Supercooling super-droplets:
on particle-based modelling of immersion freezing

S. Arabas[1,2], J.H. Curtis[1], I. Silber[3], A. Fridlind[4], D.A Knopf[5], M. West[1] & N. Riemer[1]

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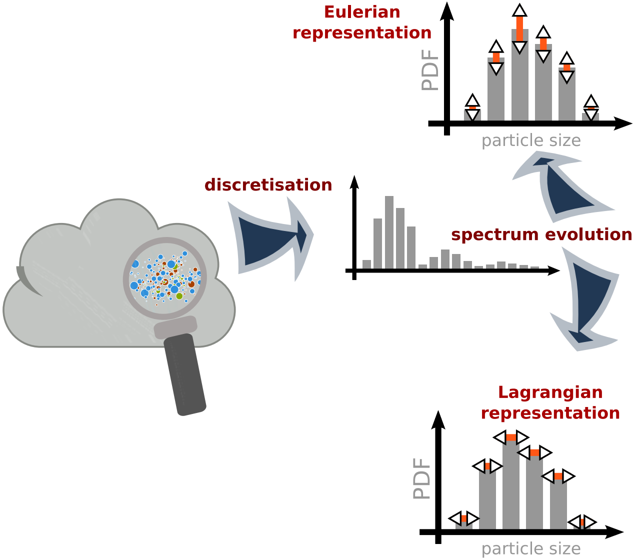
funding:
ASR
Atmospheric System Research

4th Intl Workshop on Cloud Turbulence, NITech, Nagoya (virtual), Mar 11 2022

modelling cloud μ-physics: Eulerian vs. Lagrangian approaches



Eulerian representation

discretisation

spectrum evolution

Lagrangian representation

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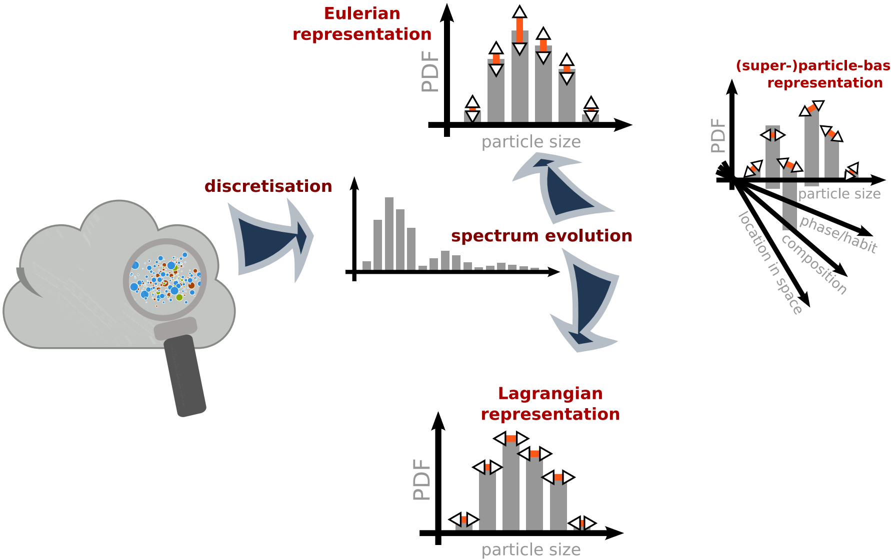

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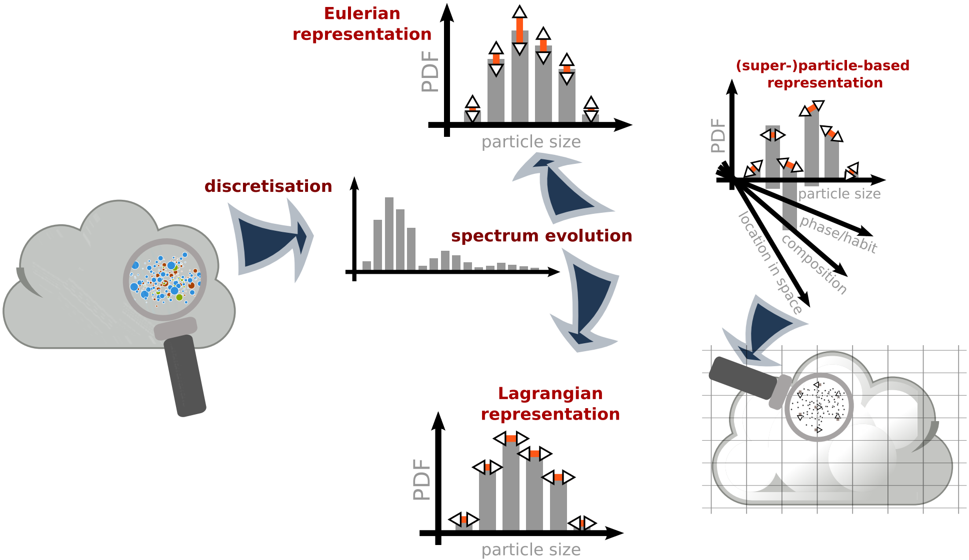

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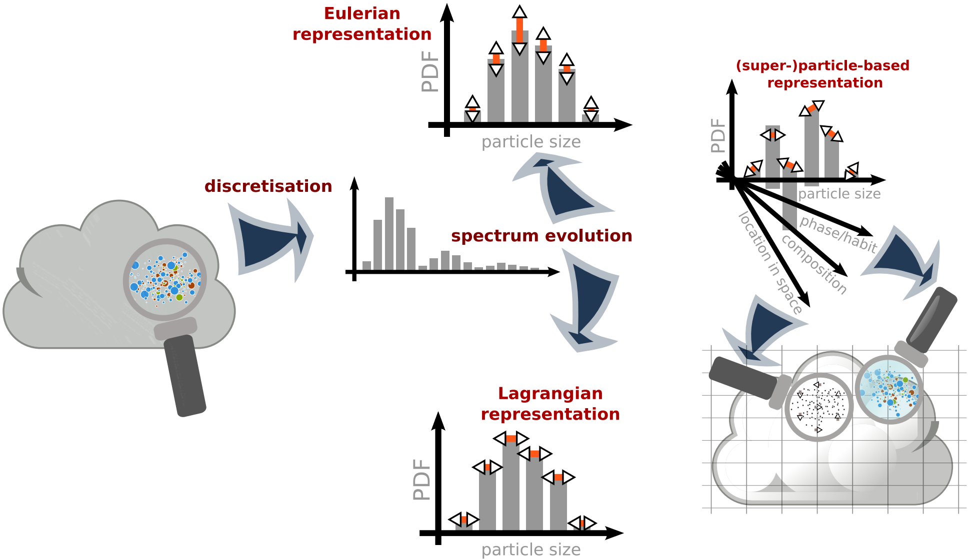

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background and agenda for this talk

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emergence of mixed-phase particle-based μ-physics models

(Shima et al.; McSnow by Brdar, Siewert, Seifert et al.; Sölch, Kärcher, Unterstrasser et al. @DLR)

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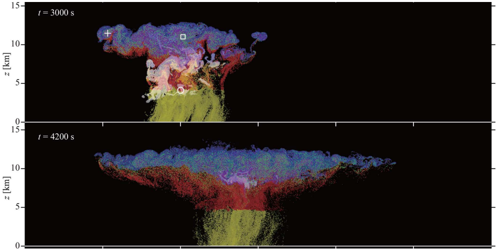

Figure 1. Typical realization of CTRL cloud spatial structures at $t = 2040$, 2460, 3000, 4200, and 5400 s. The mixing ratio of cloud water, rainwater, cloud ice, graupel, and snow aggregates are plotted in fading white, yellow, blue, red, and green, respectively. The symbols indicate examples of unrealistic predicted ice particles (Sects. 7.3 and 9.1). See also Movie 1 in the video supplement.

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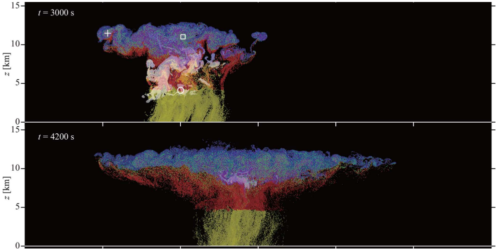

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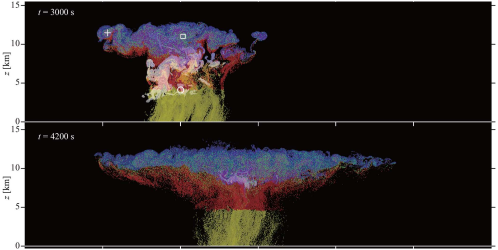

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foci of this talk

particle-based immersion freezing:

- monodisperse vs. polydisperse INP
- singular (INAS) vs. time-dependent

Marshall et al. 1961 (Nubila 4(1))

Heterogeneous Nucleations is a Stochastic Process

by

J. S. MARSHALL

McGill University, Montreal, Canad.

Presented at the International Congress on the Physics of Clouds (Hailstorms) at Verona 9-13 August 1960.

`http://cma.entecra.it/Astro2_sito/doc/Nubila_1_1961.pdf`

Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

theory (in modern notation)
(Bigg ’53, Langham & Mason ’58, Carte ’59, Marshall ’61)

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$$P^*(\text{k events in time t}) = \frac{(rt)^k \exp(-rt)}{k!}$$

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experimental $n_s(T)$ fits: e.g., Niemand et al. 2012

freezing temperature T_{fz} as a super-particle attribute

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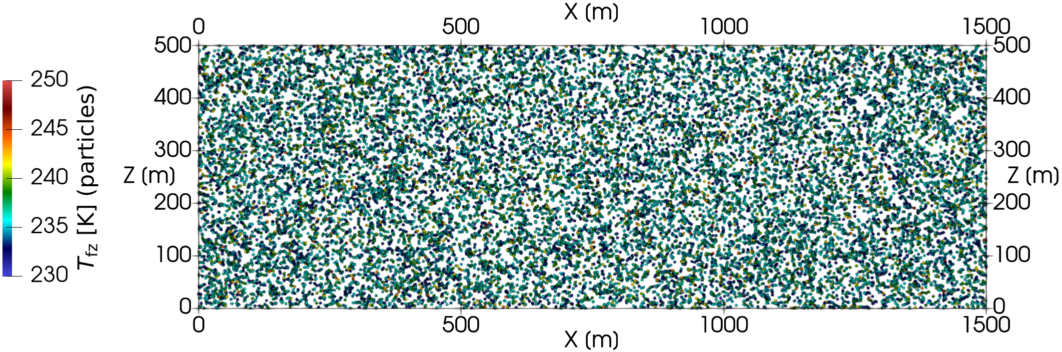

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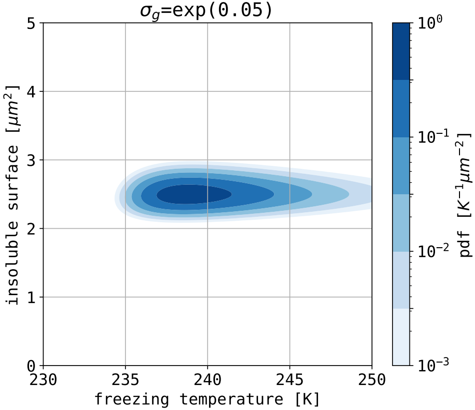

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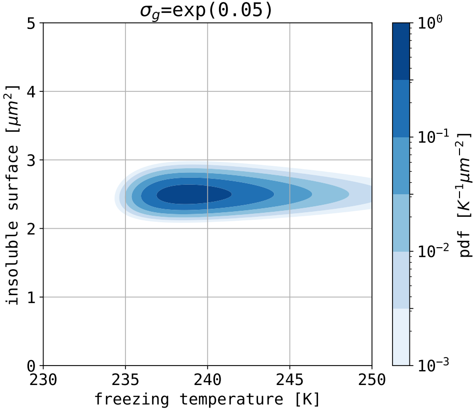

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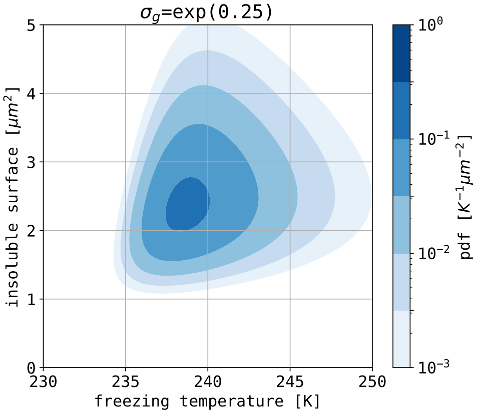

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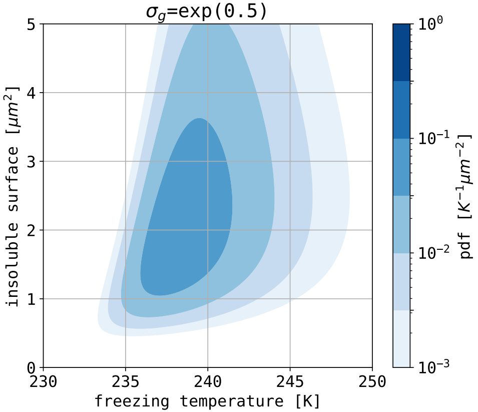

