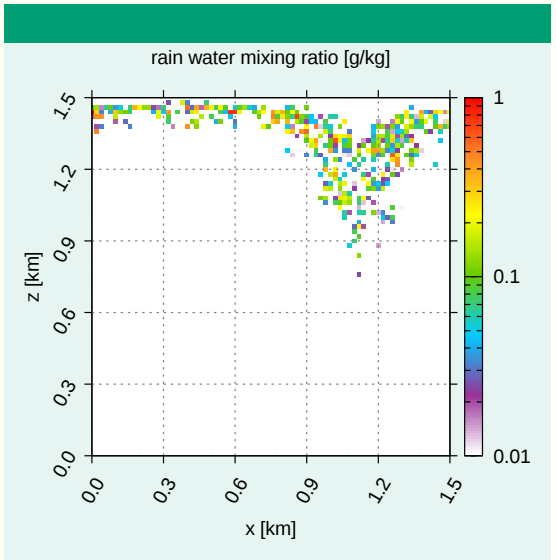
example simulation (2D, prescribed flow)



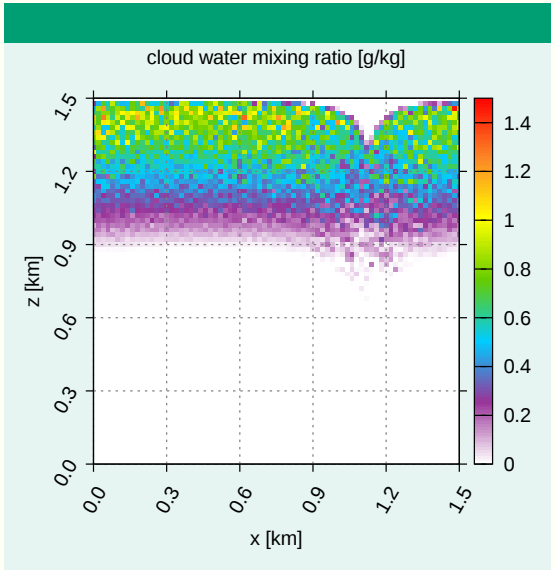


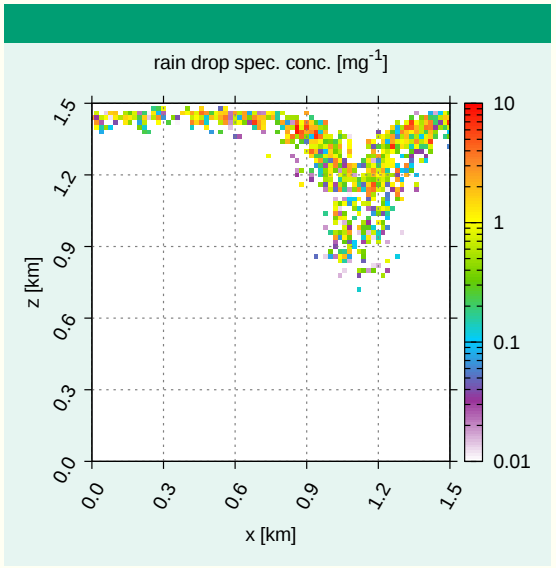


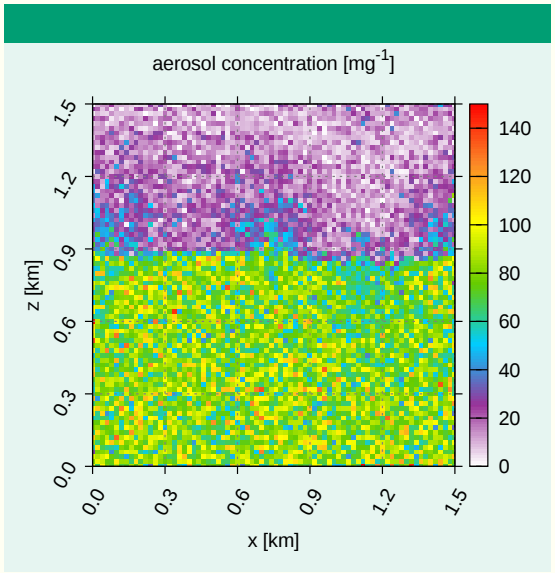


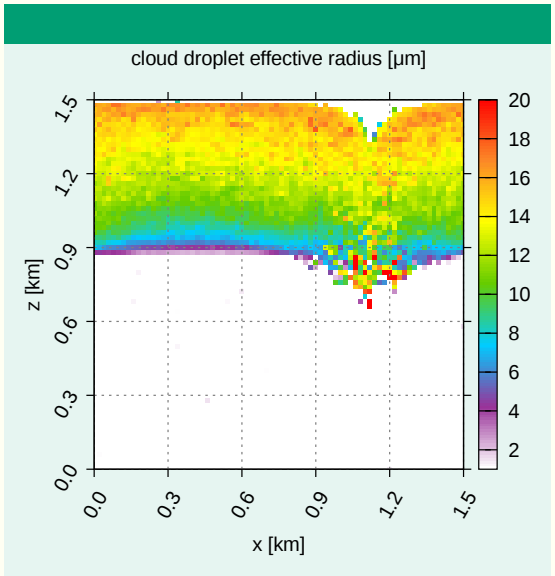


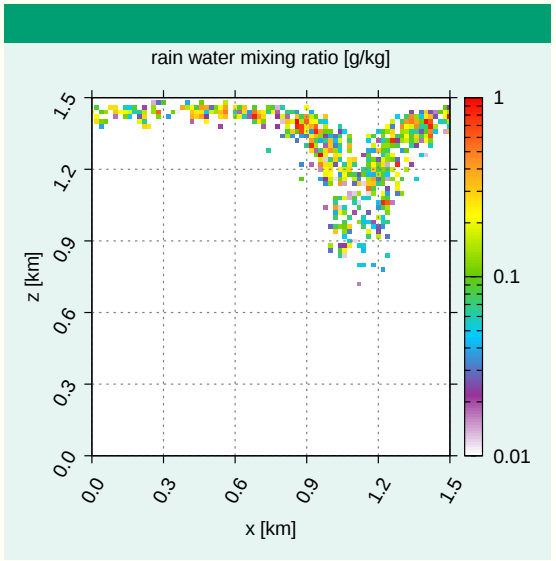
example simulation (2D, prescribed flow)



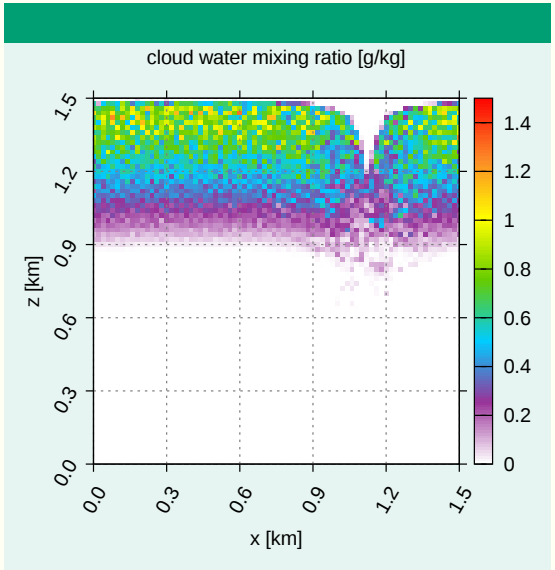


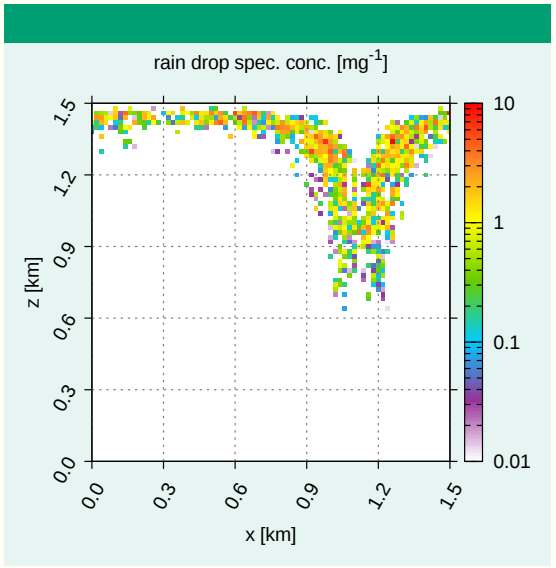


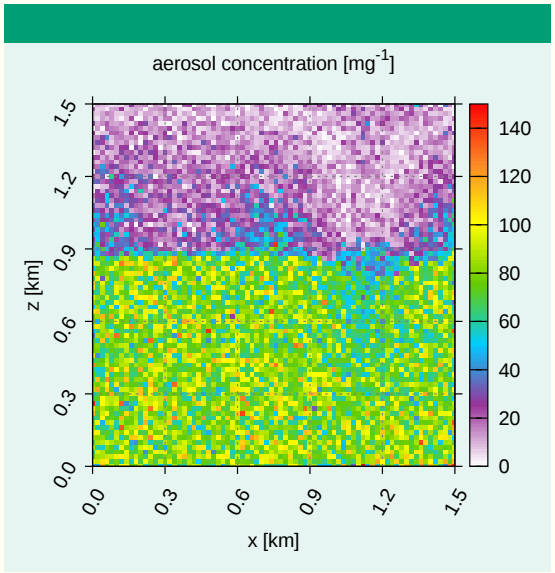


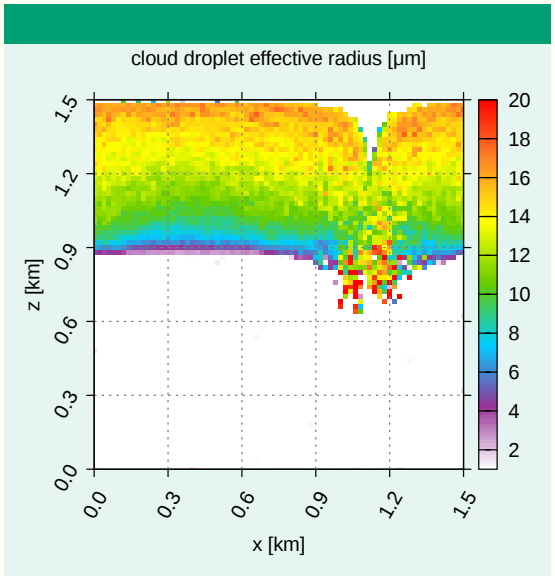


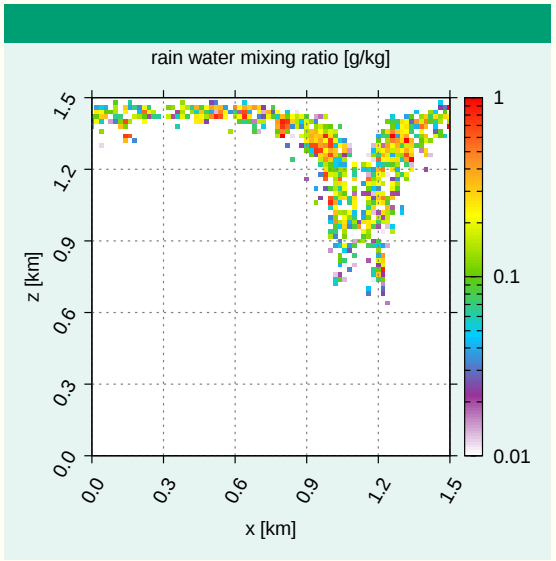
example simulation (2D, prescribed flow)



