Immersion freezing in particle-based cloud microphysics models

S. Arabas^{1,2}, J.H. Curtis¹, I. Silber³, A. Fridlind⁴, D.A Knopf⁵, M. West¹ & N. Riemer¹













super-particles as a probabilistic alternative to bulk or bin μ -physics

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COMMISSIONED MANUSCRIPT

10.1029/2019MS001689

Key Points:

 Microphysics is an important component of weather and climate models, but its representation in current models is highly uncertain

Confronting the Challenge of Modeling Cloud and Precipitation Microphysics

Hugh Morrison¹ [0], Marcus van Lier-Walqui² [0], Ann M. Fridlind³ [0], Wojciech W. Grabowski¹ [0], Jerry Y. Harrington⁴, Corinna Hoose⁵ [0], Alexei Korolev⁶ [0], Matthew R. Kumjjan⁴ [0], Jason A. Milbrandt⁷, Hanna Pawlowska⁸ [0], Derek J. Posselt⁹, Olivier P. Prat¹⁰, Karly J. Reimel⁴, Shin-Ichiro Shima¹¹ [0], Bastiaan van Diedenhoven² [0], and Lulin Xue¹ [0]

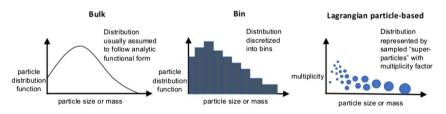


Figure 3. Representation of cloud and precipitation particle distributions in the three main types of microphysics

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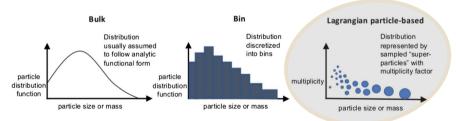


Figure 3. Representation of cloud and precipitation particle distributions in the three main types of microphysics

Shima, Sato, Hashimoto & Misumi 2020 (GMD):

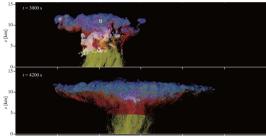


Figure 1. Typical realization of CTRL cloud spatial structures at t = 2040, 2460, 3000, 4200, and 5400 s. The mixing ratio of cloud water, rainwater, cloud ice, graupel, and snow aggregates are plotted in fading white, yellow, blue, red, and green, respectively. The symbols indicate examples of unrealistic predicted ice particles (Seex. 1) 3 and 9.1). See also Movel 1 in the video supolement.

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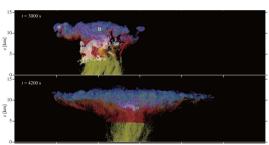


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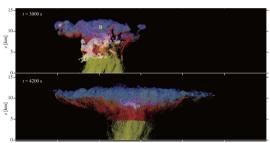


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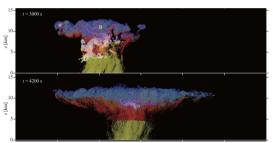


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- ► Eulerian component: momentum, heat, moisture budget
- Lagrangian component: super particles representing aerosol, water droplets, ice particles (porous spheroids)
- particle-resolved processes:
 - advection and sedimentation
 - homogeneous and immersion freezing (singular)
 - melting
 - condensation and evaporation (incl. CCN [de]activation)
 - deposition and sublimation
 - collisions (coalescence, riming, aggregation, washout)

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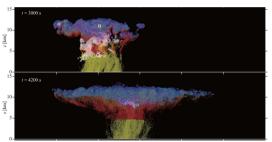


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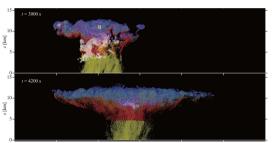


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theory (in modern notation)

(Bigg '53, Langham & Mason '58, Carte '59, Marshall '61)

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Poisson counting process with rate *r*:

$$P^*$$
 (k events in time t) = $\frac{(rt)^k \exp(-rt)}{k!}$

 $P(\text{one or more events in time t}) = 1 - P^*(k = 0, t)$

$$\ln(1-P) = -rt$$

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introducing $J_{het}(T)$, T(t) and INP surface A:

$$\ln(1-P(A,t)) = -A \int_{0}^{t} J_{\text{het}}(T(t')) dt'$$

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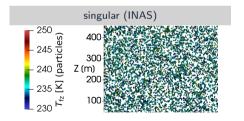
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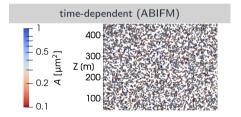
experimental $n_s(T)$ fits: e.g., Niemand et al. 2012

singular: INAS T_{fz} as attribute; initialisation by random sampling from $P(T_{fz}, A)$ with lognormal A