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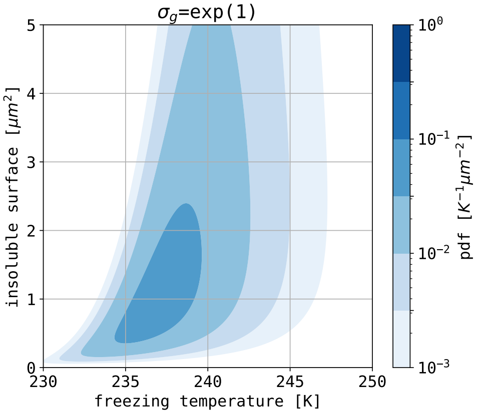

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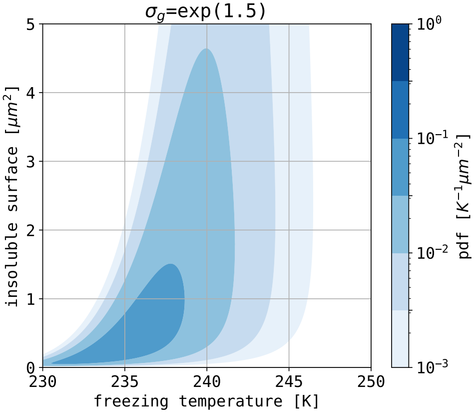

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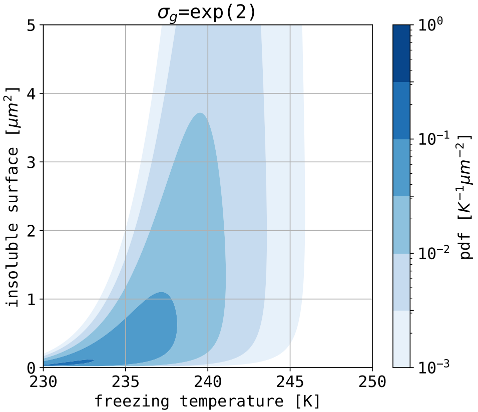

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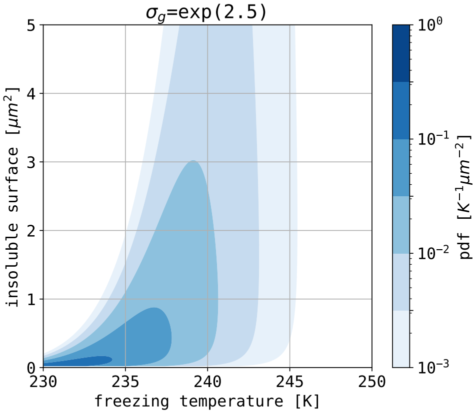

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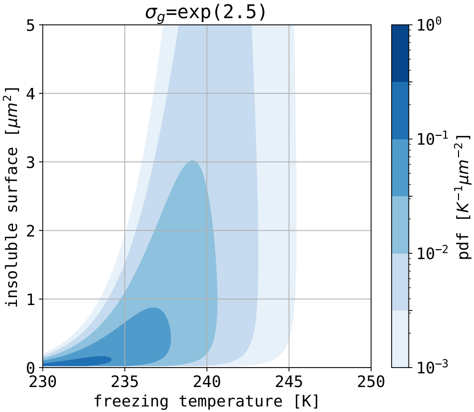
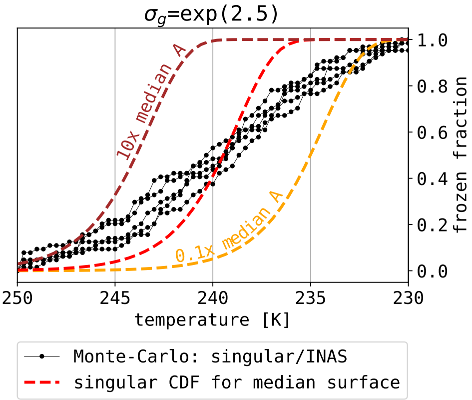


box model (or single grid cell)



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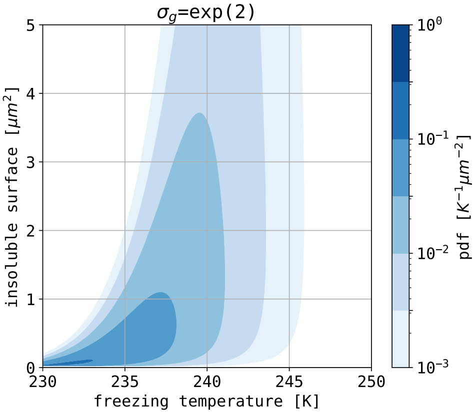

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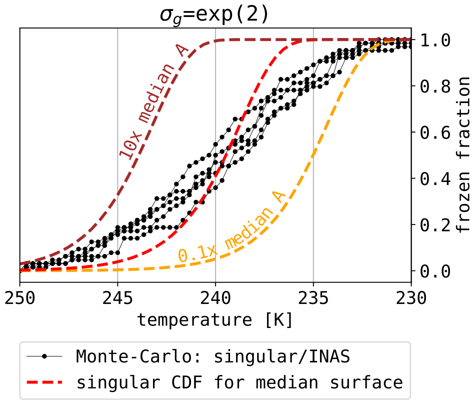

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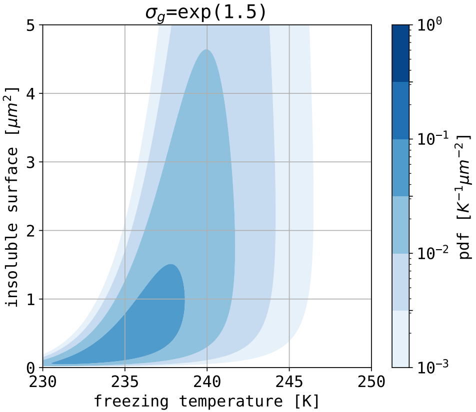

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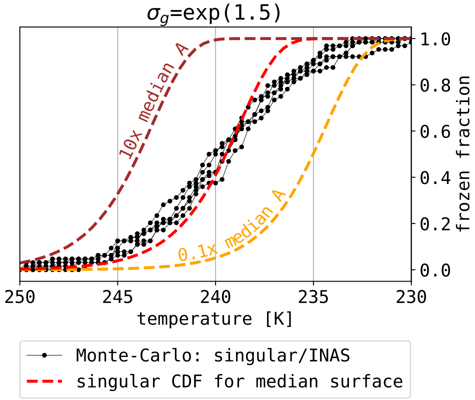

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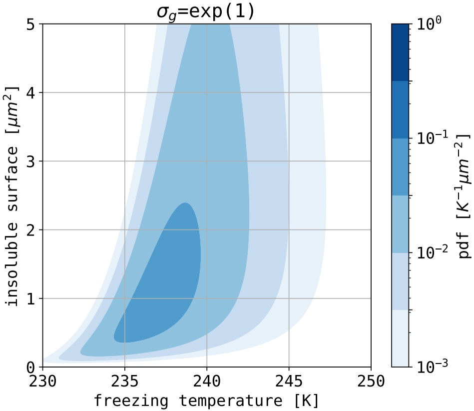

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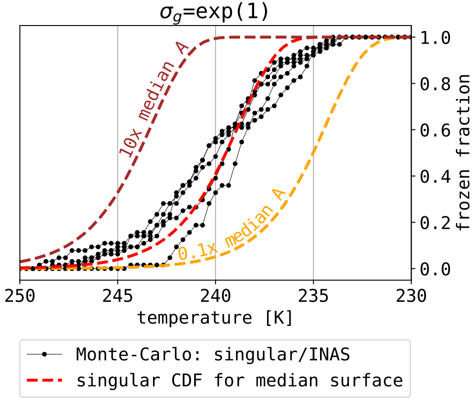

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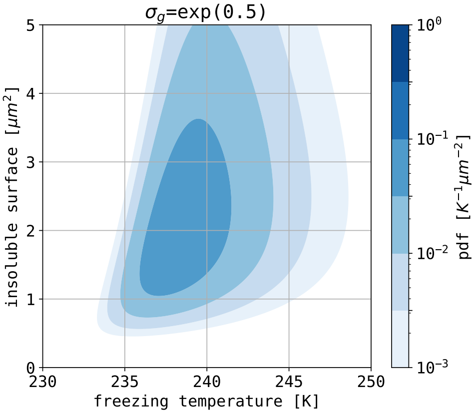

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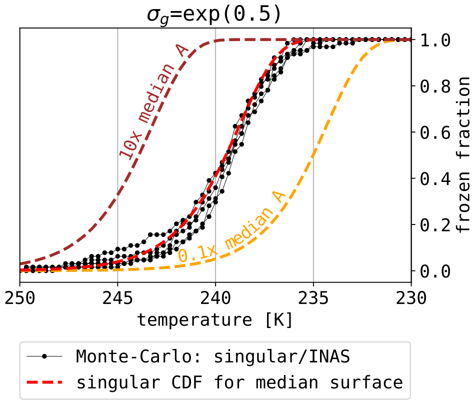

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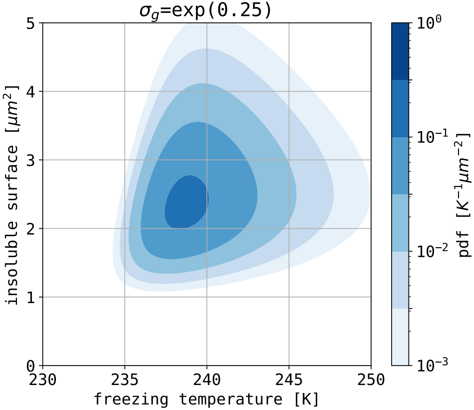

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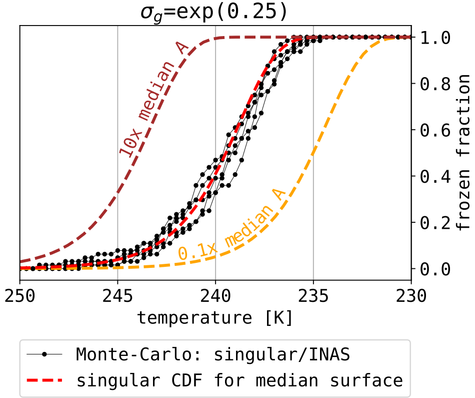

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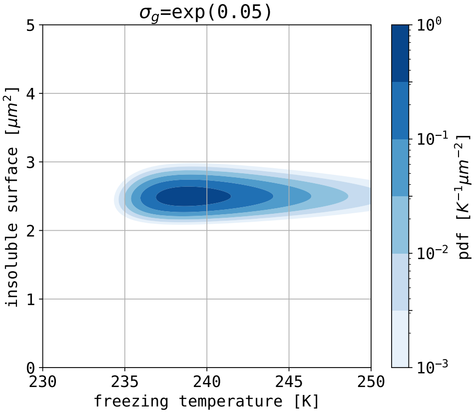

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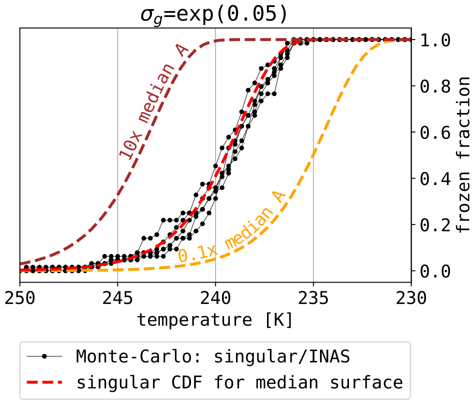

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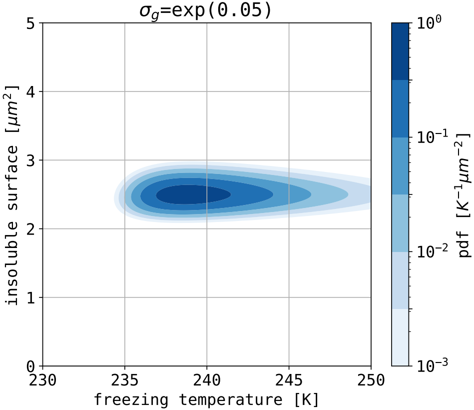

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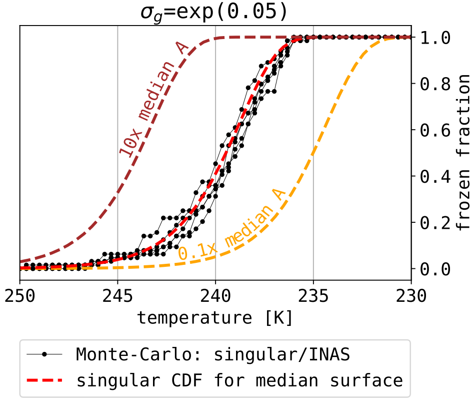


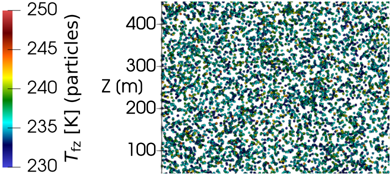
- "singular" particle-based model is capable of representing polydisperse INP
- depicted limitations stemming from monodisperse INP assumption

particle-based freezing: singular (Shima et al.) / time-dependent (this work)

singular: INAS T_{fz} **as attribute**; initialisation by random sampling from $P(T_{\text{fz}}, A)$ with lognormal A (A is not an attribute, initialisation only); freezing if $T(t) < T_{fz}(t=0)$

time-dependent: A **as attribute** (randomly sampled from the same lognormal)
Monte-Carlo freezing trigger using $P(J_{\text{het}}(T(t)))$

singular (INAS)