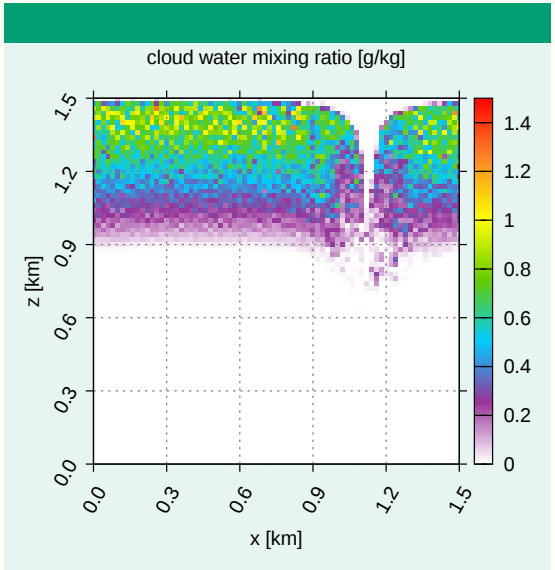


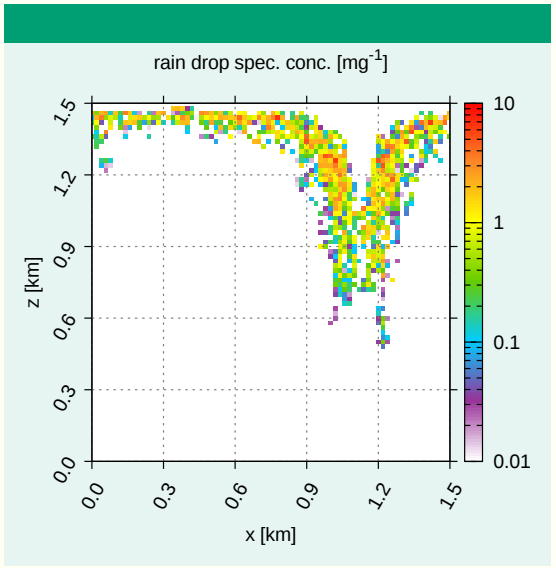


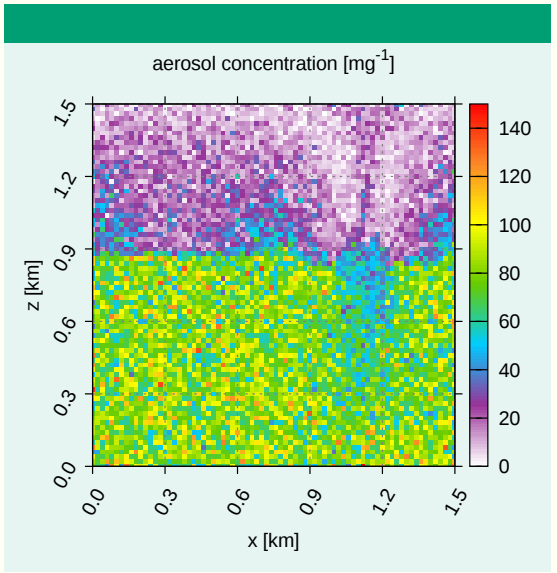


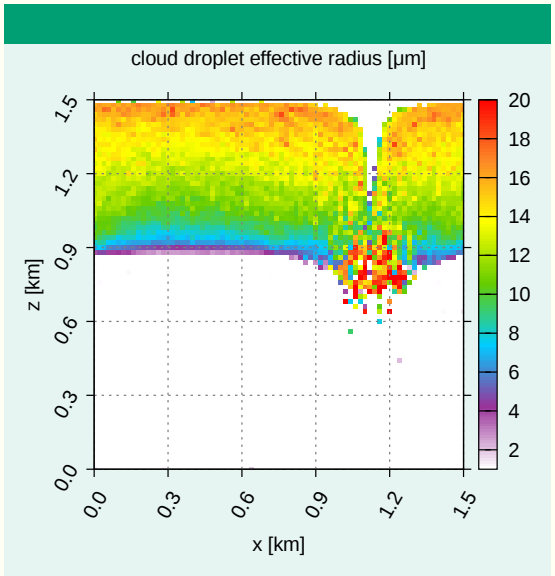


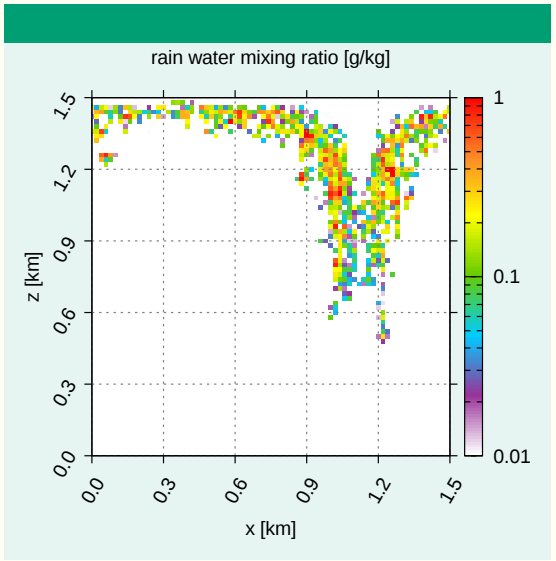
example simulation (2D, prescribed flow)



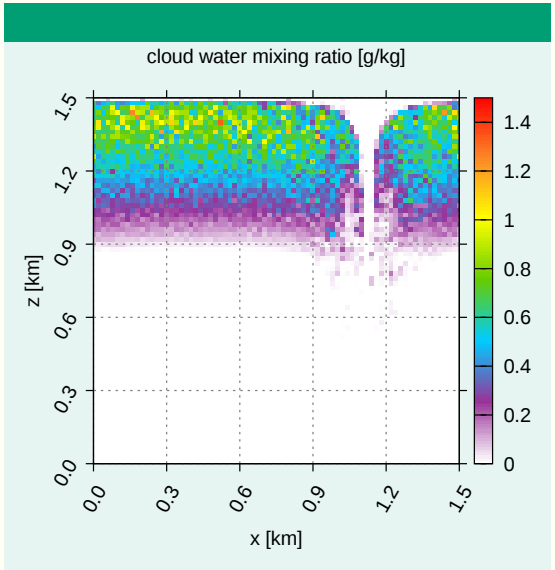


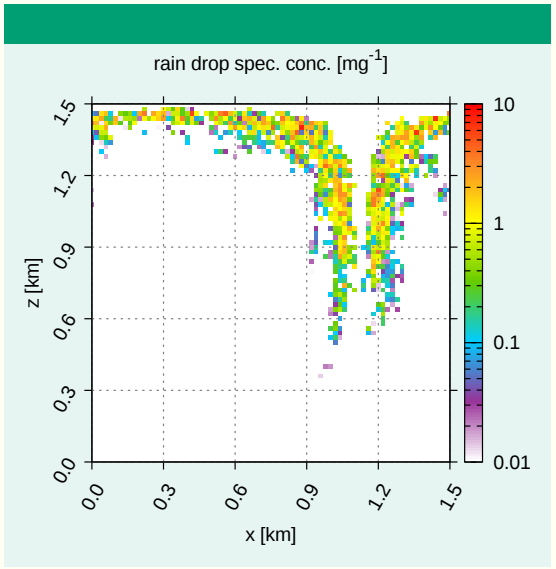


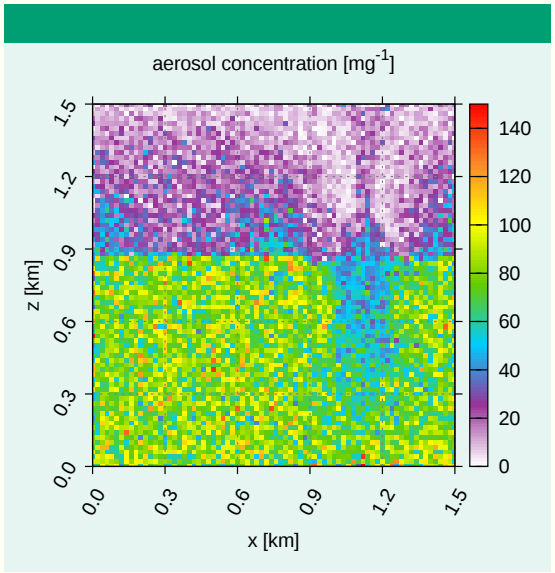


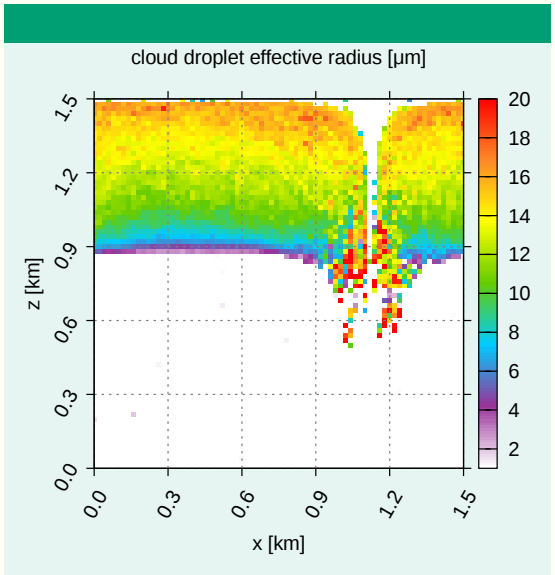


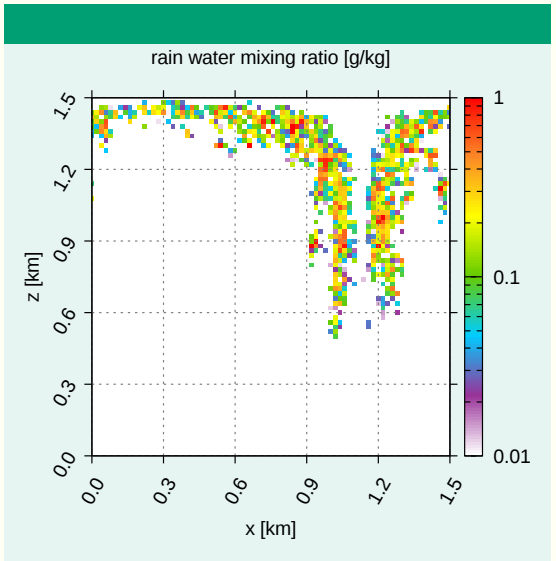
example simulation (2D, prescribed flow)