(A is not an attribute, initialisation only); freezing if $T(t) < T_{fz}(t=0)$

time-dependent: A as attribute (randomly sampled from the same lognormal)

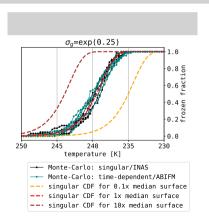




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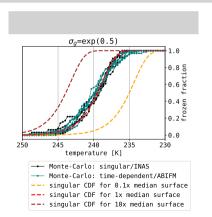
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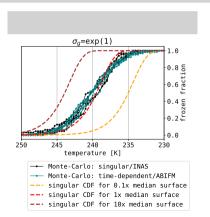
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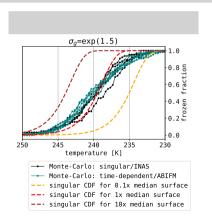
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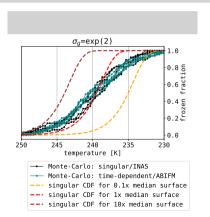
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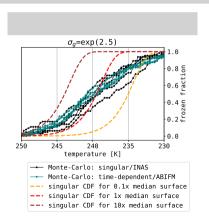
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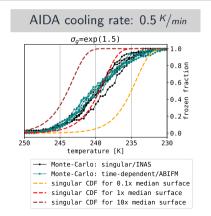
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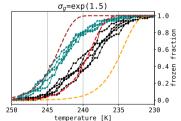
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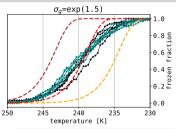
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Monte-Carlo freezing trigger using $P(J_{het}(T(t)))$

cooling rate: $0.1 \, \text{K/min}$



AIDA cooling rate: $0.5 \, \text{K/min}$



Monte-Carlo: singular/INAS
Monte-Carlo: time-dependent/ABIFM
singular CDF for 0.1x median surface
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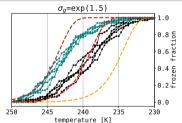
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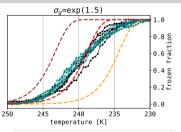
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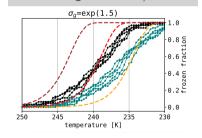


AIDA cooling rate: $0.5 \, K/min$



→ Monte-Carlo: singular/INAS
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 → singular CDF for 0.1x median surface
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cooling rate: $2.5 \, K/min$



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for a constant cooling rate c = dT/dt:

$$\ln(1 - P(A, t)) = -\frac{A}{c} \int_{T_0}^{T_0 + ct} J_{\text{het}}(T') dT' = -A \cdot I(T)$$

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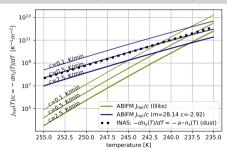
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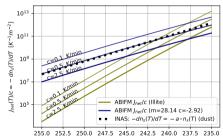
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255.0 252.5 250.0 247.5 245.0 242.5 240.0 237.5 235.0 temperature [K]

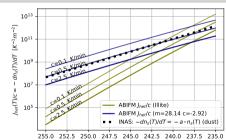
cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11) but the singular ansatz limitation of sampling $T_{\rm fz}$ at t=0 remains

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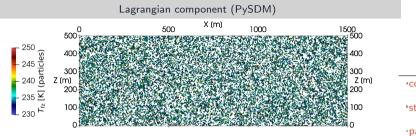


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temperature [K]

Is it a problem?

particle-based μ -physics + prescribed-flow test (aka KiD-2D) a,b,c,d,e



Eulerian component (PyMPDATA)

1000



500

400

300

200

100

1500

Z (m)

concept: Gedzelman & Arnold '93

*stratiform: Morrison & Grabowski '07

'particle-based: Arabas et al. '15

^dKiD-2D: github.com/BShipway/KiD

•here: SHEBA case (Fridlind et al. '12)

X (m)

500

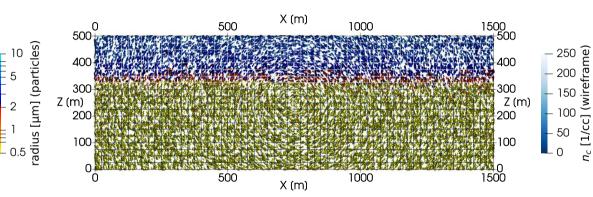
300

200

100

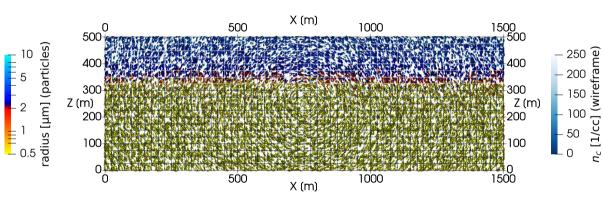
Z (m)