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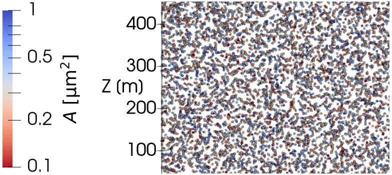

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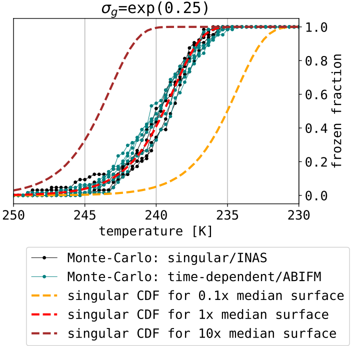

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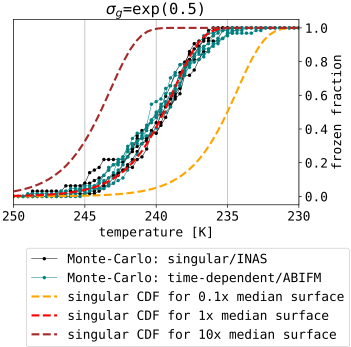

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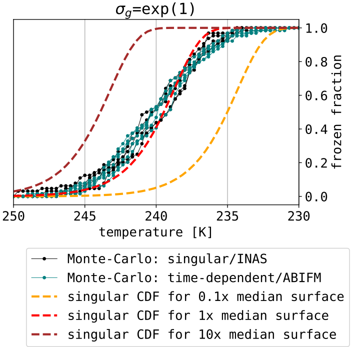

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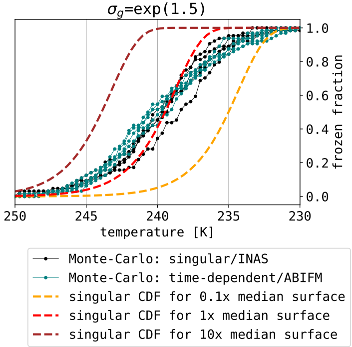

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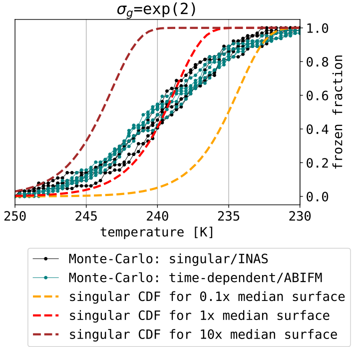

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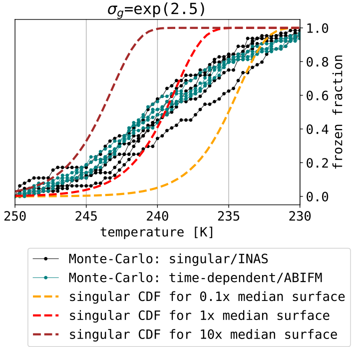

particle-based freezing: singular (Shima et al.) / time-dependent (this work)

singular: INAS T_{fz} **as attribute**; initialisation by random sampling from $P(T_{\mathrm{fz}}, A)$ with lognormal A (A is not an attribute, initialisation only); freezing if $T(t) < T_{fz}(t=0)$

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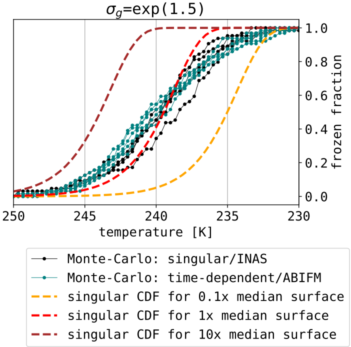
cooling rate: $0.5\ ^{K}/_{min}$


σ_g=exp(1.5)

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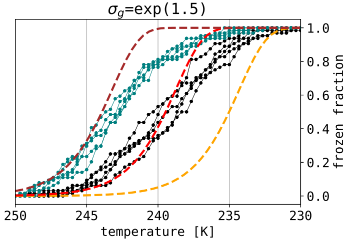
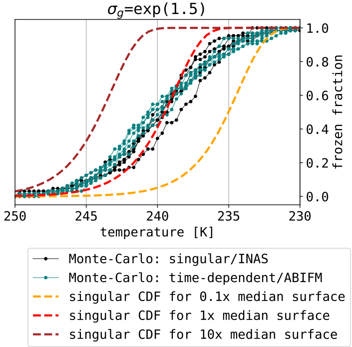
cooling rate: $0.1\ ^K/_{min}$



cooling rate: $0.5\ ^K/_{min}$



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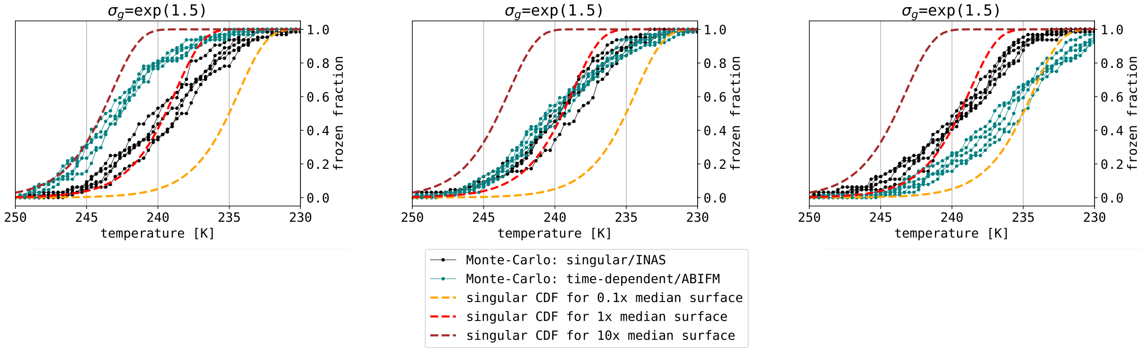
cooling rate: $0.1\ ^K/_{min}$ | cooling rate: $0.5\ ^K/_{min}$ | cooling rate: $2.5\ ^K/_{min}$



Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

theory (in modern notation)
(Bigg '53, Langham & Mason '58, Carte '59, Marshall '61)

Poisson counting process with rate r:

$$P^*(\text{k events in time t}) = \frac{(rt)^k \exp(-rt)}{k!}$$

$$P(\text{one or more events in time t}) = 1 - P^*(k = 0, t)$$

$$\ln(1 - P) = -rt$$

introducing $J_{\text{het}}(T)$, $T(t)$ and INP surface A:

$$\ln\left(1 - P(A, t)\right) = -A \underbrace{\int_0^t J_{\text{het}}(T(t'))\, dt'}_{I(T)}$$

INAS: $I(T) = n_s(T) = \exp(a \cdot (T - T_{0°C}) + b)$

experimental $n_s(T)$ fits: e.g., Niemand et al. 2012

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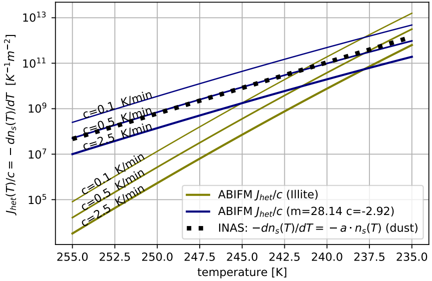

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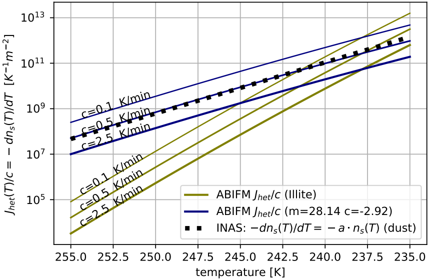

cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11)
but the singular ansatz limitation of sampling T_{fz} at t=0 remains

Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

Is it a problem?

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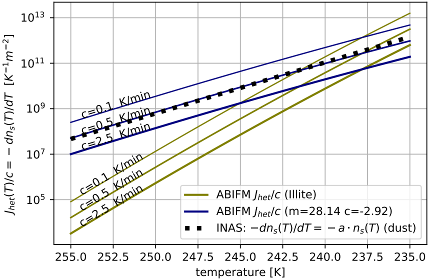

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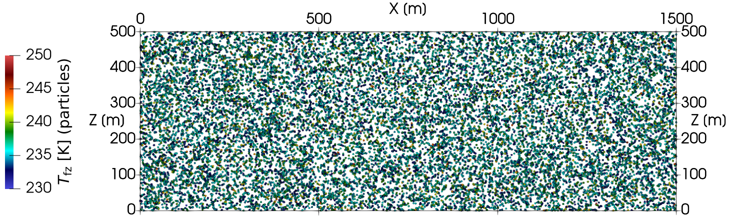
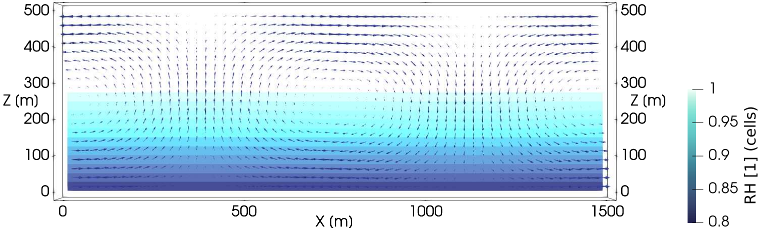
particle-based μ-physics + prescribed-flow test (aka KiD-2D)^{a,b,c,d,e}

Lagrangian component (PySDM)



Eulerian component (PyMPDATA)



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[a] concept: Gedzelman & Arnold '93

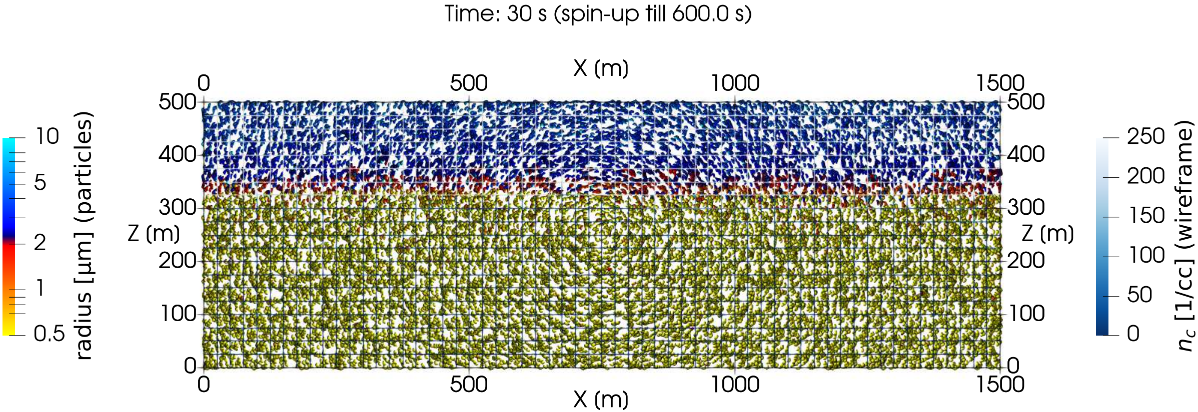
[b] stratiform: Morrison & Grabowski '07

[c] particle-based: Arabas et al. '15

[d] KiD-2D: github.com/BShipway/KiD

[e] here: SHEBA case (Fridlind et al. '12)

particle-based μ-physics + prescribed-flow test

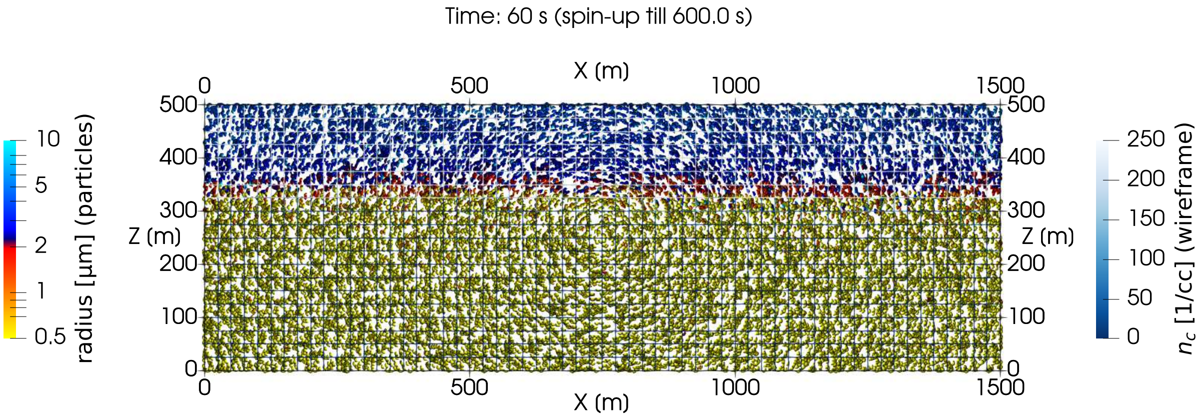


16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\mathrm{aer}} = 300/cc$ (two-mode lognormal) $N_{\mathrm{INP}} = 150/L$ (lognormal, $D_g = 0.74$ μm, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

particle-based μ-physics + prescribed-flow test



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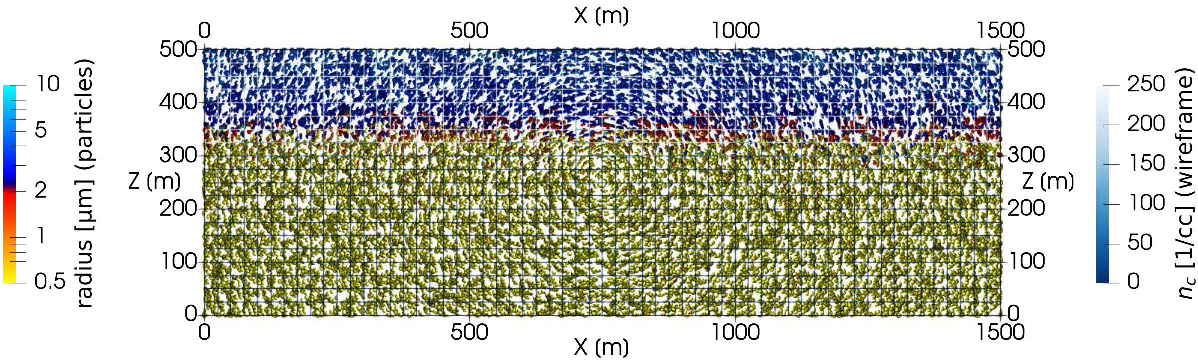
particle-based μ-physics + prescribed-flow test