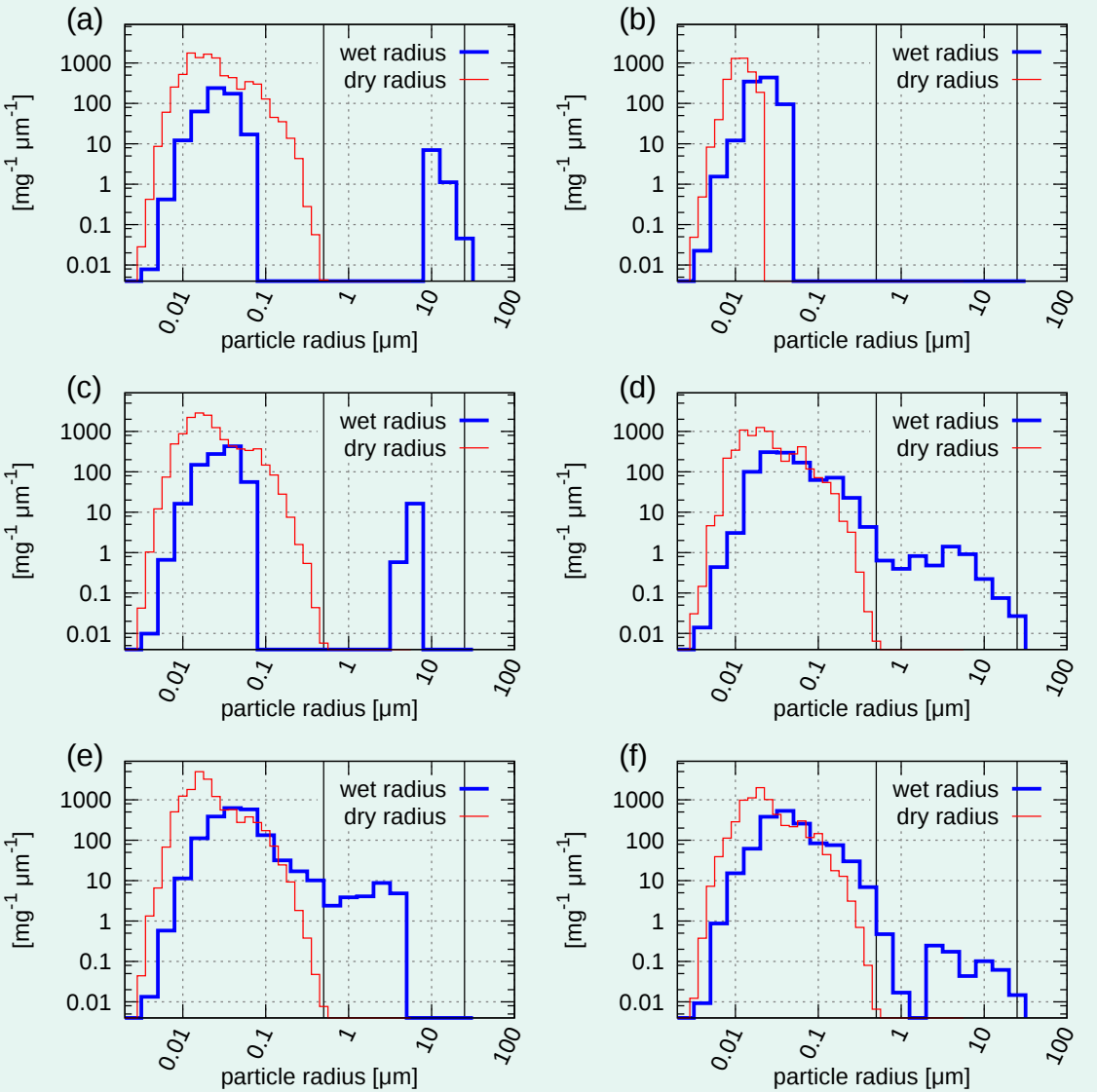


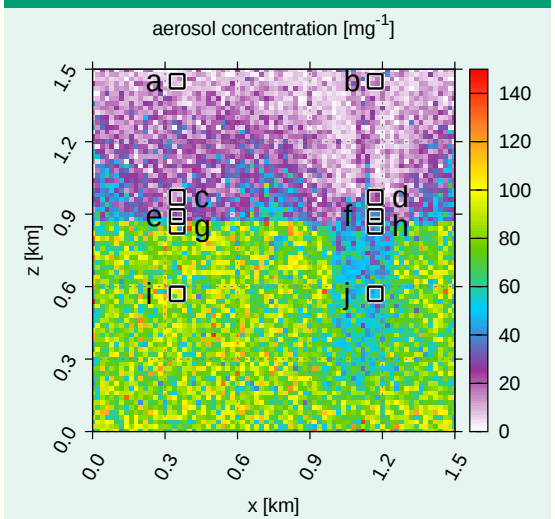


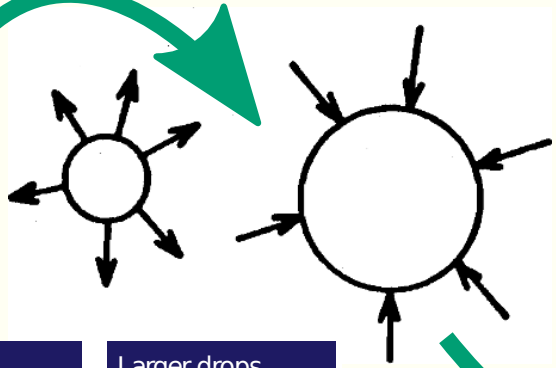


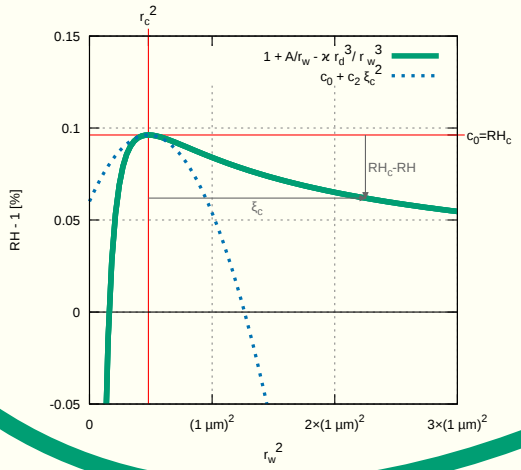
example simulation (2D, prescribed flow)