Time: 30 s (spin-up till 600.0 s)



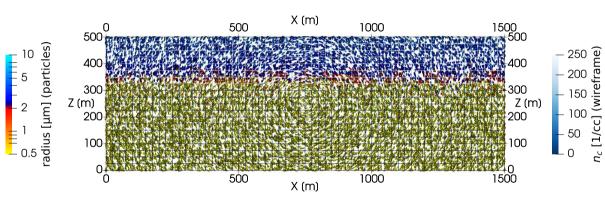
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 60 s (spin-up till 600.0 s)



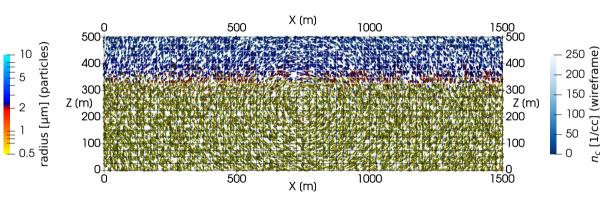
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 90 s (spin-up till 600.0 s)



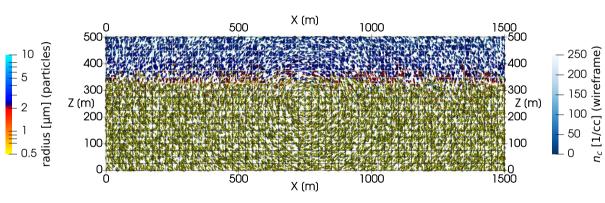
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 120 s (spin-up till 600.0 s)



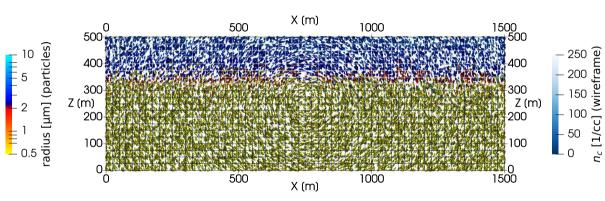
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 150 s (spin-up till 600.0 s)



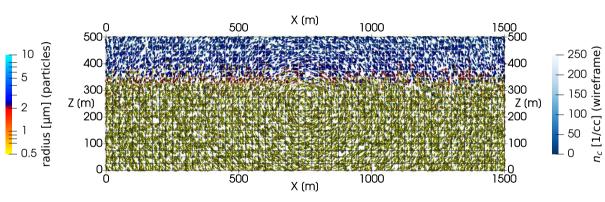
 $N_{\rm aer} = 300/cc \; {\rm (two-mode\ lognormal)} \quad N_{\rm INP} = 150/L \; {\rm (lognormal,} \; D_g = 0.74 \; \mu {\rm m}, \; \sigma_g = 2.55) \\ {\rm spin-up} = {\rm freezing\ off;} \; {\rm subsequently\ frozen\ particles\ act\ as\ tracers}$

Time: 180 s (spin-up till 600.0 s)



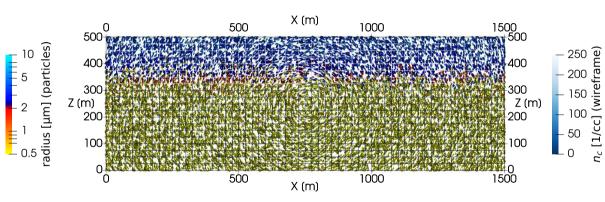
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 210 s (spin-up till 600.0 s)



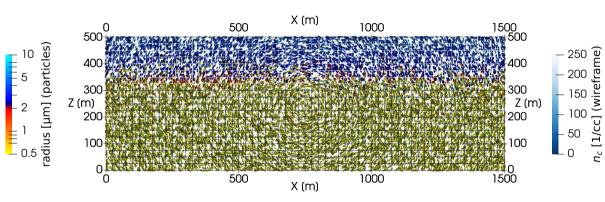
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 240 s (spin-up till 600.0 s)