Time: 90 s (spin-up till 600.0 s)



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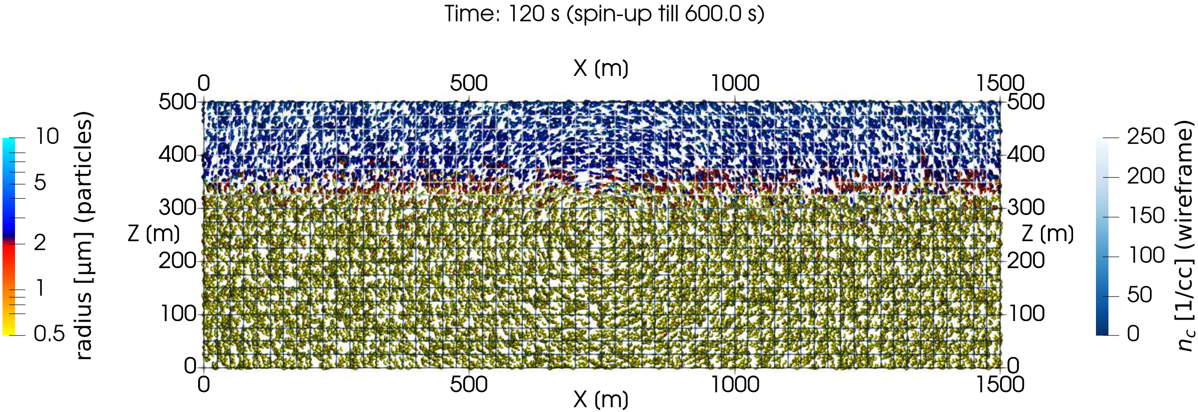

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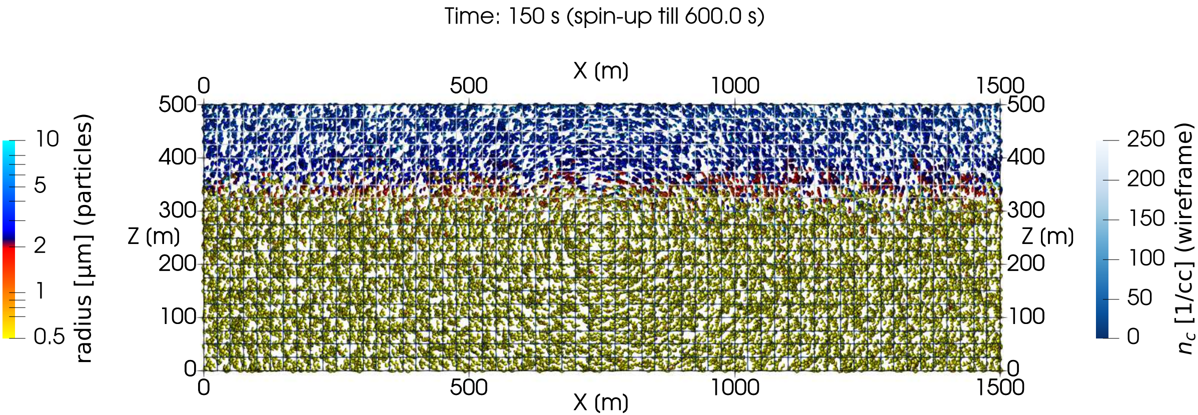

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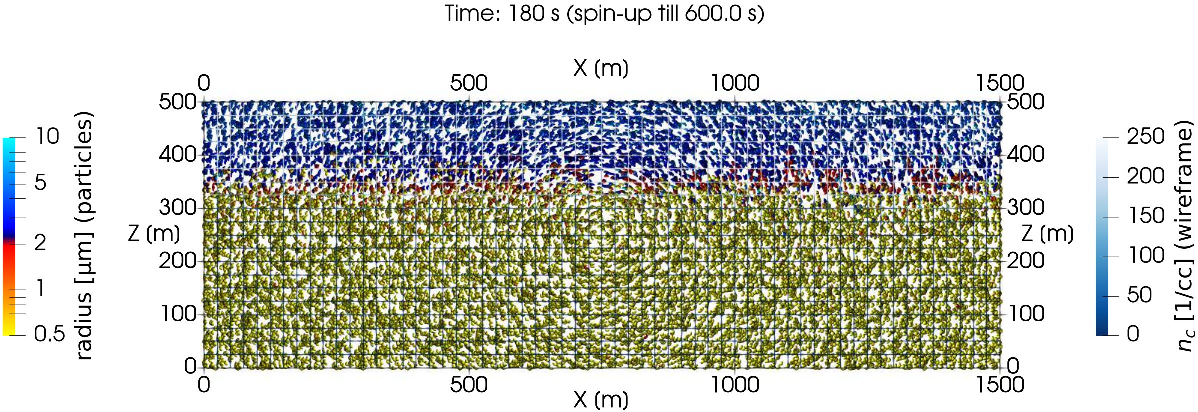

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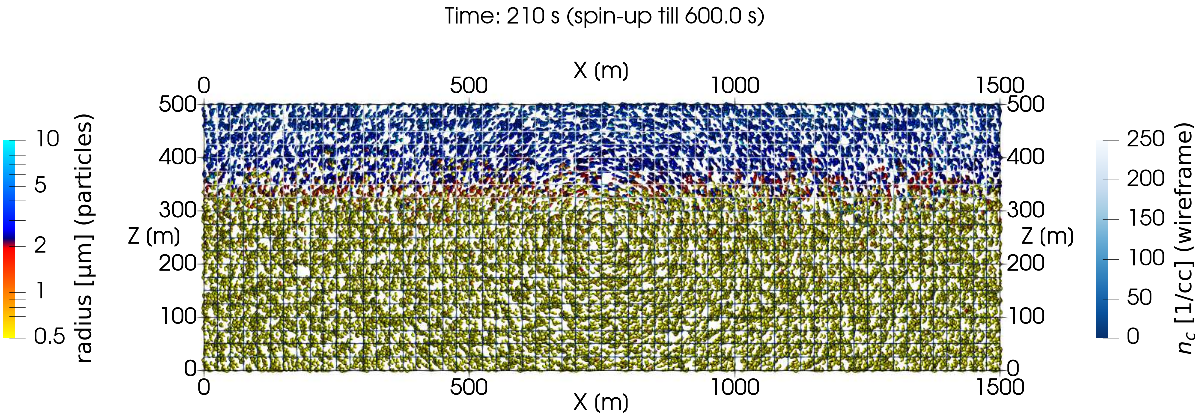

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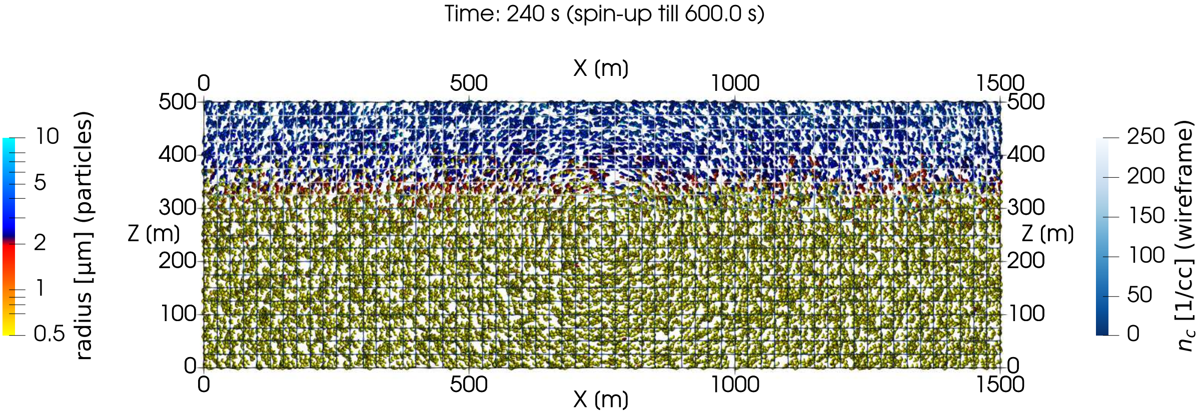

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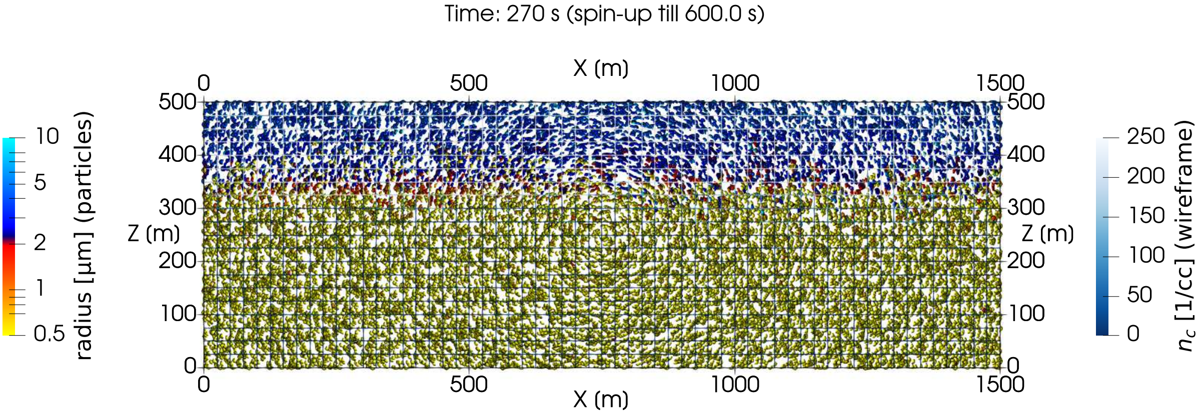

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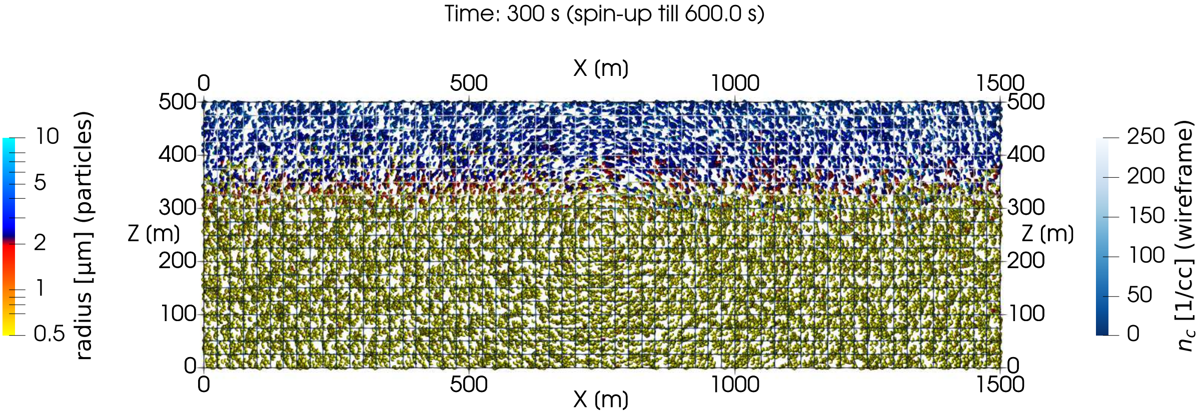

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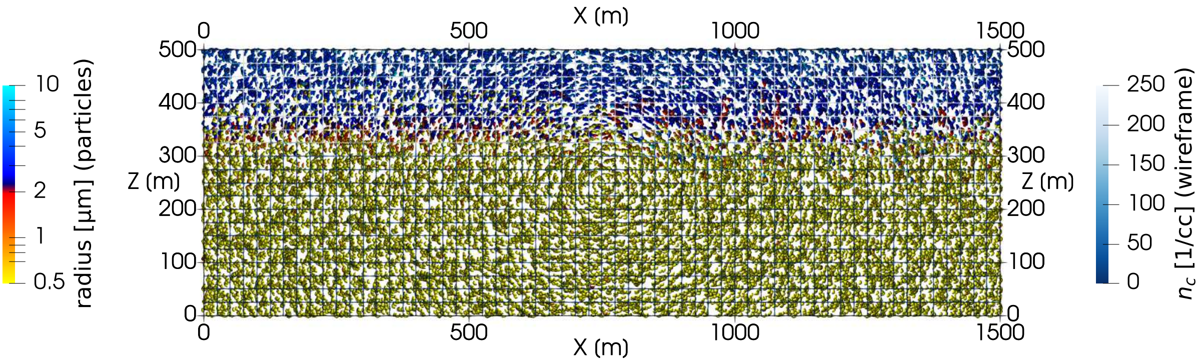
Time: 330 s (spin-up till 600.0 s)