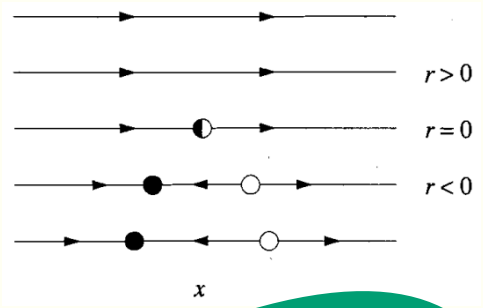
particle size spectra

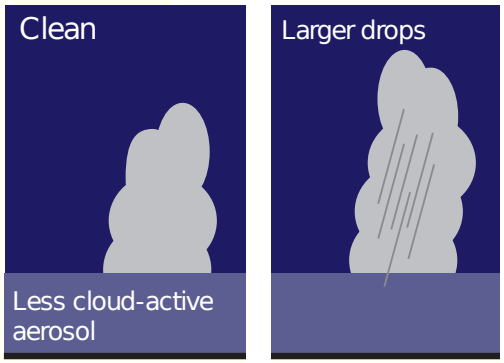






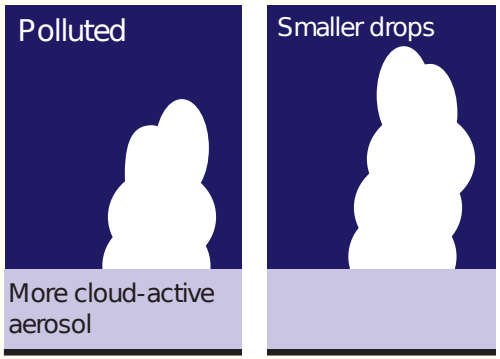


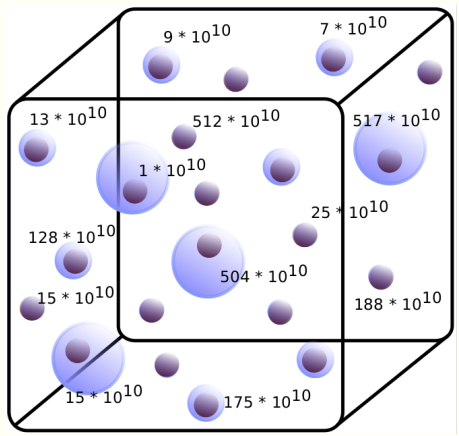


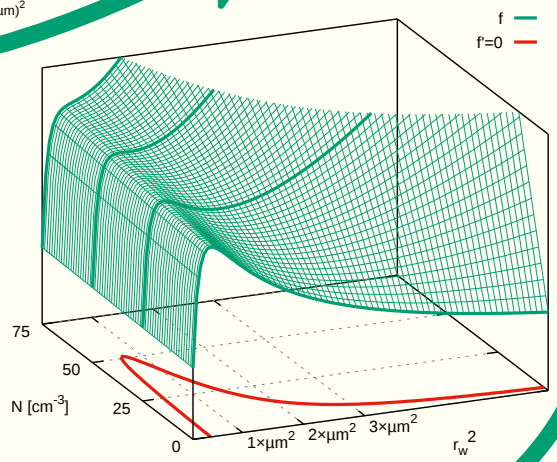


t = 10 min

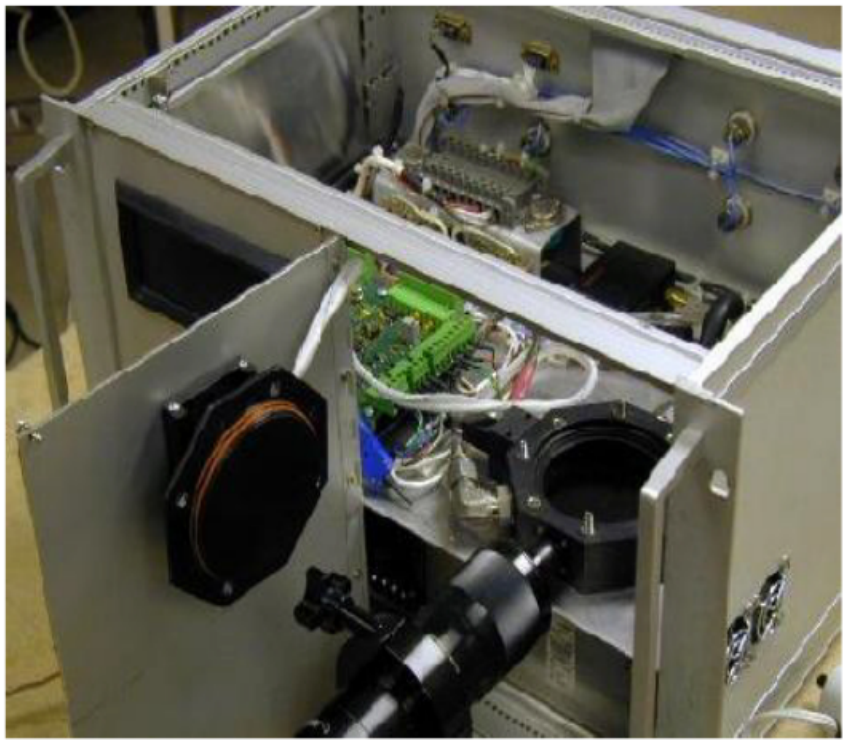
t = 20 min

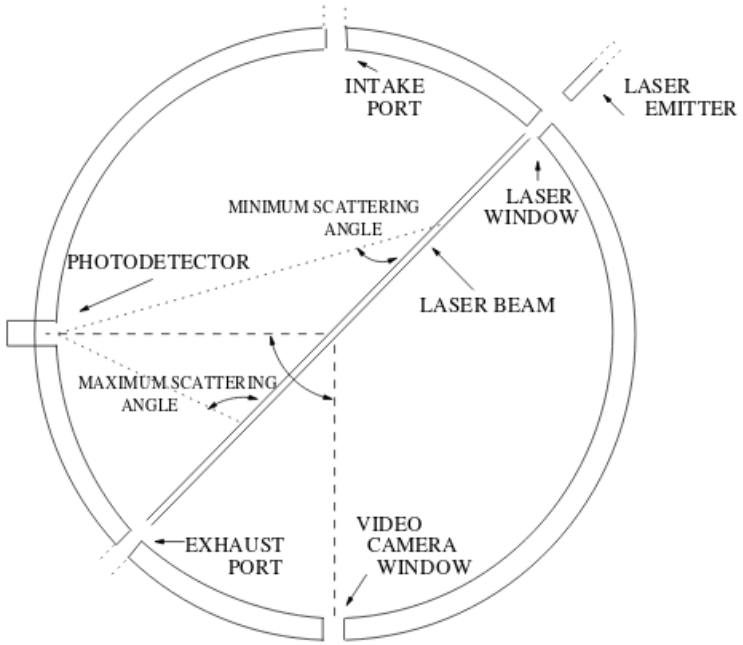






model applicability: CCN instruments? (hypothesis...)



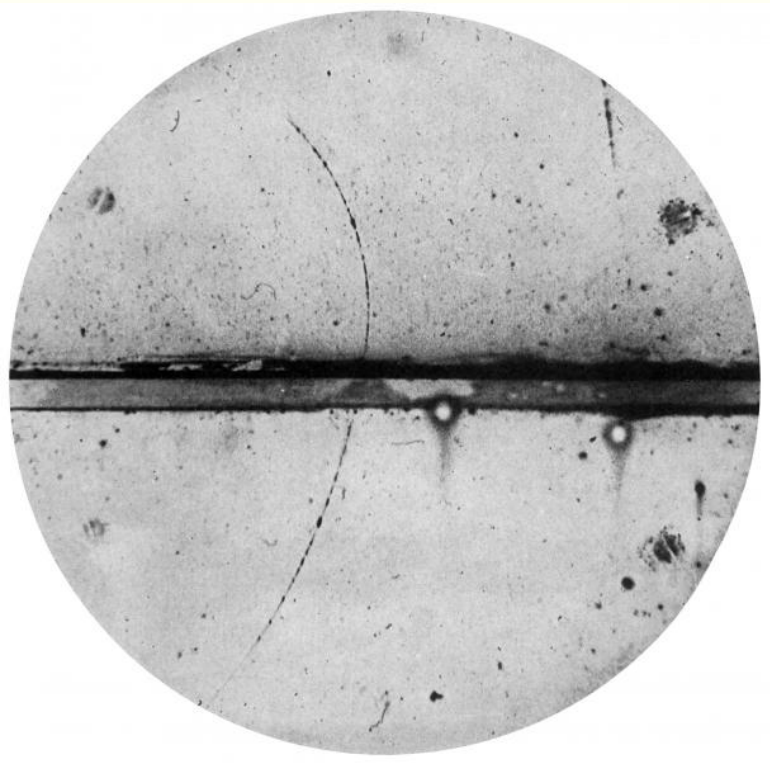


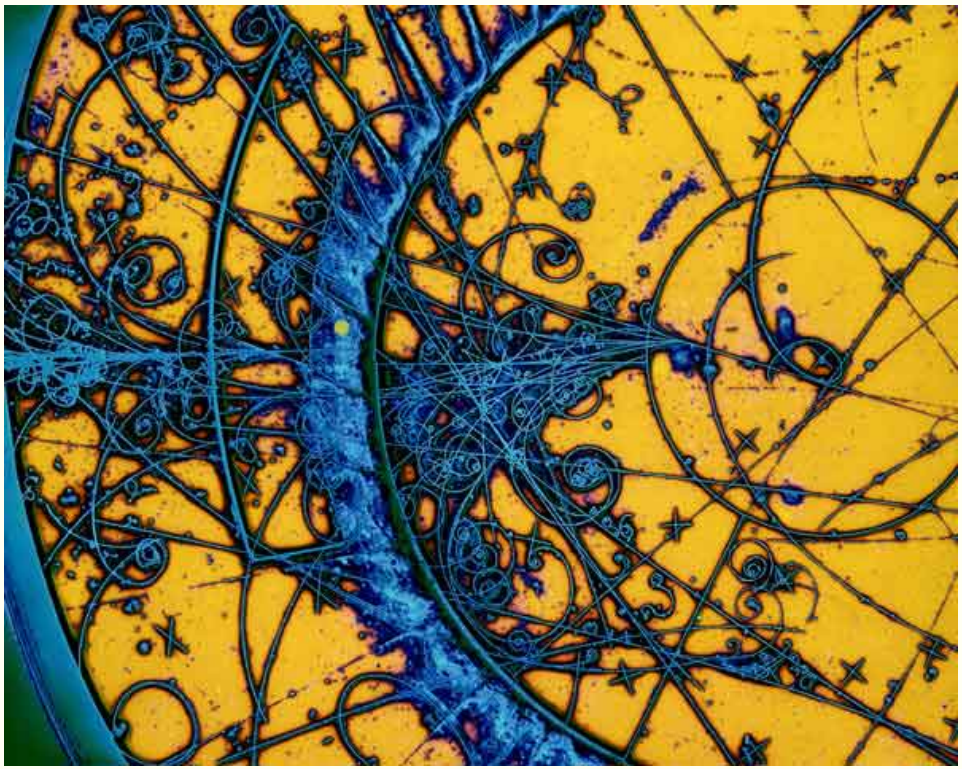
pictured: UWyoming WyoCCN instrument

(photo from DYCOMS-II CCN data report by Jeff Snider et al.)

https://www.eol.ucar.edu/projects/dycoms/dm/archive/docs/snider_ccnreadme.pdf

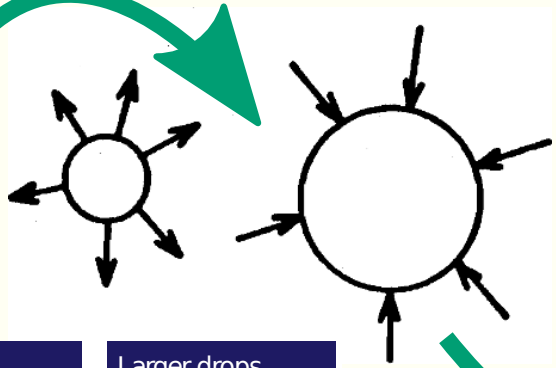
applicability beyond cloud physics (hypothesis...)

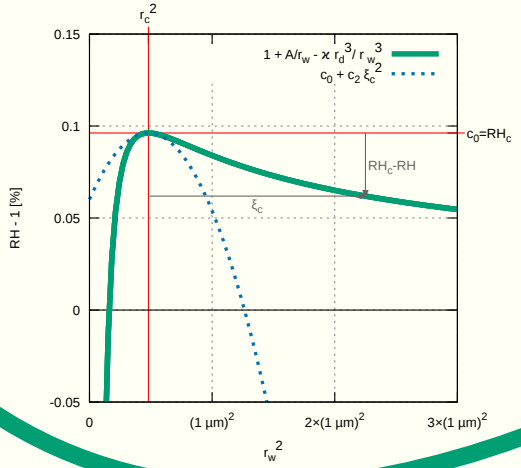


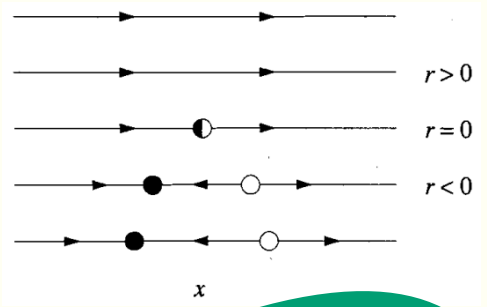


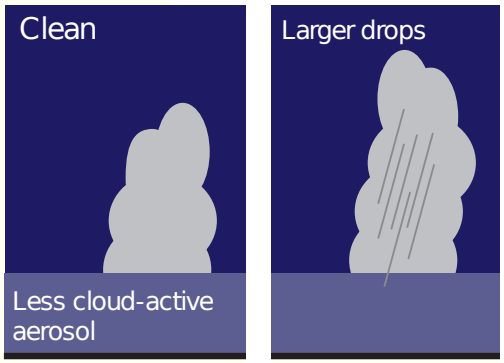
Wilson & bubble chambers

https://home.cern/about/updates/2015/06/seeing-invisible-event-displays-particle-physics



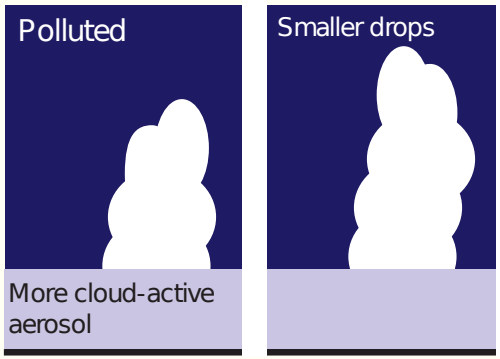




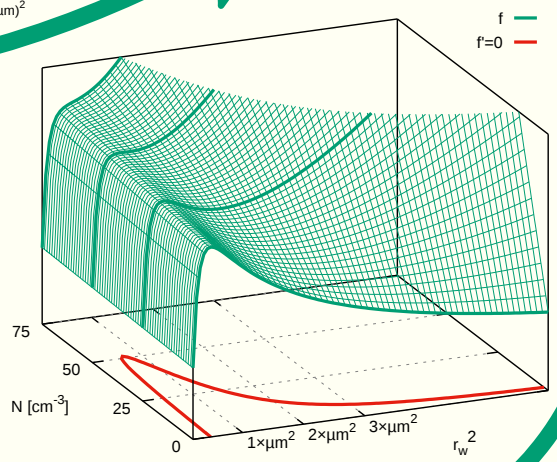


t = 10 min

t = 20 min



?



particle-based μ-physics: last decade

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),
- UCLA-LES (`http://github.com/uclales`) from UCLA/MPI-M,

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),
- UCLA-LES (`http://github.com/uclales`) from UCLA/MPI-M,
- Pencil-Code (`http://pencil-code.nordita.org`) from Nordita/UC,

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),
- UCLA-LES (`http://github.com/uclales`) from UCLA/MPI-M,
- Pencil-Code (`http://pencil-code.nordita.org`) from Nordita/UC,
- SCALE (`http://scale.aics.riken.jp/`) from RIKEN,
  (Monte-Carlo ice microphysics, deep convection: Shima et al., in prep.)