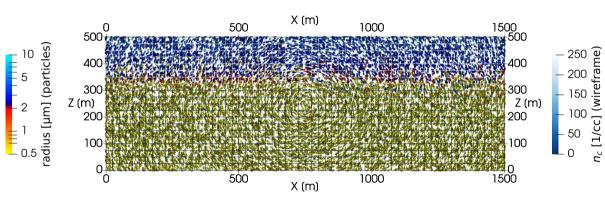
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 270 s (spin-up till 600.0 s)



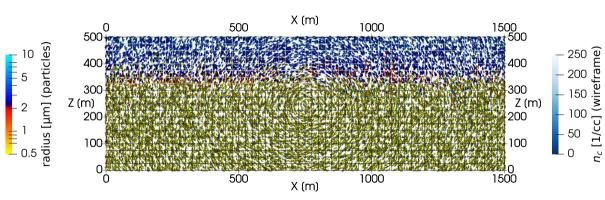
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 300 s (spin-up till 600.0 s)



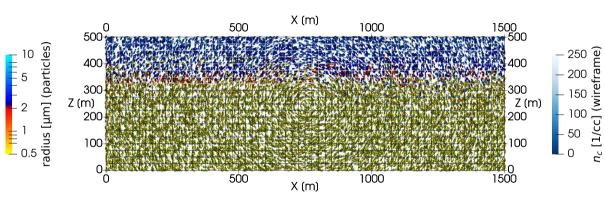
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 330 s (spin-up till 600.0 s)



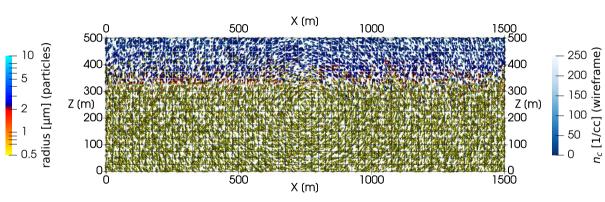
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 360 s (spin-up till 600.0 s)



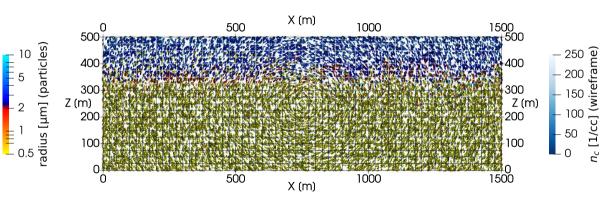
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 390 s (spin-up till 600.0 s)



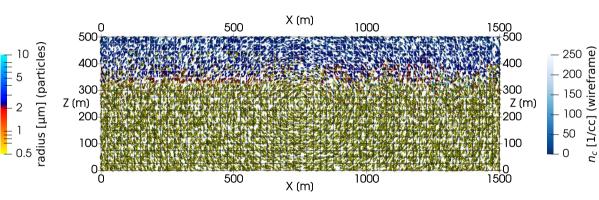
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 420 s (spin-up till 600.0 s)



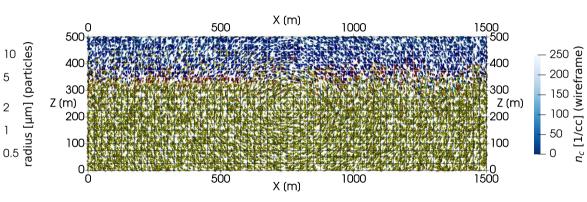
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 450 s (spin-up till 600.0 s)



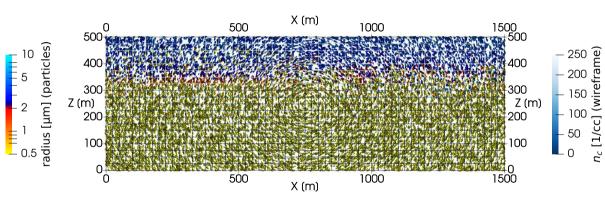
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 480 s (spin-up till 600.0 s)



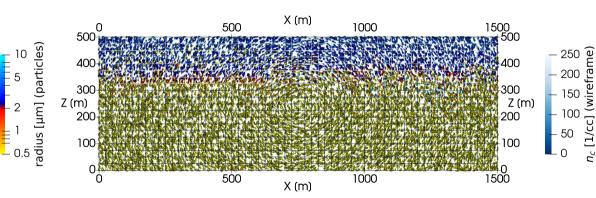
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 510 s (spin-up till 600.0 s)