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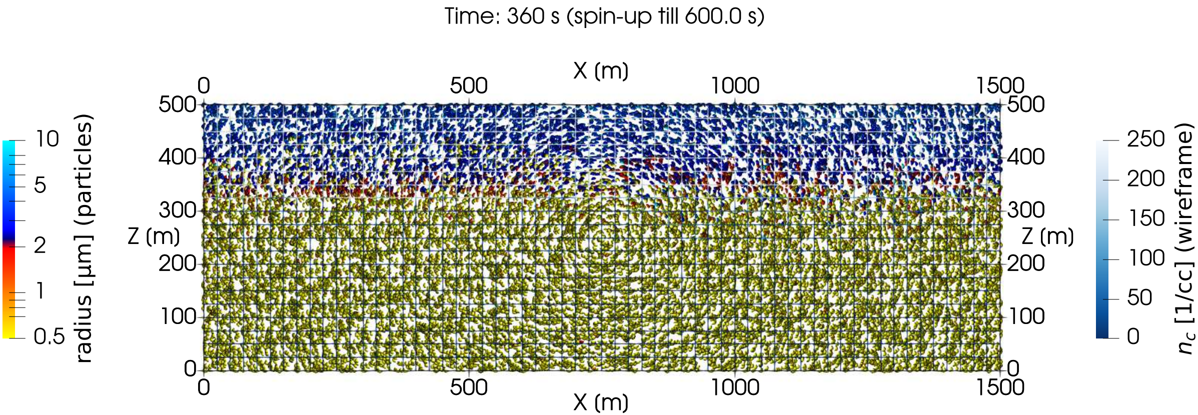

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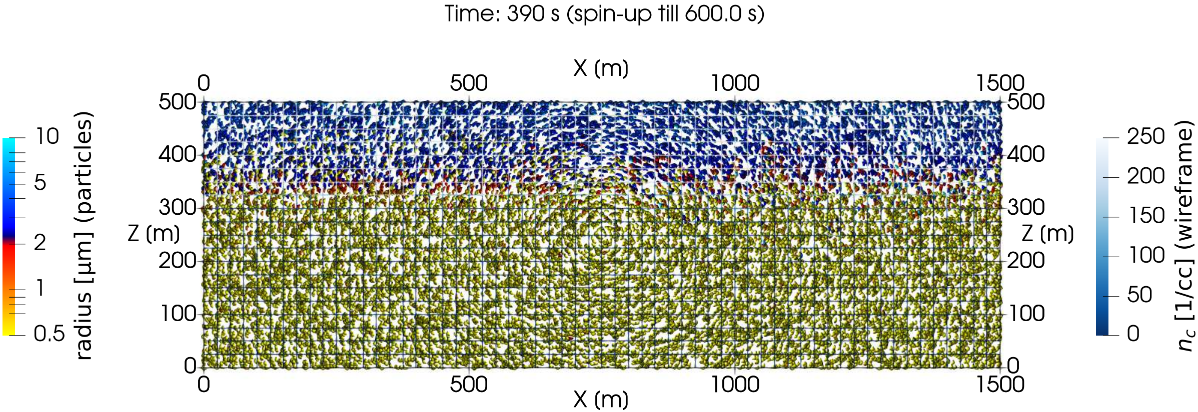

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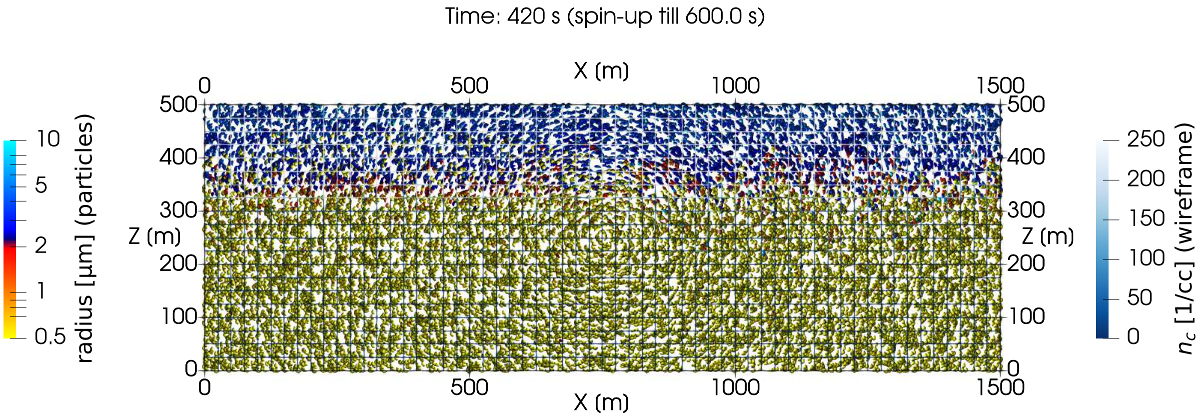

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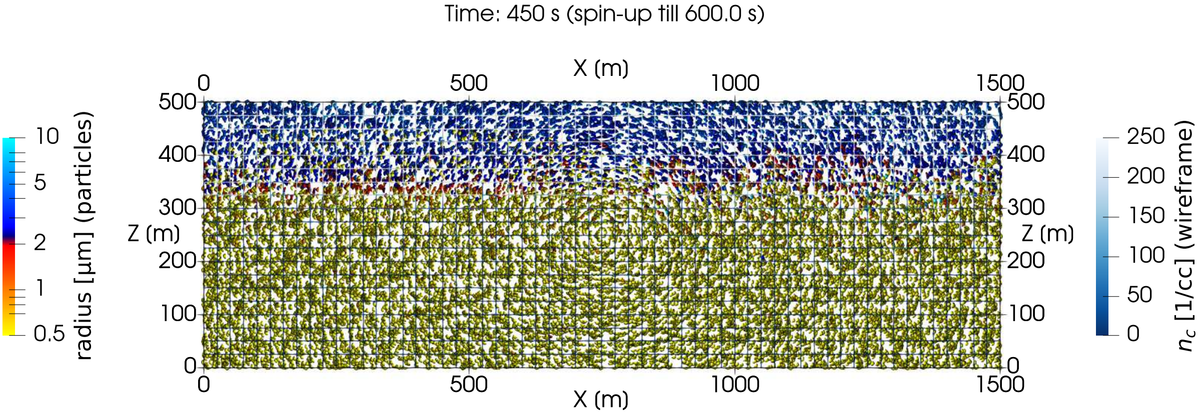

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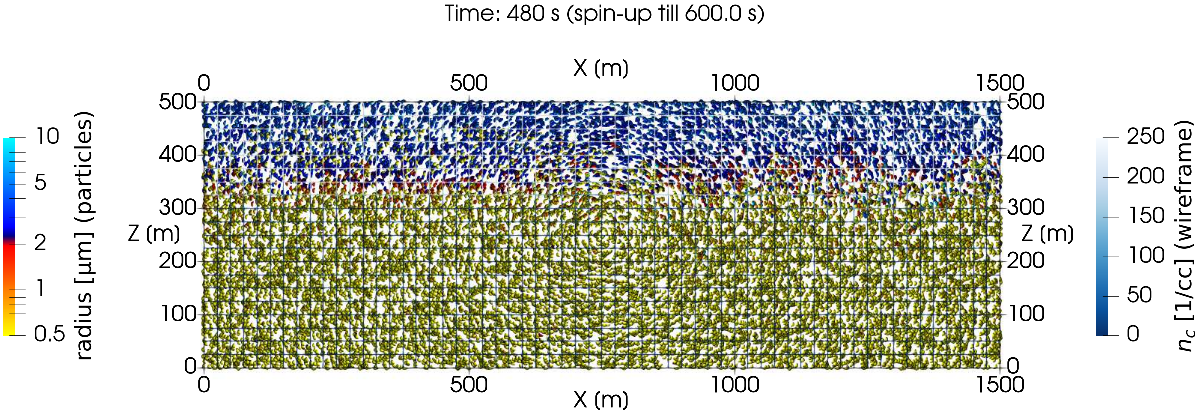

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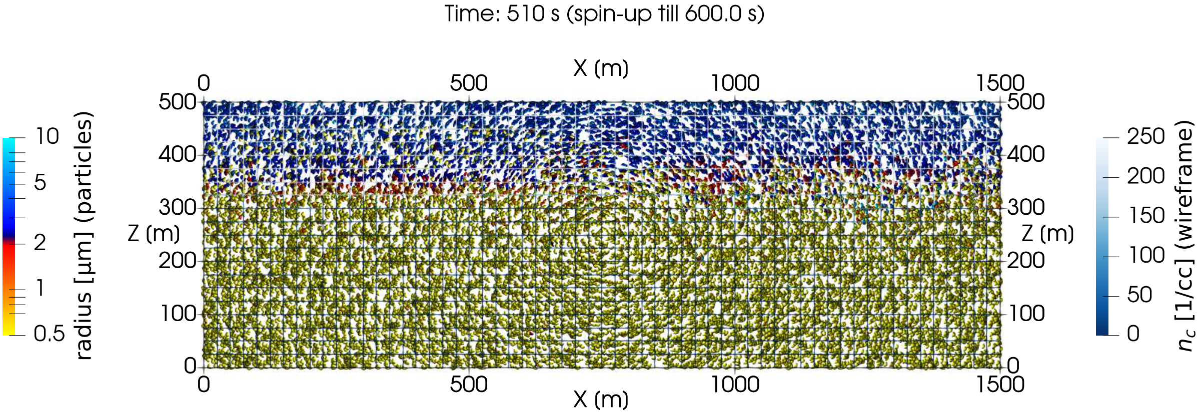

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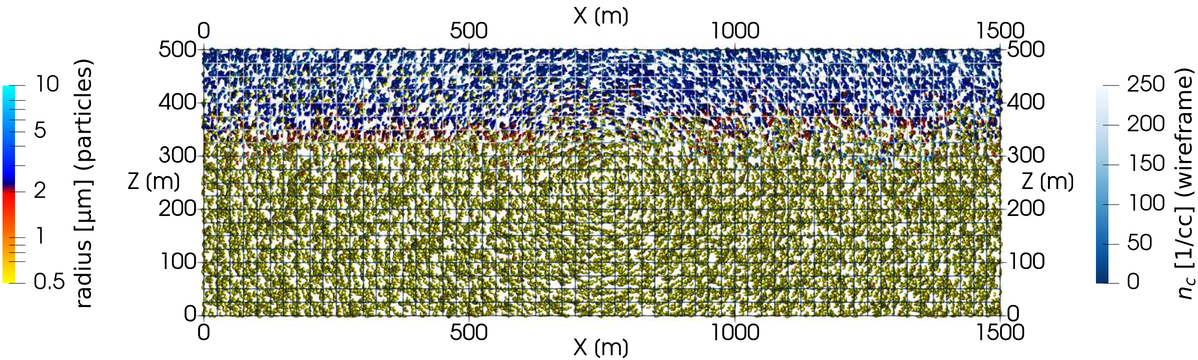
particle-based μ-physics + prescribed-flow test

Time: 540 s (spin-up till 600.0 s)



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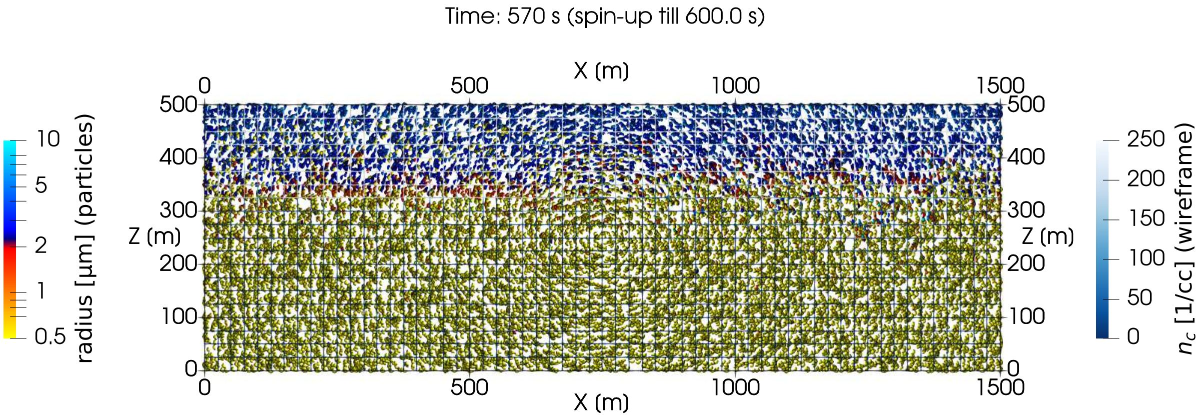