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),
- UCLA-LES (`http://github.com/uclales`) from UCLA/MPI-M,
- Pencil-Code (`http://pencil-code.nordita.org`) from Nordita/UC,
- SCALE (`http://scale.aics.riken.jp/`) from RIKEN,
  (Monte-Carlo ice microphysics, deep convection: Shima et al., in prep.)
- UWLCM (`http://github.com/igfuw/UWLCM`) from Univ. Warsaw
  (GPU-resident implementation: Arabas et al. 2015
  Hoppel-gap resolving aqueous chemistry: Jaruga & Pawlowska 2018),

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),
- UCLA-LES (`http://github.com/uclales`) from UCLA/MPI-M,
- Pencil-Code (`http://pencil-code.nordita.org`) from Nordita/UC,
- SCALE (`http://scale.aics.riken.jp/`) from RIKEN,
  (Monte-Carlo ice microphysics, deep convection: Shima et al., in prep.)
- UWLCM (`http://github.com/igfuw/UWLCM`) from Univ. Warsaw
  (GPU-resident implementation: Arabas et al. 2015
  Hoppel-gap resolving aqueous chemistry: Jaruga & Pawlowska 2018),
- ICON/McSnow (`http://gitlab.com/sbrdar/mcsnow`) from DWD,

particle-based μ-physics: last decade

- INC/LCM from LANL/Leeds,
- EULAG-LCM (`http://www.mmm.ucar.edu/eulag/`) from NCAR/DLR,
  (contrail-to-cirrus transitions: Unterstrasser 2014)
- PALM-LES (`http://palm.muk.uni-hannover.de/`) from Univ. Hannover,
  (Linear Eddy Model (LEM): Hoffmann et al. 2018),
- CReSS (`http://www.rain.hyarc.nagoya-u.ac.jp/`) from Univ. Nagoya,
  (24h 3D LES of trade-wind cumuli field: Arabas & Shima 2013),
- UCLA-LES (`http://github.com/uclales`) from UCLA/MPI-M,
- Pencil-Code (`http://pencil-code.nordita.org`) from Nordita/UC,
- SCALE (`http://scale.aics.riken.jp/`) from RIKEN,
  (Monte-Carlo ice microphysics, deep convection: Shima et al., in prep.)
- UWLCM (`http://github.com/igfuw/UWLCM`) from Univ. Warsaw
  (GPU-resident implementation: Arabas et al. 2015
  Hoppel-gap resolving aqueous chemistry: Jaruga & Pawlowska 2018),
- ICON/McSnow (`http://gitlab.com/sbrdar/mcsnow`) from DWD,
- ASAM (`http://asam.tropos.de/`) from TROPOS.

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)
- **by-design non-negativity** of the derived density/concentration fields

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)
- **by-design non-negativity** of the derived density/concentration fields
- **ab-initio** (particle-level) vs. parametrised (bulk/moment/bin) formulations

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)
- **by-design non-negativity** of the derived density/concentration fields
- **ab-initio** (particle-level) vs. parametrised (bulk/moment/bin) formulations
- **favourable scaling** (particle attributes vs. Eulerian *curse of dimensionality*)

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)
- **by-design non-negativity** of the derived density/concentration fields
- **ab-initio** (particle-level) vs. parametrised (bulk/moment/bin) formulations
- **favourable scaling** (particle attributes vs. Eulerian *curse of dimensionality*)
- **lifetime tracing** of aerosol particles (coalescence: props:yes; identity:no)

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)
- **by-design non-negativity** of the derived density/concentration fields
- **ab-initio** (particle-level) vs. parametrised (bulk/moment/bin) formulations
- **favourable scaling** (particle attributes vs. Eulerian *curse of dimensionality*)
- **lifetime tracing** of aerosol particles (coalescence: props:yes; identity:no)
- **hybrid supercomputing** adaptable (GPU-resident particles)

particle-based microphysics: recap/takeaways

- **no numerical diffusion** in radius space (also for coalesc. if Monte-Carlo)
- **subgrid cloud fraction** is effectively represented (robust wrt Eulerian grid)
- **by-design non-negativity** of the derived density/concentration fields
- **ab-initio** (particle-level) vs. parametrised (bulk/moment/bin) formulations
- **favourable scaling** (particle attributes vs. Eulerian *curse of dimensionality*)
- **lifetime tracing** of aerosol particles (coalescence: props:yes; identity:no)
- **hybrid supercomputing** adaptable (GPU-resident particles)
- **lucky-droplet & GCCN friendly** Monte-Carlo (non-SCE) coalescence

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)
- **ensemble analysis** (multiple realisations, probabilistic "thinking")

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)
- **ensemble analysis** (multiple realisations, probabilistic "thinking")
- **(de)activation nonlinearities** $\rightsquigarrow$ numerical/resolution challenges

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)
- **ensemble analysis** (multiple realisations, probabilistic "thinking")
- **(de)activation nonlinearities** $\rightsquigarrow$ numerical/resolution challenges
- **Eulerian/Lagrangian** dynamics consistency (resolved and subgrid)

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)
- **ensemble analysis** (multiple realisations, probabilistic "thinking")
- **(de)activation nonlinearities** $\rightsquigarrow$ numerical/resolution challenges
- **Eulerian/Lagrangian** dynamics consistency (resolved and subgrid)
- **radiative transfer** $\rightsquigarrow$ visualisations & radiative cooling

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)
- **ensemble analysis** (multiple realisations, probabilistic "thinking")
- **(de)activation nonlinearities** $\rightsquigarrow$ numerical/resolution challenges
- **Eulerian/Lagrangian** dynamics consistency (resolved and subgrid)
- **radiative transfer** $\rightsquigarrow$ visualisations & radiative cooling
- **commensurable comparisons** wrt bin/bulk: "aerosol water",
  cannot "switch off" aerosol processing, ripening, etc (ab-initio)

particle-based microphysics: challenges/opportunities

- **aerosol budget** (precipication/scavenging sinks vs. long-term LES)
- **ensemble analysis** (multiple realisations, probabilistic "thinking")
- **(de)activation nonlinearities** $\rightsquigarrow$ numerical/resolution challenges
- **Eulerian/Lagrangian** dynamics consistency (resolved and subgrid)
- **radiative transfer** $\rightsquigarrow$ visualisations & radiative cooling
- **commensurable comparisons** wrt bin/bulk: "aerosol water",
  cannot "switch off" aerosol processing, ripening, etc (ab-initio)
- **charge, isotopic ratio, ...**

news: BAMS super-droplet review (Grabowski et al. '19)

MODELING OF CLOUD MICROPHYSICS

Can We Do Better?

Wojciech W. Grabowski, Hugh Morrison, Shin-Ichiro Shima, Gustavo C. Abade, Piotr Dziekan, and Hanna Pawlowska

The Lagrangian particle-based approach is an emerging technique to model cloud microphysics and its coupling with dynamics, offering significant advantages over Eulerian approaches typically used in cloud models.

doi:10.1175/BAMS-D-18-0005.1

`http://particle-based-cloud-modelling.network`

particle-based-cloud-modelling.network

View on GitHub

Particle-Based Cloud Modelling Network Initiative

Mailing List

Venue for communications relevant to the development and applications of particle-based models of atmospheric clouds: announcements of meetings, calls for submissions, funding opportunities, scholarships, openings, software/data releases, publications and other notices warranting community-wide dissemination.

Archives and subscription management:
https://mailing.uj.edu.pl/sympa/info/particle-based-cloud-modelling

Event Calendar

Database of events announced on the mailing list:

Thank you for your attention!

hysteresis: activation/deactivation cycle

hysteresis: activation/deactivation cycle

- nomenclature:

hysteresis: activation/deactivation cycle

- nomenclature:
  - CCN activation
  - (heterogeneous) nucleation

hysteresis: activation/deactivation cycle

- nomenclature:
  - CCN activation
  - (heterogeneous) nucleation

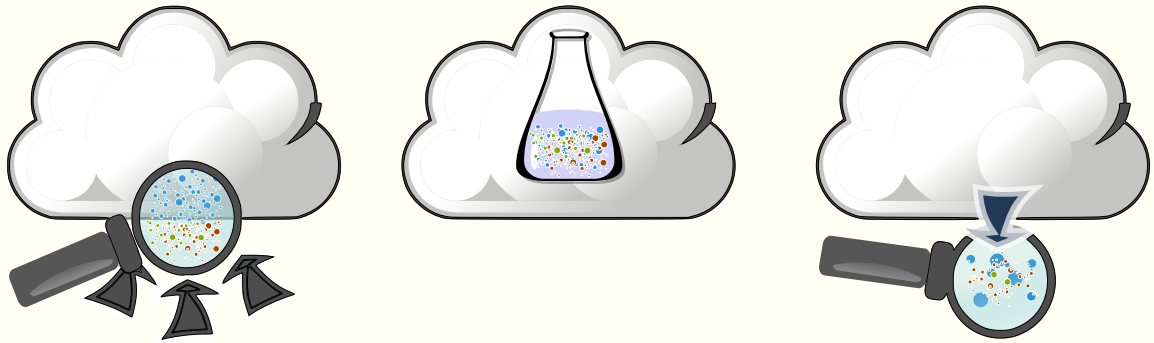
  - CCN deactivation
  - aerosol regeneration / resuspension / recycling
  - drop-to-particle conversion
  - droplet evaporation

hysteresis: activation/deactivation cycle

- nomenclature:
  - CCN activation
  - (heterogeneous) nucleation

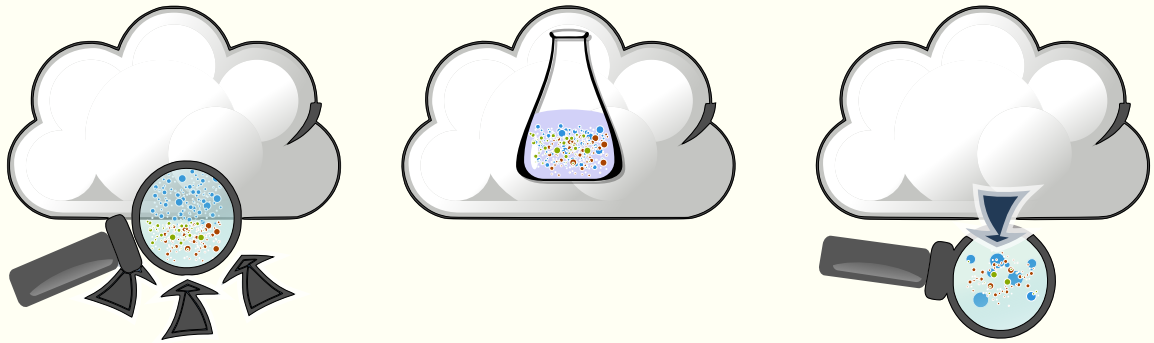
  - CCN deactivation
  - aerosol regeneration / resuspension / recycling
  - drop-to-particle conversion
  - droplet evaporation
- significance:

hysteresis: activation/deactivation cycle



- nomenclature:
  - CCN activation
  - (heterogeneous) nucleation
  - CCN deactivation
  - aerosol regeneration / resuspension / recycling
  - drop-to-particle conversion
  - droplet evaporation
- significance:
  - aerosol processing by clouds (aqueous chemistry, coalescence)

hysteresis: activation/deactivation cycle



- nomenclature:
  - CCN activation
  - (heterogeneous) nucleation
  - CCN deactivation
  - aerosol regeneration / resuspension / recycling
  - drop-to-particle conversion
  - droplet evaporation
- significance:
  - aerosol processing by clouds (aqueous chemistry, coalescence)
  - spectral broadening (mixing, parcel history, ...)