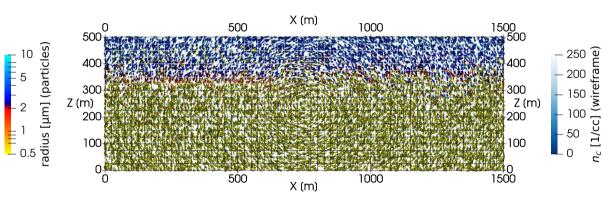
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 540 s (spin-up till 600.0 s)



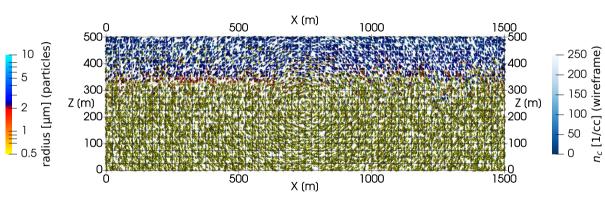
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 570 s (spin-up till 600.0 s)



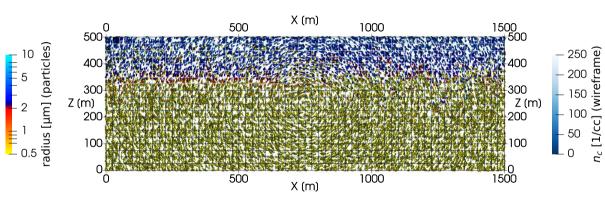
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 600 s (spin-up till 600.0 s)



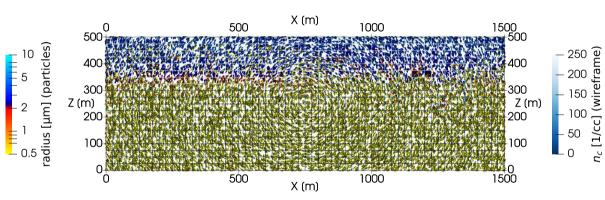
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 630 s (spin-up till 600.0 s)



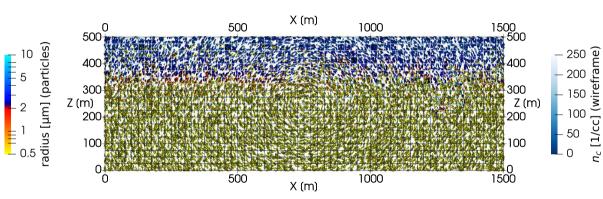
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 660 s (spin-up till 600.0 s)



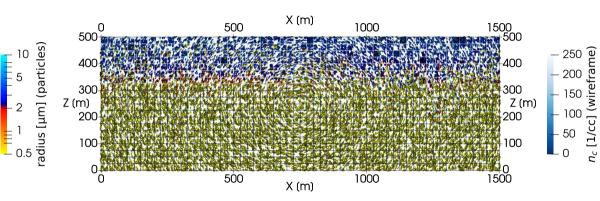
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 690 s (spin-up till 600.0 s)



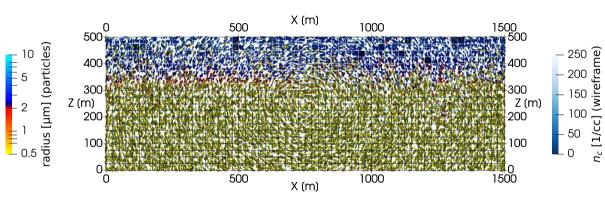
 $16+16 \ \text{super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \ \text{(two-mode lognormal)} \quad N_{\text{INP}} = 150/L \ \text{(lognormal)} \quad D_g = 0.74 \ \mu\text{m}, \ \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers}$

Time: 720 s (spin-up till 600.0 s)



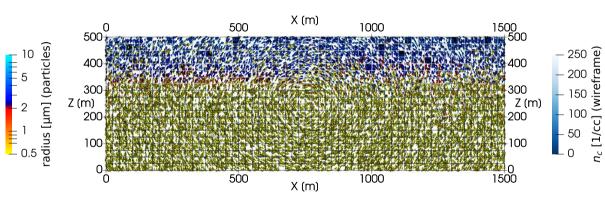
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 750 s (spin-up till 600.0 s)



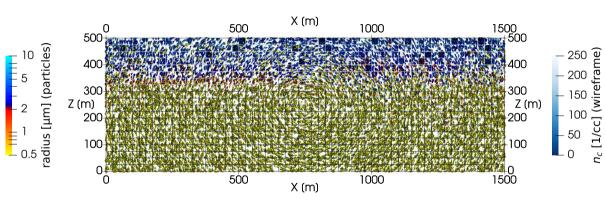
 $N_{\rm aer} = 300/cc \; {\rm (two-mode\ lognormal)} \quad N_{\rm INP} = 150/L \; {\rm (lognormal,} \; D_g = 0.74 \; \mu {\rm m}, \; \sigma_g = 2.55) \\ {\rm spin-up} = {\rm freezing\ off;} \; {\rm subsequently\ frozen\ particles\ act\ as\ tracers}$