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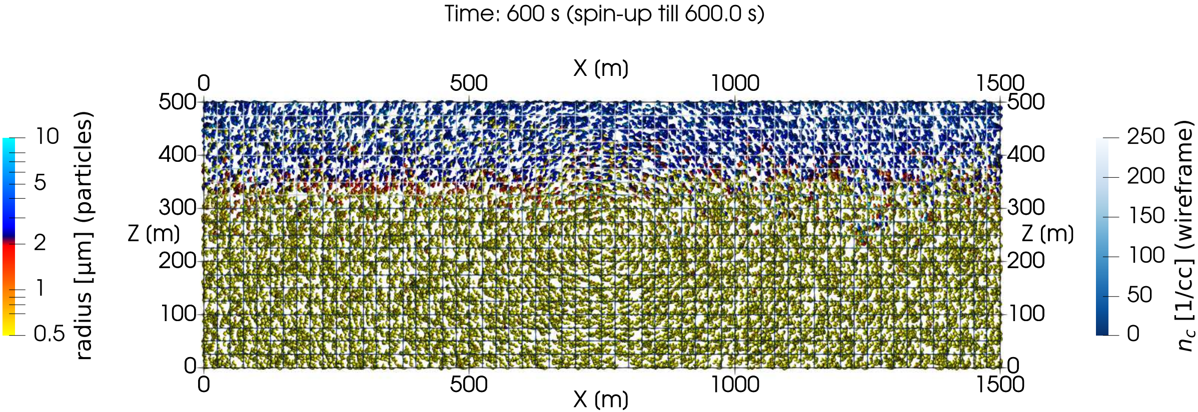

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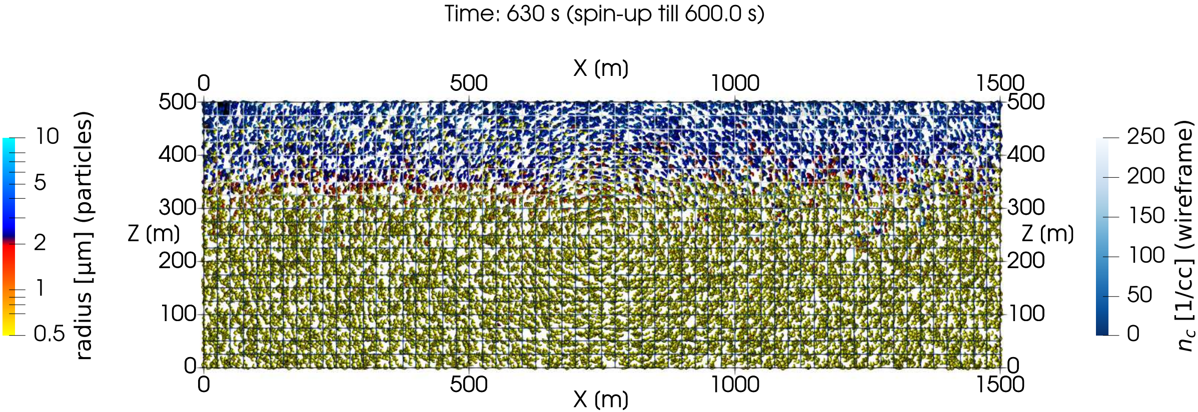
particle-based μ-physics + prescribed-flow test



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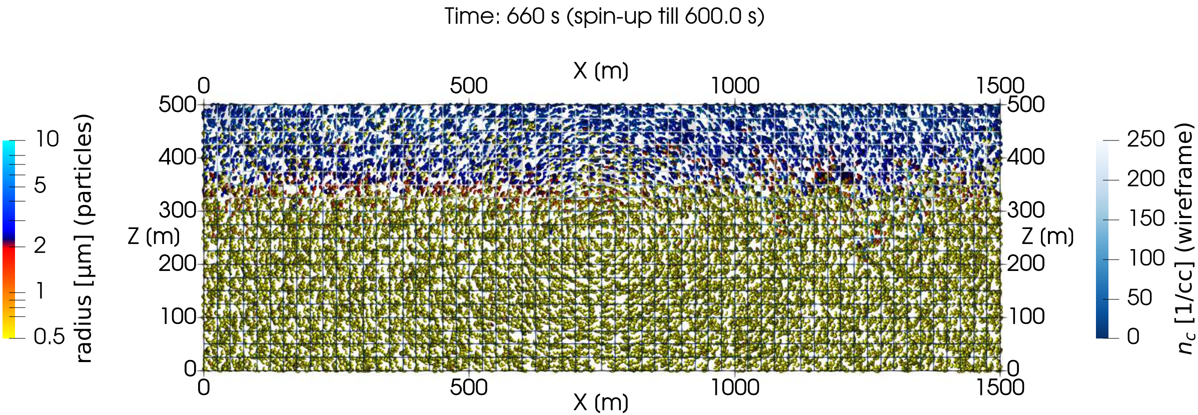

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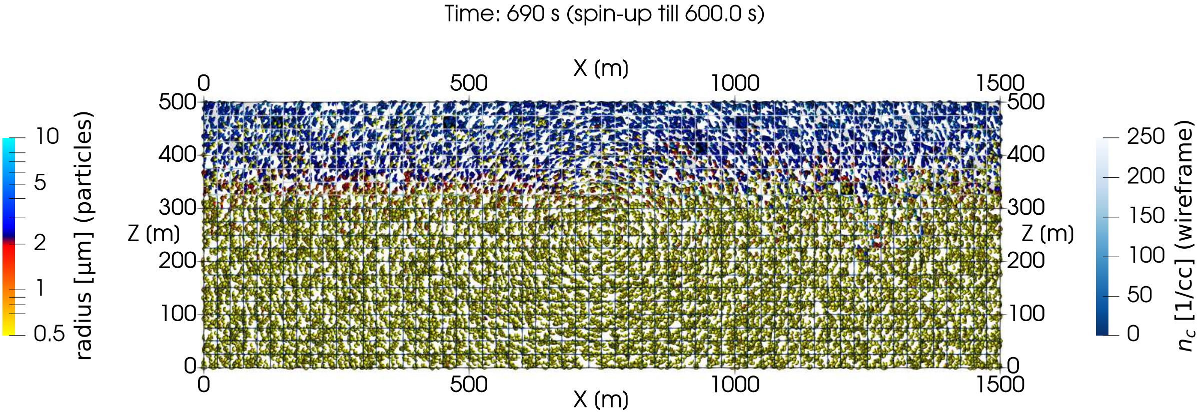

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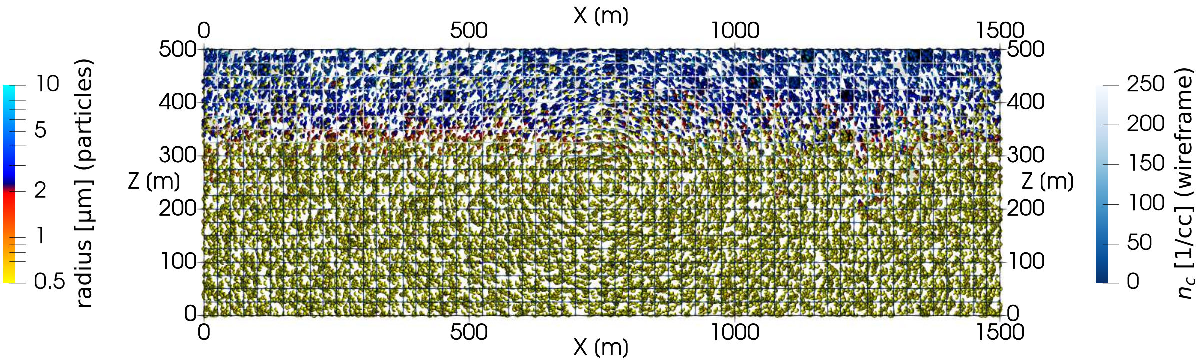
Time: 720 s (spin-up till 600.0 s)



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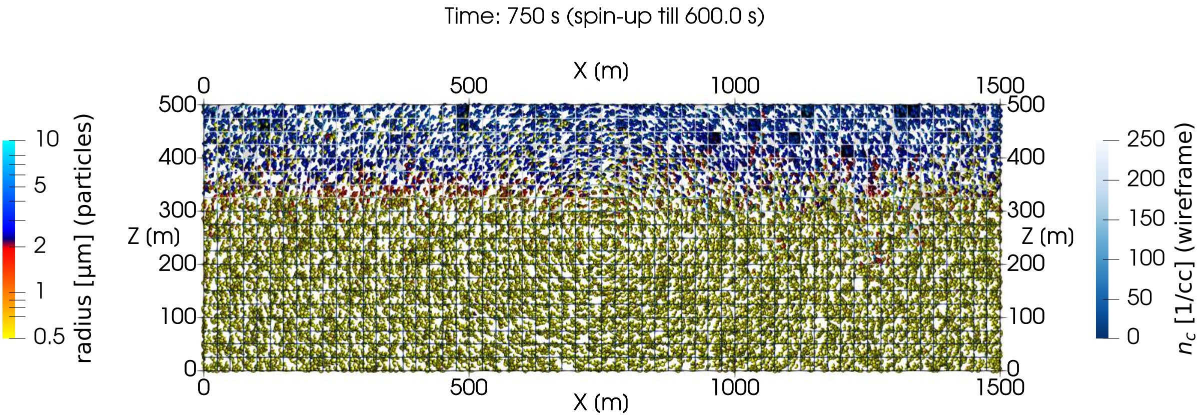

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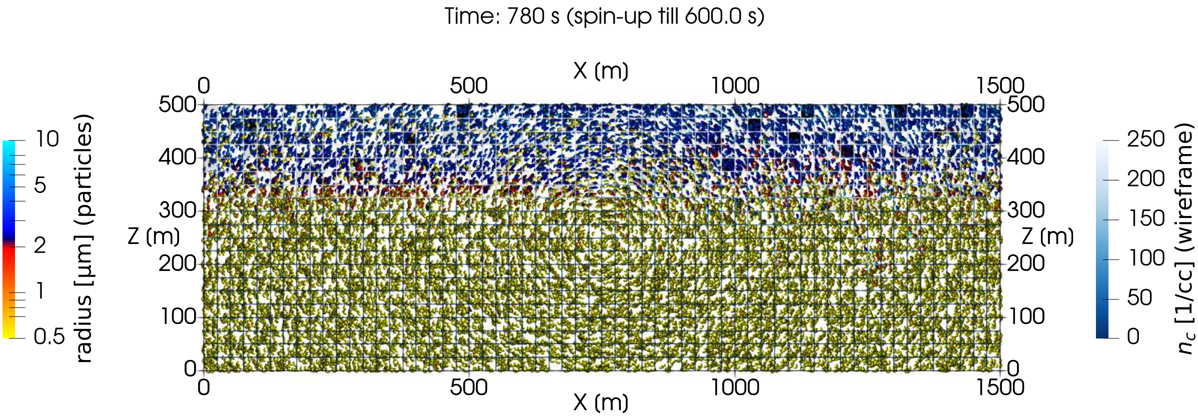

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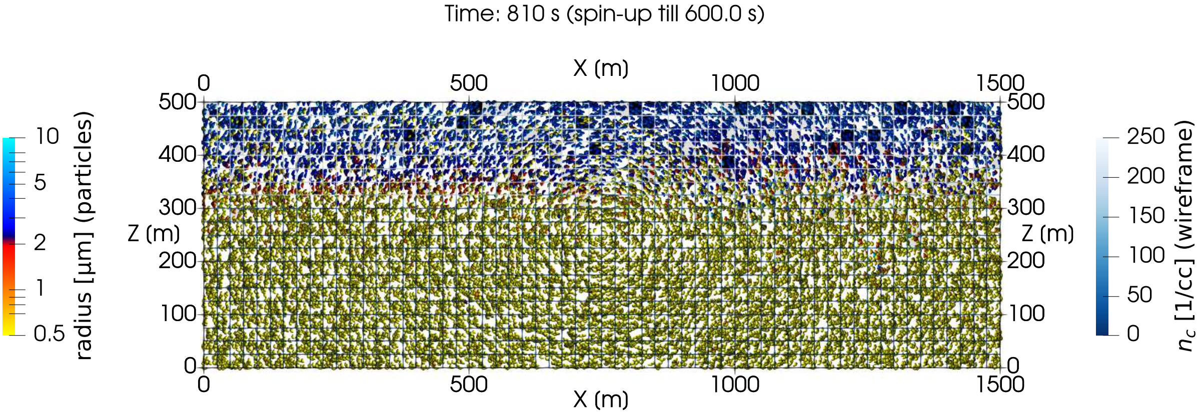

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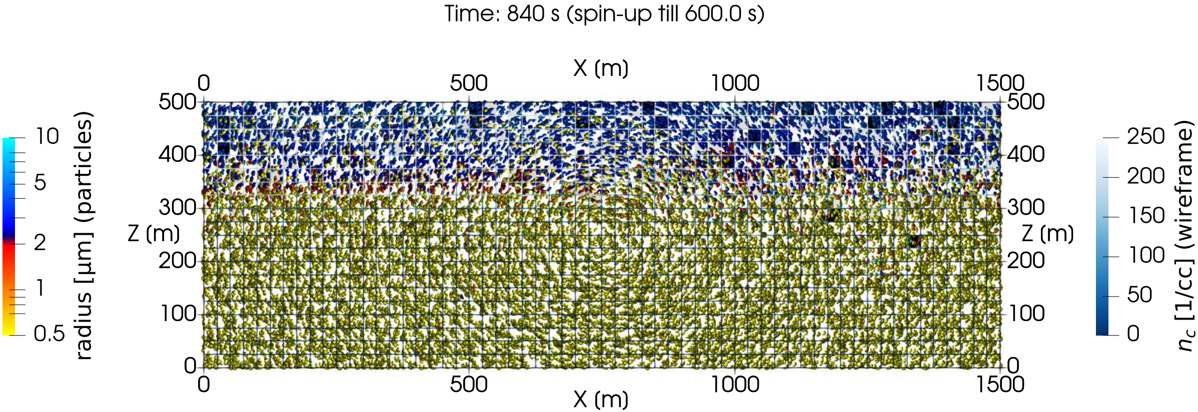