lifting the constant T-p assumptions: parcel model

lifting the constant T-p assumptions: parcel model

vertically displaced (velocity w, hydrostatic background) adiabatic parcel:
(q: mixing ratio, p_d: bgnd pressure, ρ_d bgnd density, g,l_v,c_pd: constants)

$$\begin{bmatrix} \dot{p}_d \\ \dot{T} \\ \dot{r}_w \end{bmatrix} = \begin{bmatrix} -\rho_\mathrm{d} g w \\ (\dot{p}_\mathrm{d}/\rho_\mathrm{d} - \dot{q} l_\mathrm{v})/c_\mathrm{pd} \\ (D_\mathrm{eff}/\rho_\mathrm{w})(\rho_\mathrm{v} - \rho_\circ)/r_\mathrm{w} \end{bmatrix}$$

lifting the constant T-p assumptions: parcel model

vertically displaced (velocity w, hydrostatic background) adiabatic parcel:
(q: mixing ratio, p_d: bgnd pressure, ρ_d bgnd density, g,l_v,c_pd: constants)

$$\begin{bmatrix} \dot{p}_d \\ \dot{T} \\ \dot{r}_w \end{bmatrix} = \begin{bmatrix} -\rho_\mathrm{d} g w \\ (\dot{p}_\mathrm{d}/\rho_\mathrm{d} - \dot{q} l_\mathrm{v})/c_\mathrm{pd} \\ (D_\mathrm{eff}/\rho_\mathrm{w})(\rho_\mathrm{v} - \rho_\circ)/r_\mathrm{w} \end{bmatrix}$$

- $w \longrightarrow 0$ (and hence $\dot{p}_\mathrm{d} \approx 0$) i.e., slow, close-to-equilibrium evolution of the system relevant to fixed-point analysis (by some means pertinent to formation of non-convective clouds such as fog)

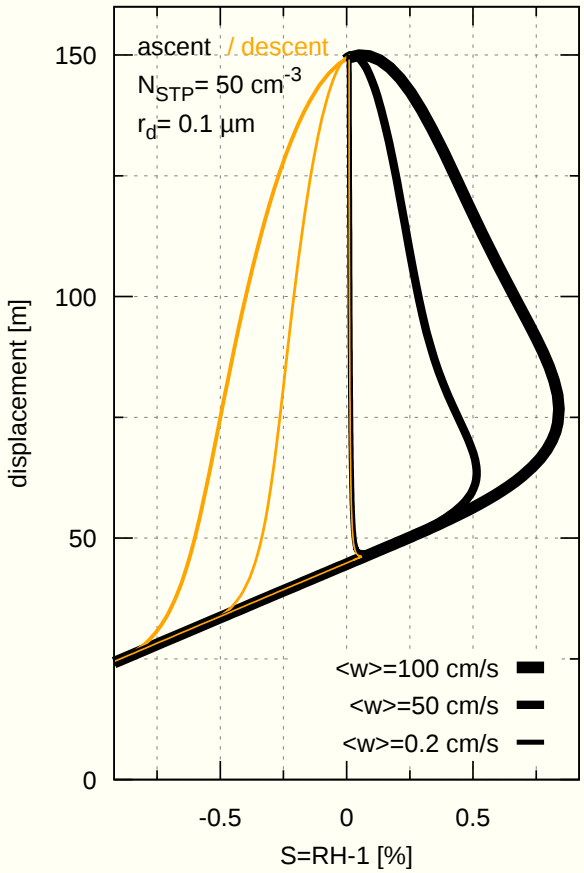
lifting the constant T-p assumptions: parcel model

vertically displaced (velocity w, hydrostatic background) adiabatic parcel:
(q: mixing ratio, p_d: bgnd pressure, ρ_d bgnd density, g,l_v,c_pd: constants)

$$\begin{bmatrix} \dot{p}_d \\ \dot{T} \\ \dot{r}_w \end{bmatrix} = \begin{bmatrix} -\rho_\mathrm{d} g w \\ (\dot{p}_\mathrm{d}/\rho_\mathrm{d} - \dot{q} l_\mathrm{v})/c_\mathrm{pd} \\ (D_\mathrm{eff}/\rho_\mathrm{w})(\rho_\mathrm{v} - \rho_\circ)/r_\mathrm{w} \end{bmatrix}$$

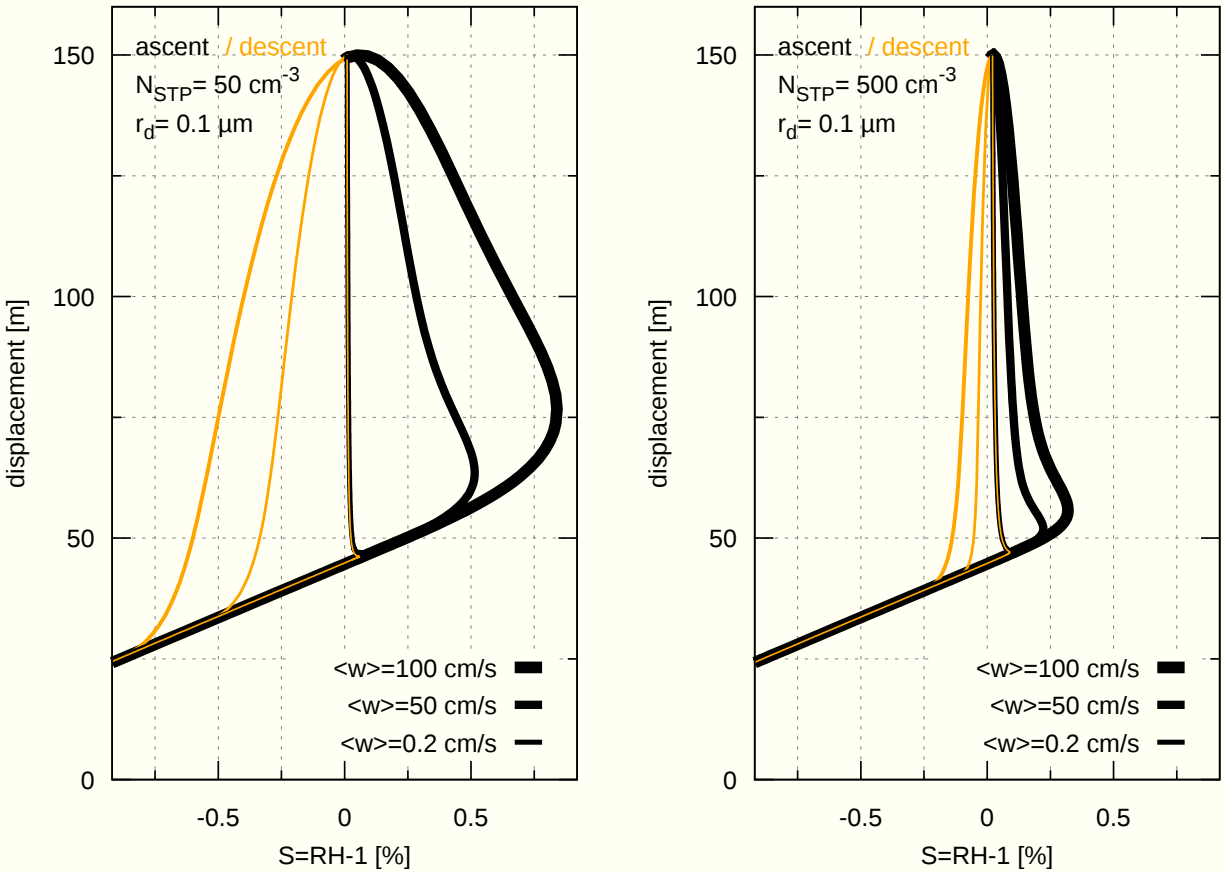
- $w \longrightarrow 0$ (and hence $\dot{p}_\mathrm{d} \approx 0$) i.e., slow, close-to-equilibrium evolution of the system relevant to fixed-point analysis (by some means pertinent to formation of non-convective clouds such as fog)
- $N \longrightarrow 0$ (and hence $\dot{q} \approx 0$) i.e., weak coupling between particle size evolution and ambient thermodynamics (pertinent to the case of low particle concentration).

parcel model: numerical integration (sinusoidal w)



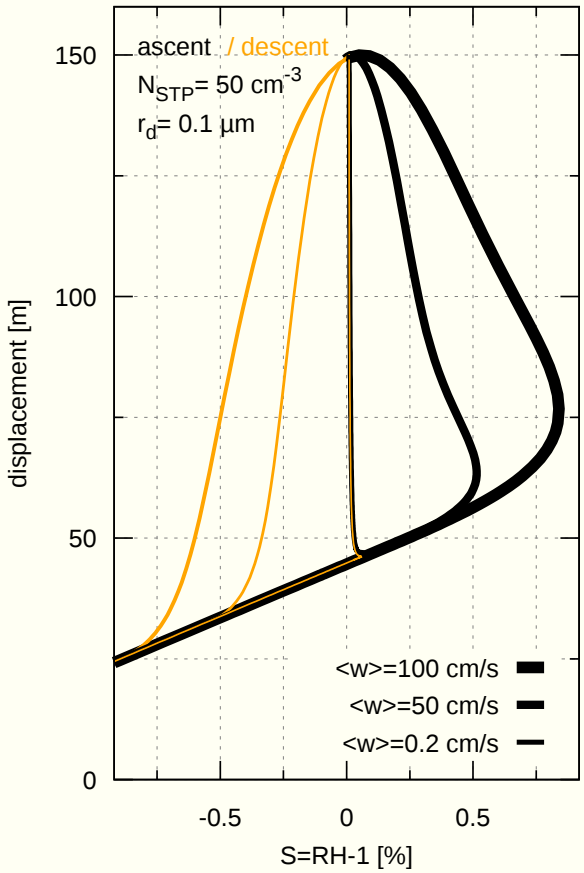
integration using CVODE adaptive solver
open source code (based on libcloudph++) as electronic paper supplement