Time: 780 s (spin-up till 600.0 s)



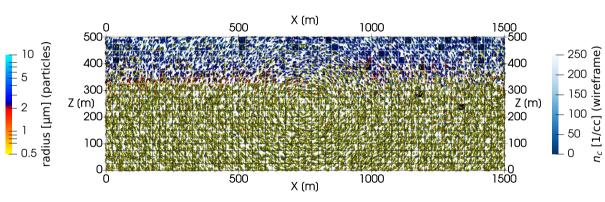
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 810 s (spin-up till 600.0 s)



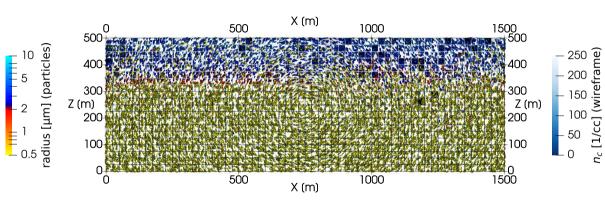
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 840 s (spin-up till 600.0 s)



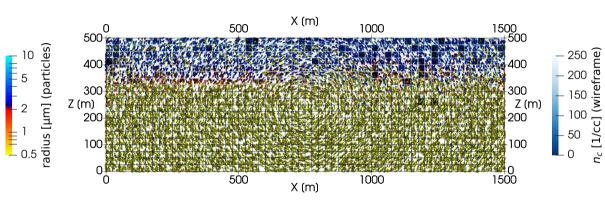
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 870 s (spin-up till 600.0 s)



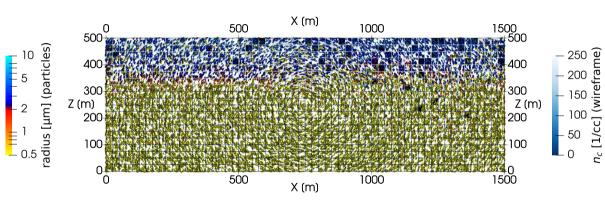
 $N_{\rm aer} = 300/cc \; \mbox{(two-mode lognormal)} \quad N_{\rm INP} = 150/L \; \mbox{(lognormal, } D_g = 0.74 \; \mbox{\mu m, } \sigma_{\rm g} = 2.55) \\ {\rm spin-up} = {\rm freezing \; off; \; subsequently \; frozen \; particles \; act \; as \; tracers}$

Time: 900 s (spin-up till 600.0 s)



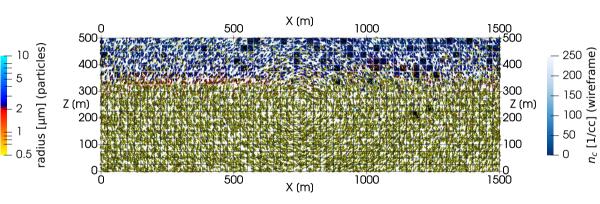
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 930 s (spin-up till 600.0 s)



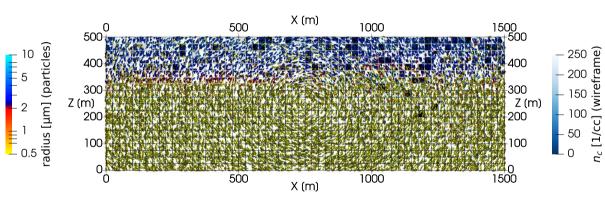
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 960 s (spin-up till 600.0 s)



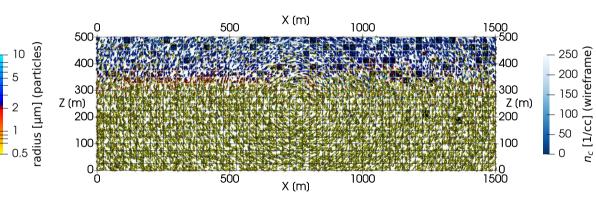
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 990 s (spin-up till 600.0 s)