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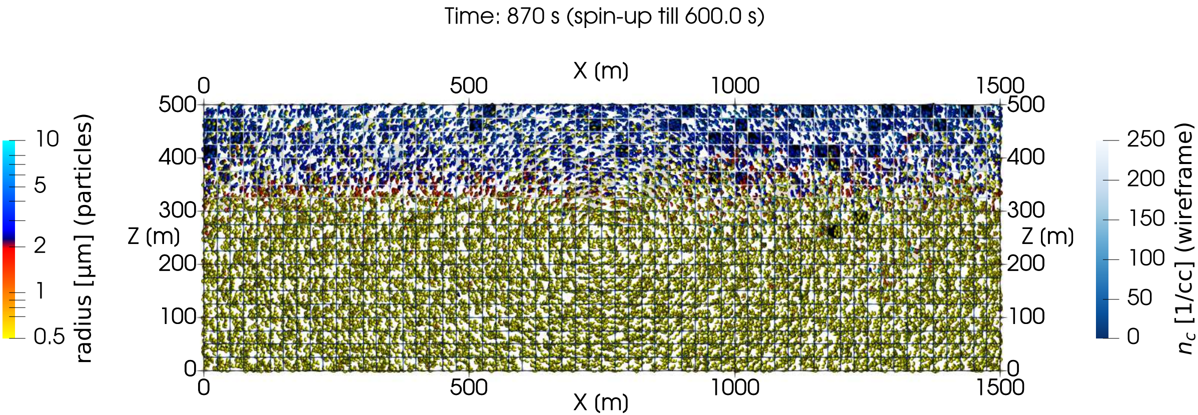

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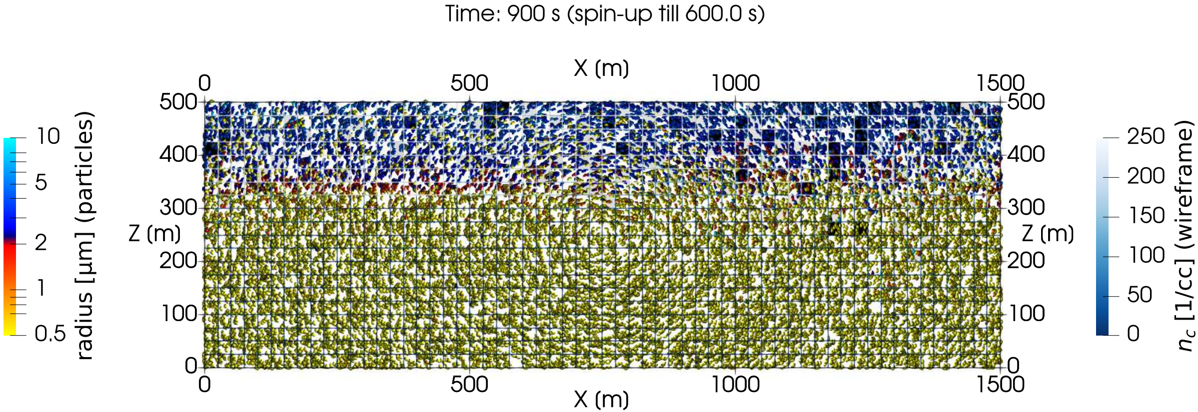

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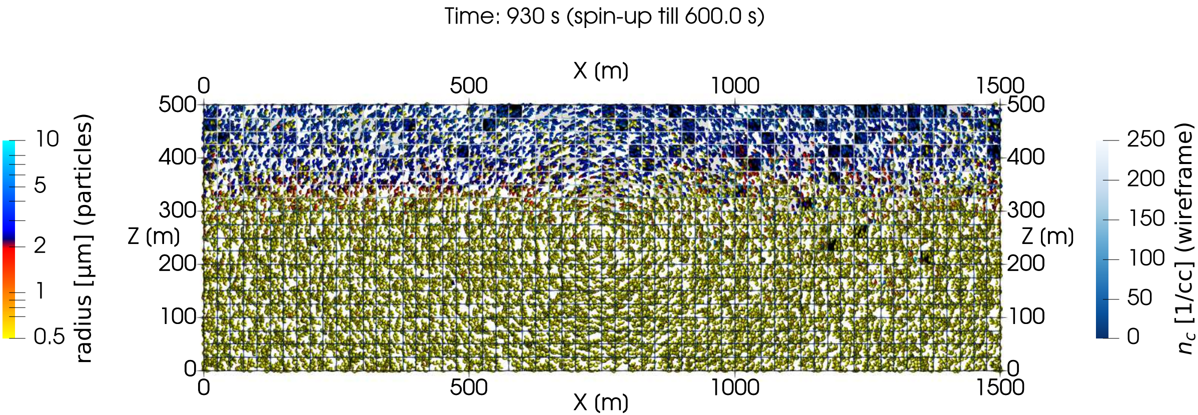

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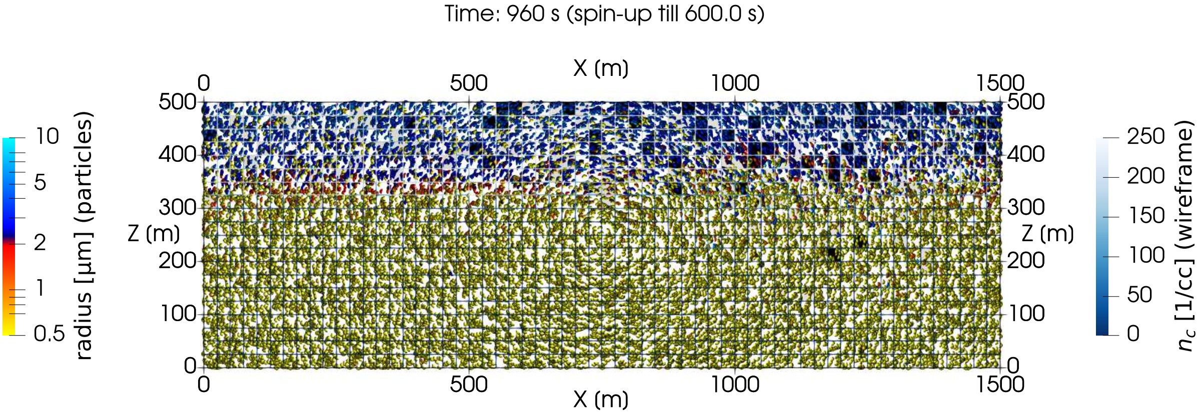

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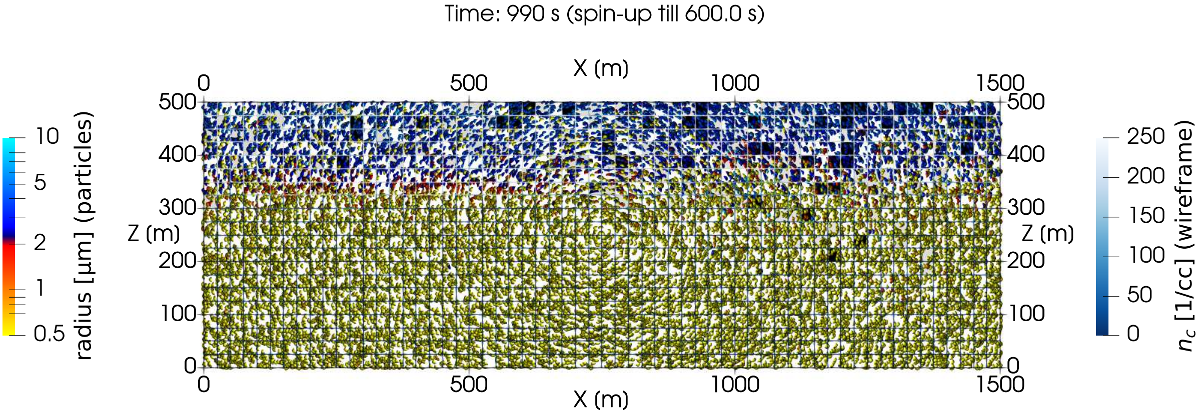

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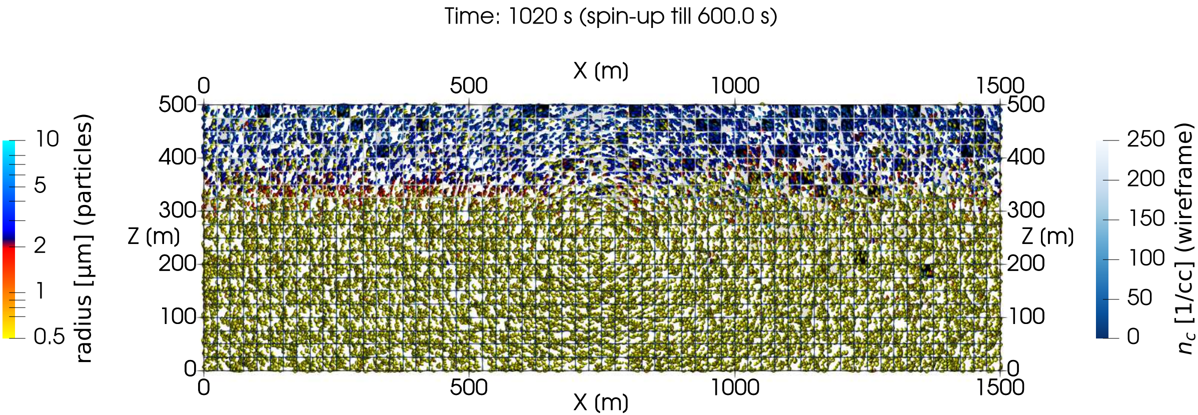

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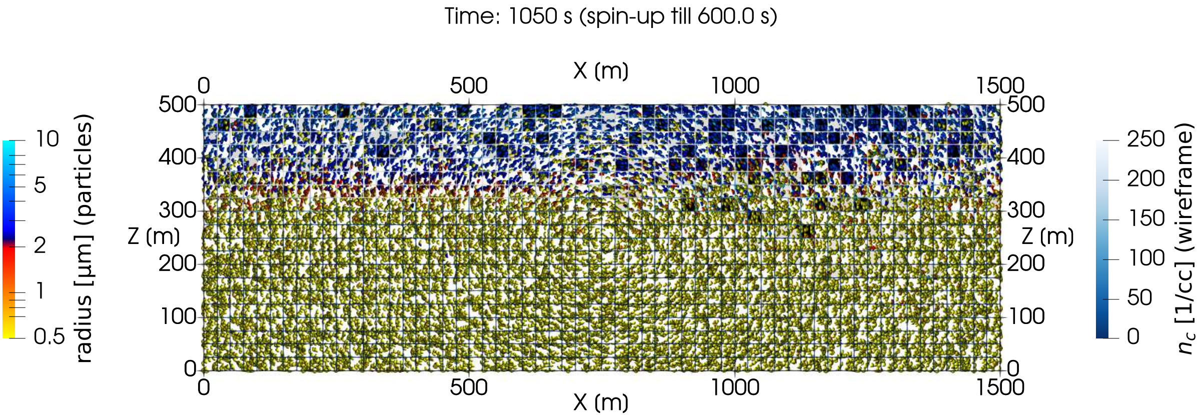

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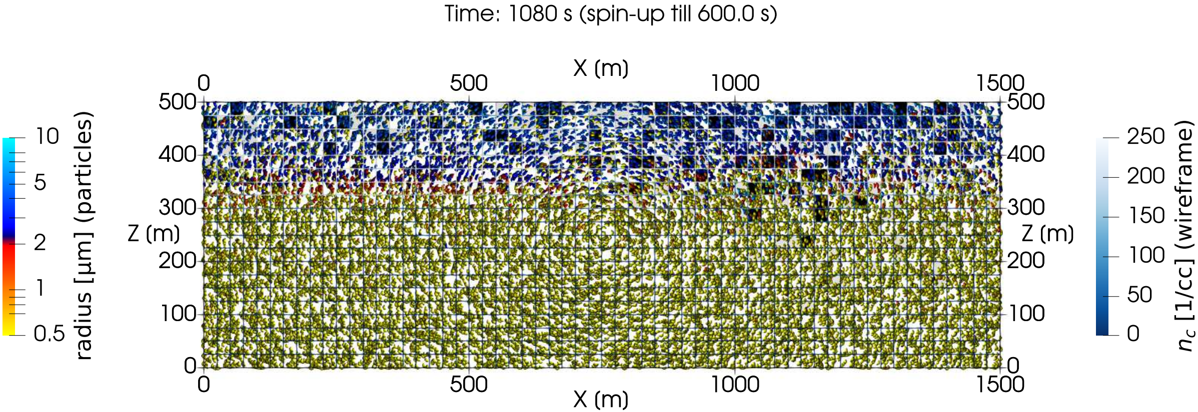