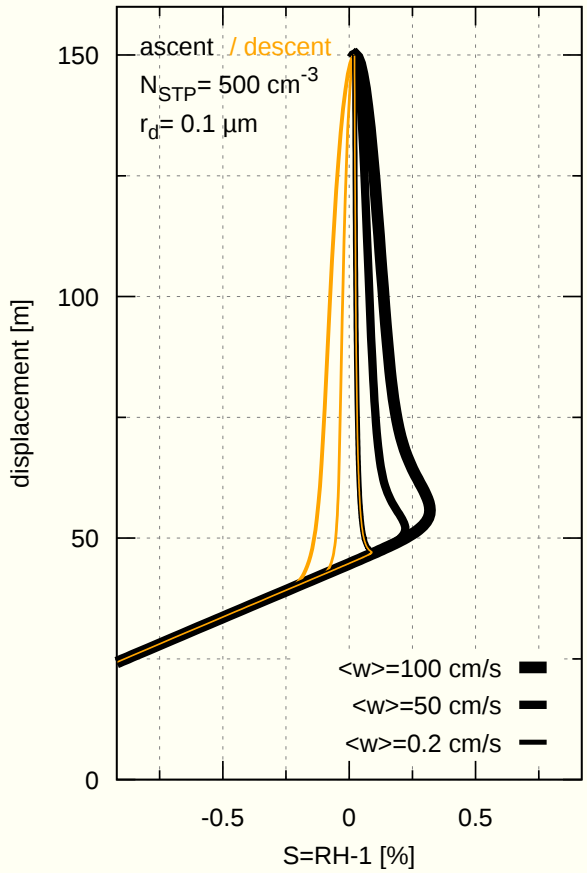
parcel model: numerical integration (sinusoidal w)

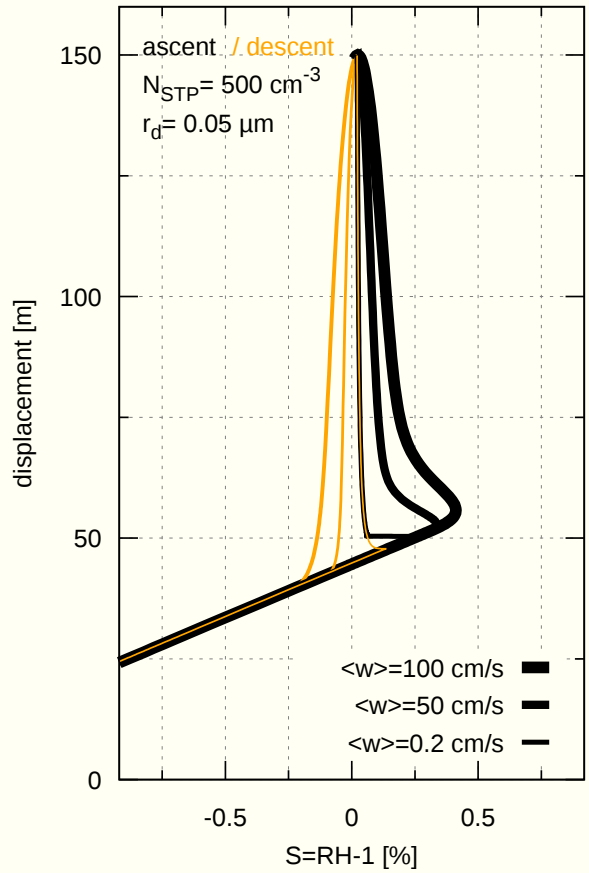


integration using CVODE adaptive solver
open source code (based on libcloudph++) as electronic paper supplement

parcel model: numerical integration (sinusoidal w)



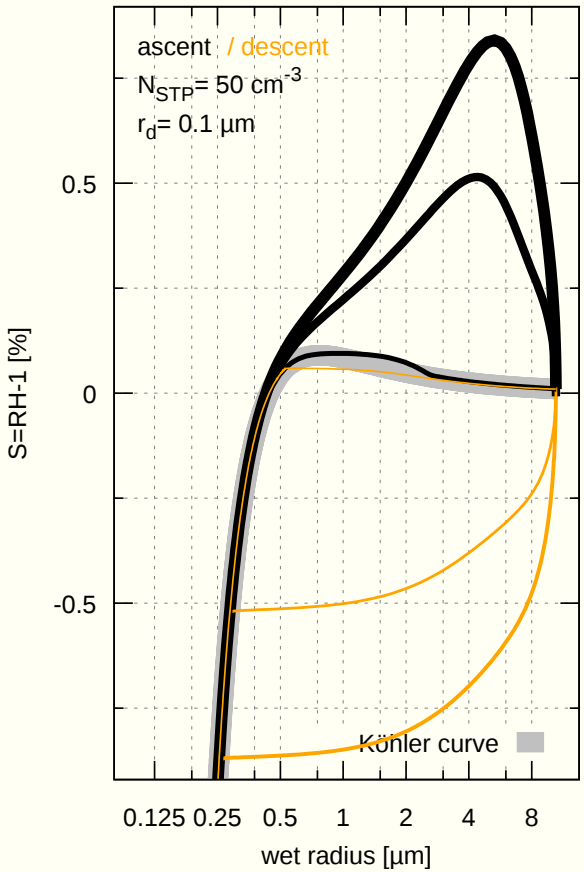




integration using CVODE adaptive solver

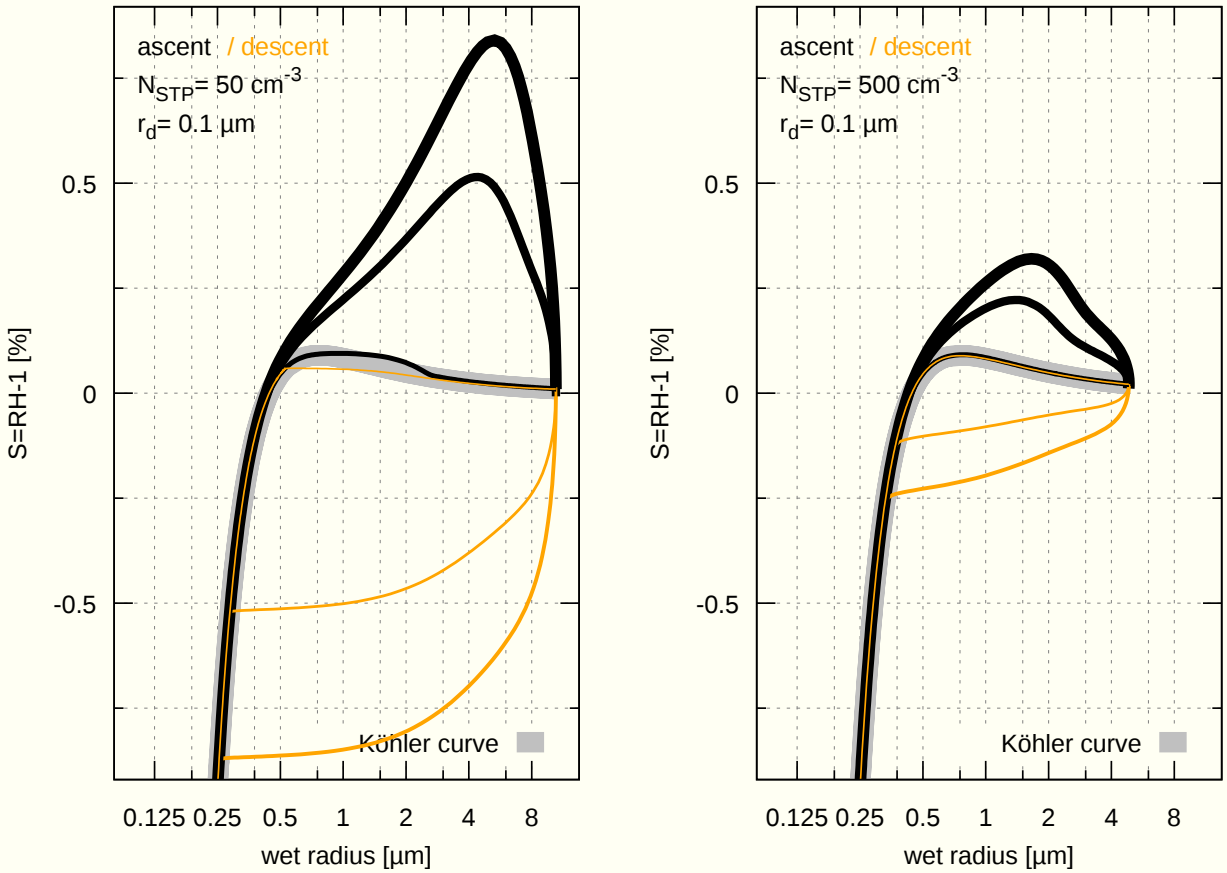
open source code (based on libcloudph++) as electronic paper supplement

parcel model: numerical integration (sinusoidal w)



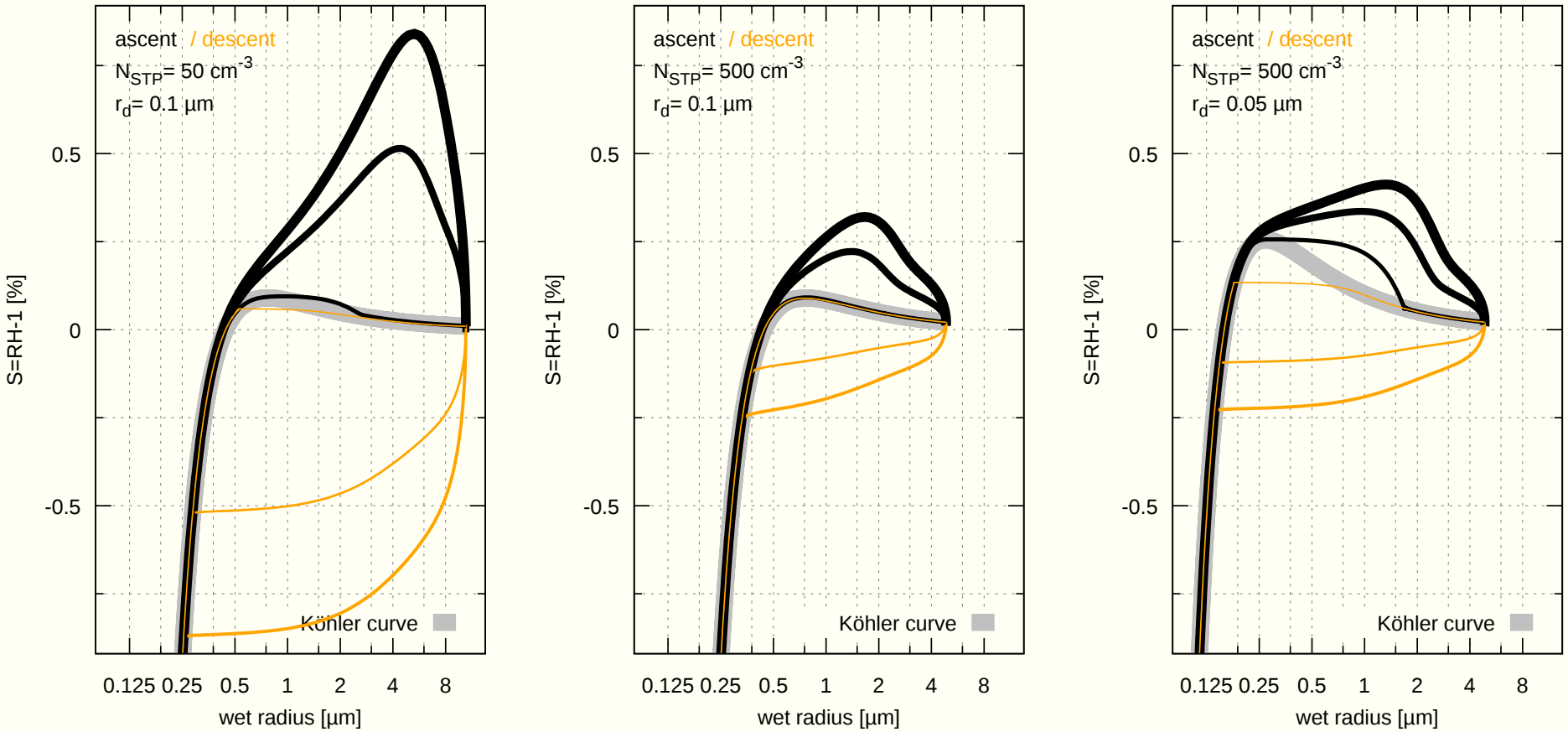
integration using CVODE adaptive solver
open source code (based on libcloudph++) as electronic paper supplement

parcel model: numerical integration (sinusoidal w)



integration using CVODE adaptive solver
open source code (based on libcloudph++) as electronic paper supplement