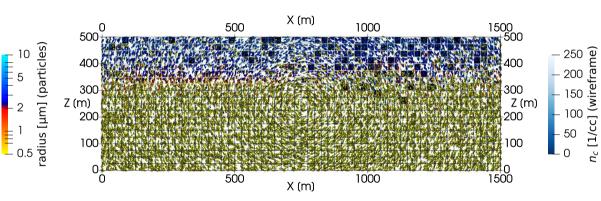
 $N_{\rm aer} = 300/cc \; {\rm (two-mode\ lognormal)} \quad N_{\rm INP} = 150/L \; {\rm (lognormal,} \; D_g = 0.74 \; \mu {\rm m}, \; \sigma_g = 2.55) \\ {\rm spin-up} = {\rm freezing\ off;} \; {\rm subsequently\ frozen\ particles\ act\ as\ tracers}$

Time: 1020 s (spin-up till 600.0 s)



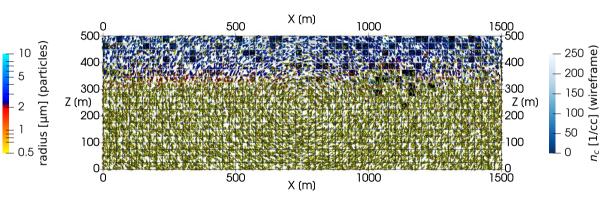
 $N_{\rm aer} = 300/cc \; {\rm (two-mode\ lognormal)} \quad N_{\rm INP} = 150/L \; {\rm (lognormal,} \; D_g = 0.74 \; \mu {\rm m}, \; \sigma_g = 2.55) \\ {\rm spin-up} = {\rm freezing\ off;} \; {\rm subsequently\ frozen\ particles\ act\ as\ tracers}$

Time: 1050 s (spin-up till 600.0 s)



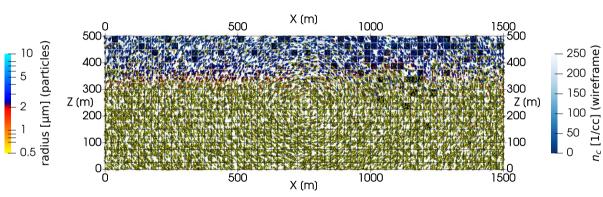
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 1080 s (spin-up till 600.0 s)



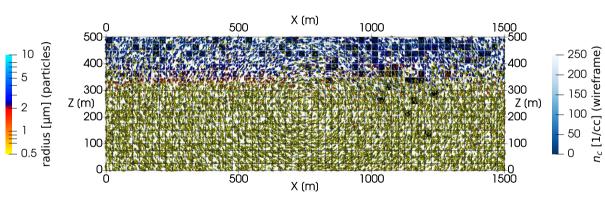
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 1110 s (spin-up till 600.0 s)



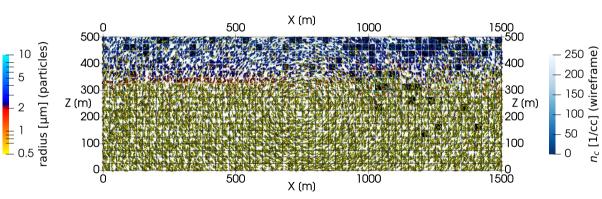
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 1140 s (spin-up till 600.0 s)



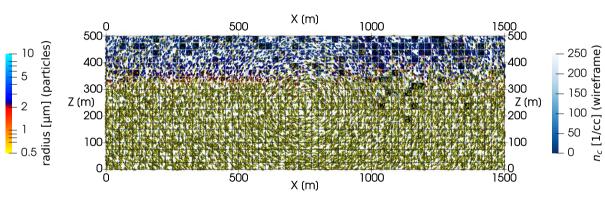
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 1170 s (spin-up till 600.0 s)

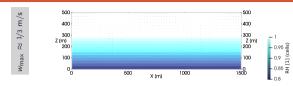


 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

Time: 1200 s (spin-up till 600.0 s)

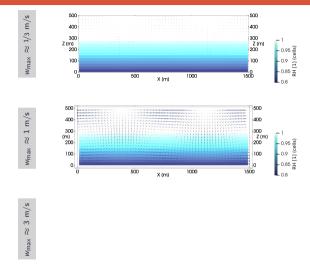


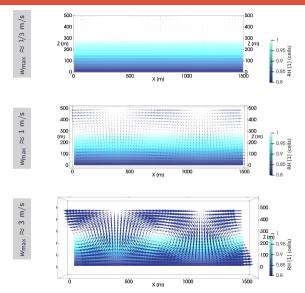
 $\begin{array}{c} 16+16 \text{ super-particles/cell for INP-rich} + \text{INP-free particles} \\ N_{\text{aer}} = 300/cc \text{ (two-mode lognormal)} & N_{\text{INP}} = 150/L \text{ (lognormal, } D_g = 0.74 \text{ } \mu\text{m}, \text{ } \sigma_g = 2.55) \\ \text{spin-up} = \text{freezing off; subsequently frozen particles act as tracers} \end{array}$