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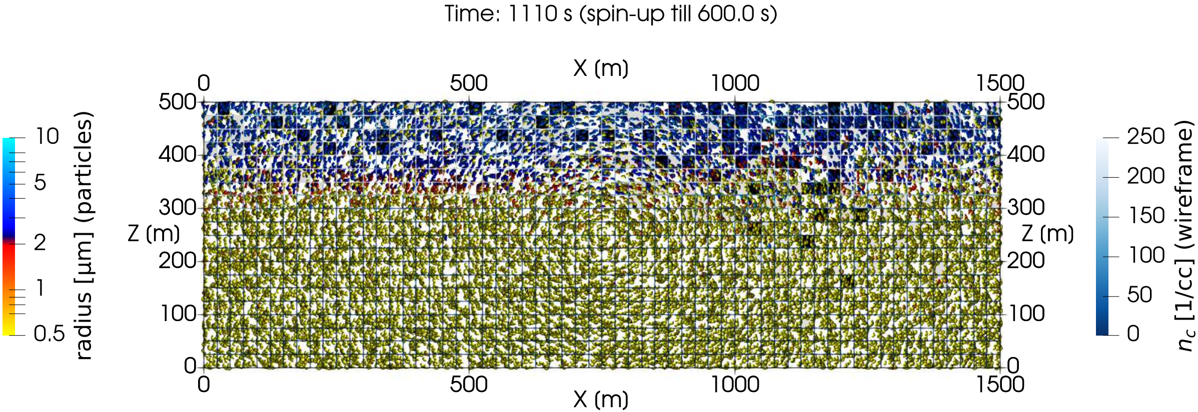

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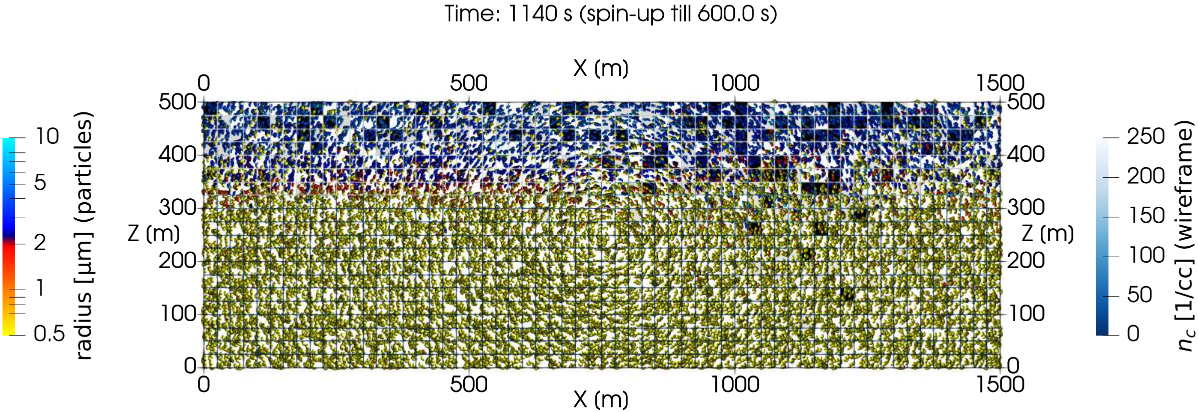

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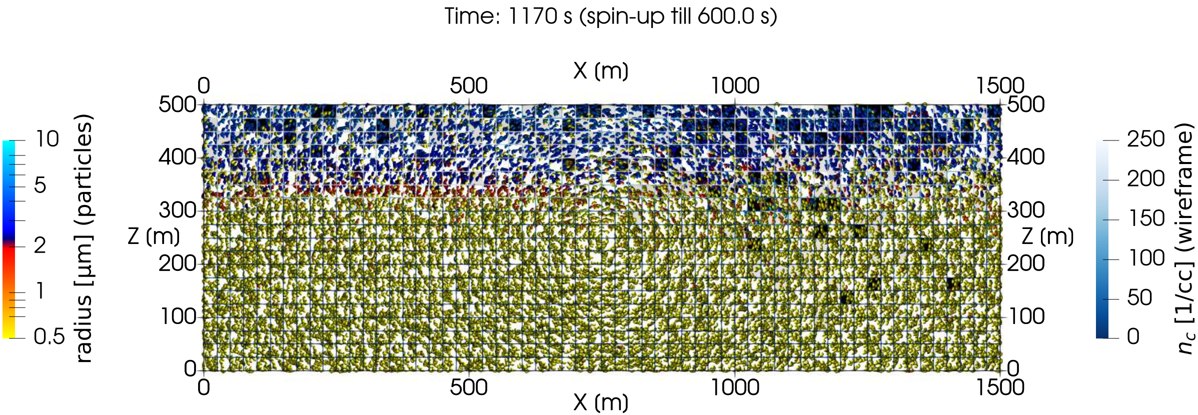

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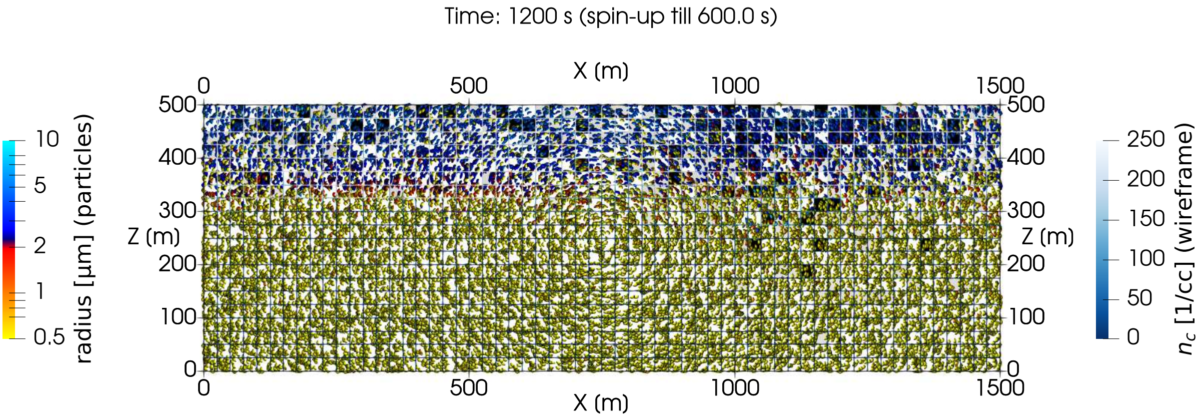

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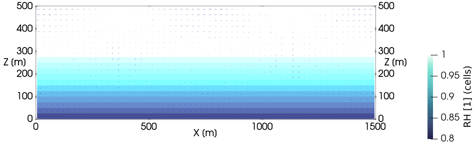
testing three flow regimes and two immersion freezing representations

$w_{max} \approx 1/3$ m/s



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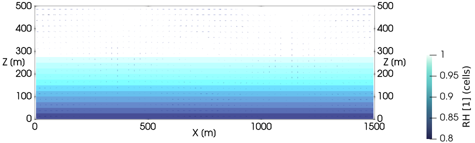

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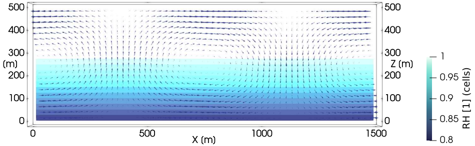

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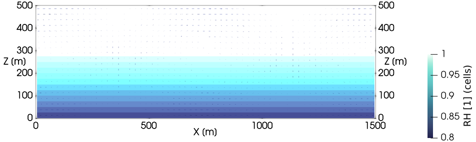

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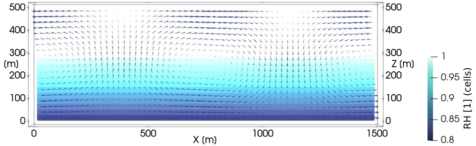

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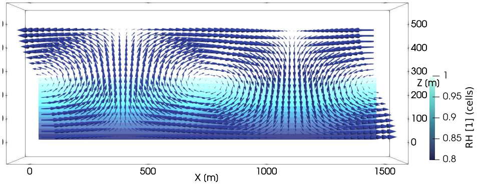

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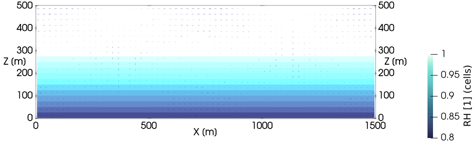

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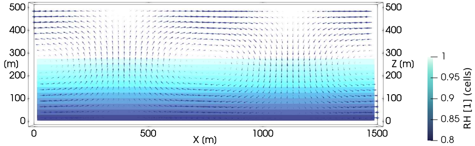

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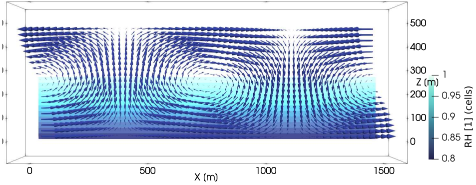

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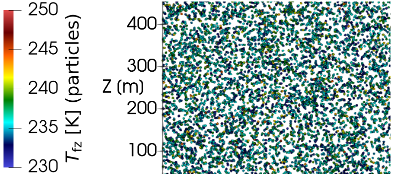
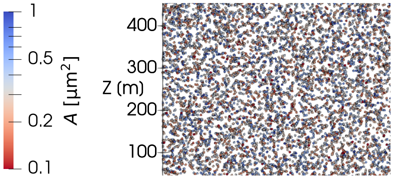


time-dependent (J_{het})



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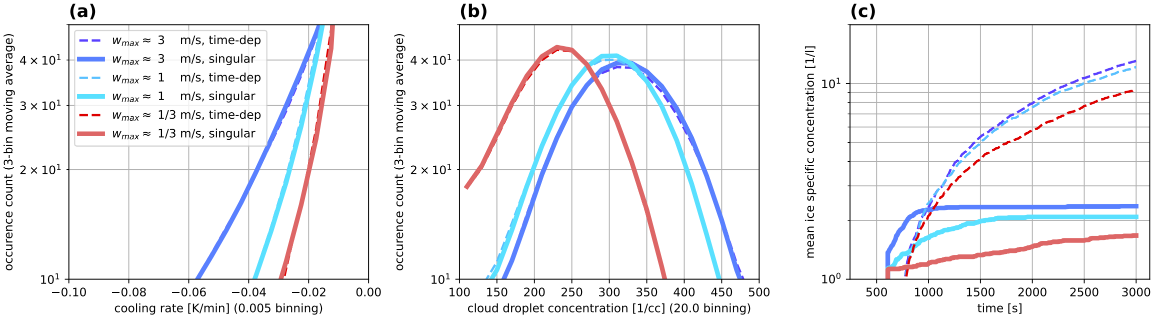

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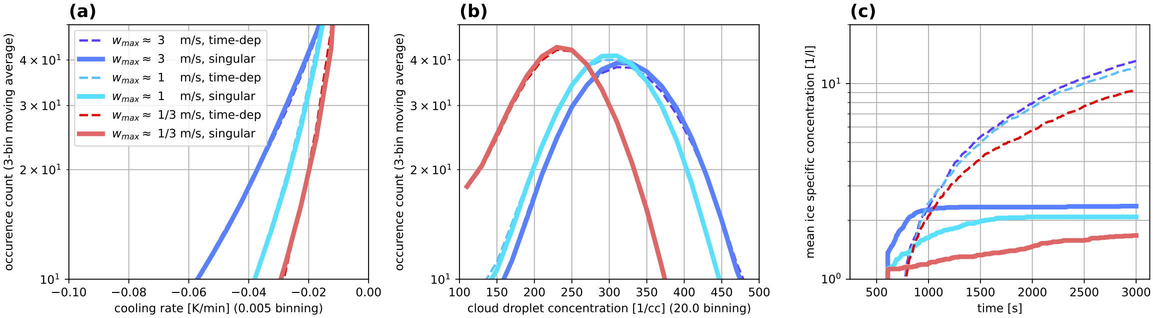

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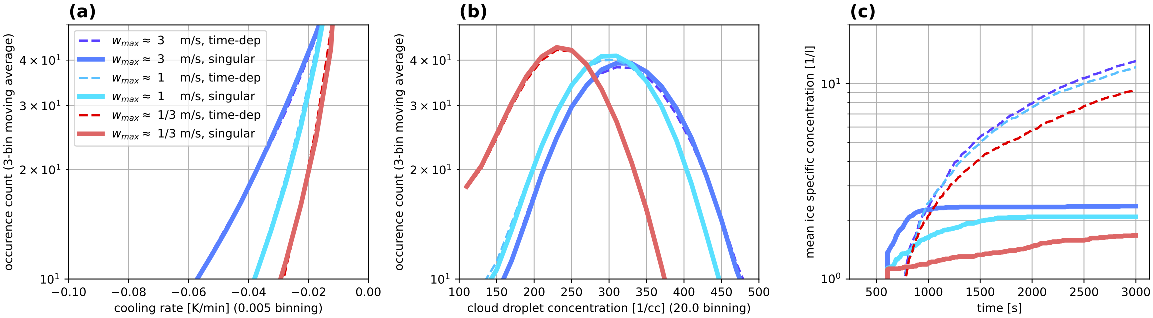

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DOE ASR grant no.
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🙏 Thank you
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