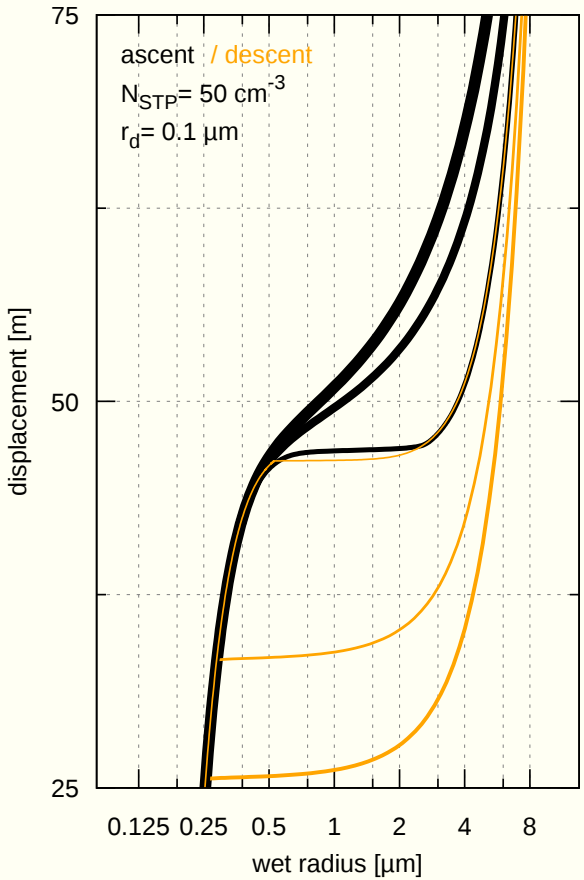
parcel model: numerical integration (sinusoidal w)



integration using CVODE adaptive solver

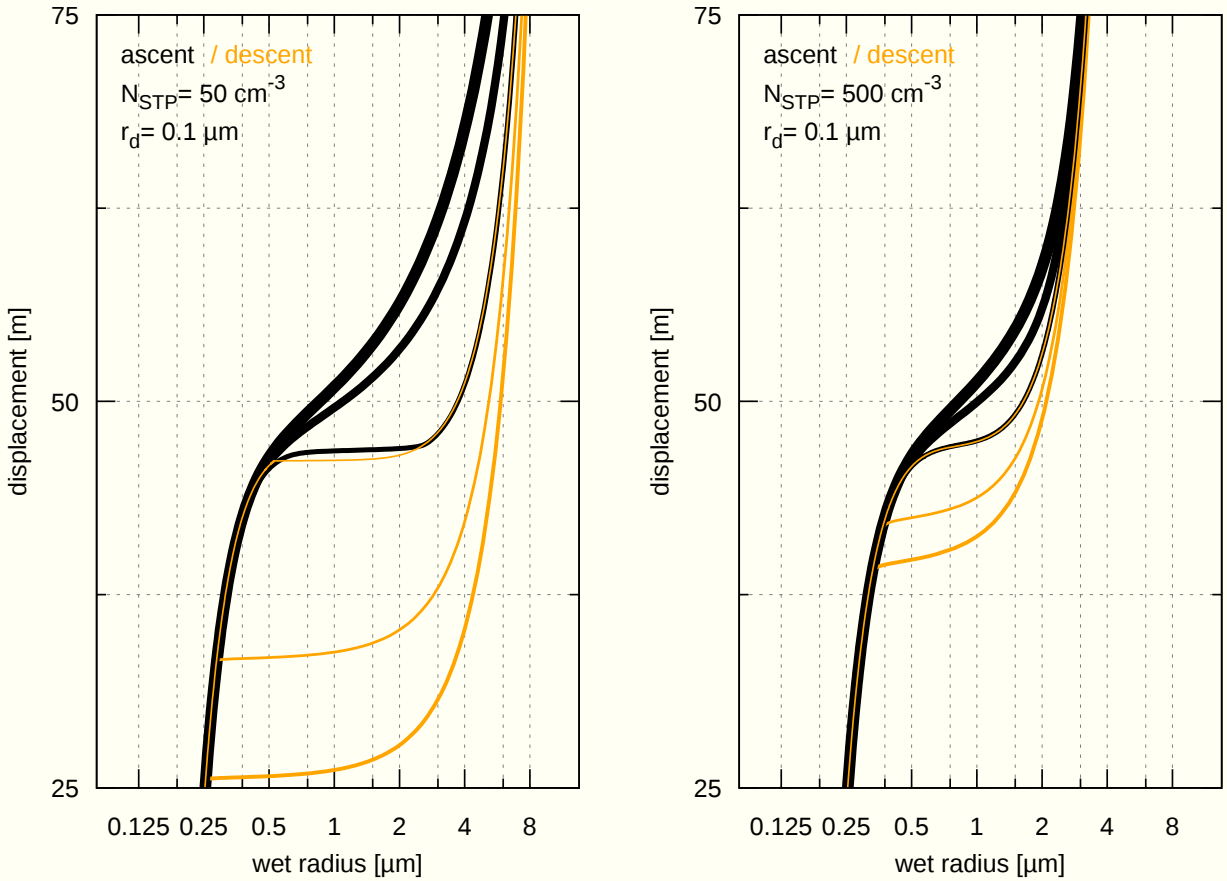
open source code (based on libcloudph++) as electronic paper supplement

parcel model: numerical integration (sinusoidal w)



integration using CVODE adaptive solver
open source code (based on libcloudph++) as electronic paper supplement

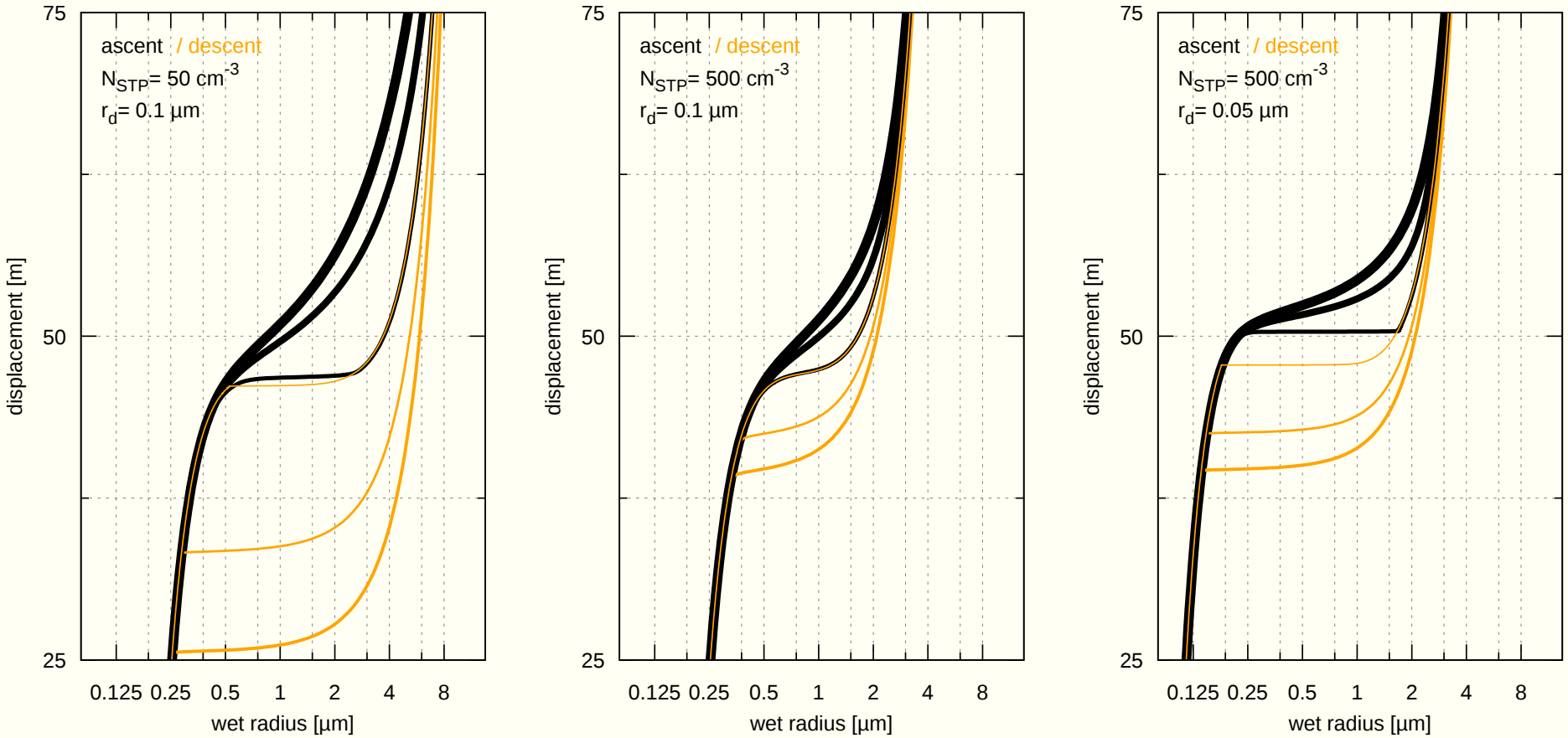
parcel model: numerical integration (sinusoidal w)



integration using CVODE adaptive solver

open source code (based on libcloudph++) as electronic paper supplement

parcel model: numerical integration (sinusoidal w)



integration using CVODE adaptive solver
open source code (based on libcloudph++) as electronic paper supplement