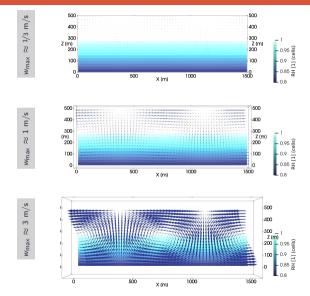


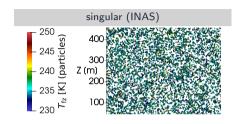


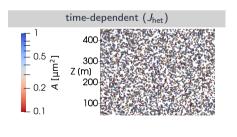


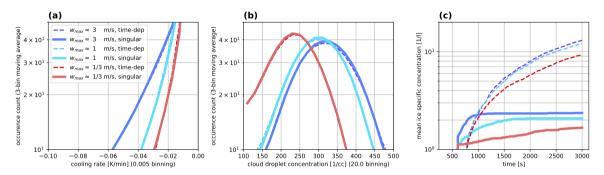




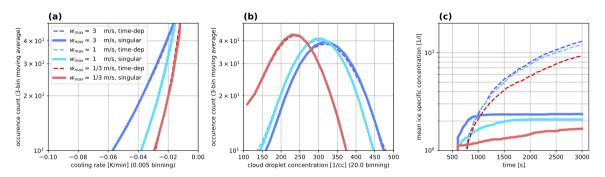




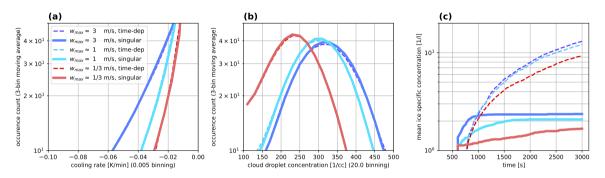




ightharpoonup range of cooling rates in simple flow (far from $c\sim 1$ K/min for AIDA as in Niemand et al. 2012)

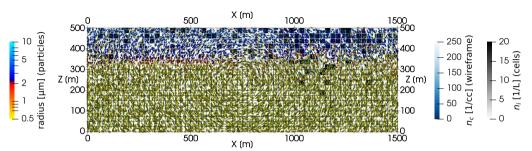


- range of cooling rates in simple flow (far from $c \sim 1$ K/min for AIDA as in Niemand et al. 2012)
- ightharpoonup singular vs. time-dependent markedly different (consistent with box model for $c \ll 1K/min$)

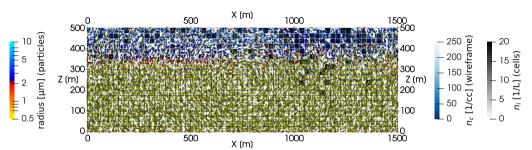


- lacktriangle range of cooling rates in simple flow (far from $c\sim 1$ K/min for AIDA as in Niemand et al. 2012)
- lacktriangle singular vs. time-dependent markedly different (consistent with box model for $c \ll 1 K/min$)
- ▶ CPU time trade off: time dependent ca. 3-4 times costlier



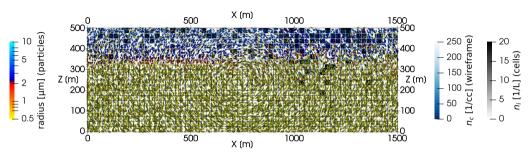


- emergence of comprehensive mixed-phase particle-based aerosol/cloud μ -physics models
- cooling rate embedded in INAS fits \leadsto limited robustness to different flow regimes



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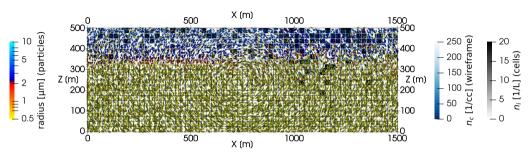




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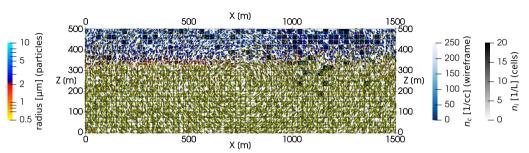


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