Super-cooling super-droplets: on particle-based modelling of heterogeneous freezing

S. Arabas[1,2], J.H. Curtis[1], I. Silber[3], A. Fridlind[4], D.A Knopf[5], M. West[1] & N. Riemer[1]

1: atmos.illinois.edu
2: uj.edu.pl
3: met.psu.edu
4: giss.nasa.gov
5: stonybrook.edu

funding:

ASR Atmospheric System Research

Brookhaven National Lab, NY, June 8 2022

aerosol-cloud interactions: a conceptual picture

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background image: vitsly.ru / Hokusai

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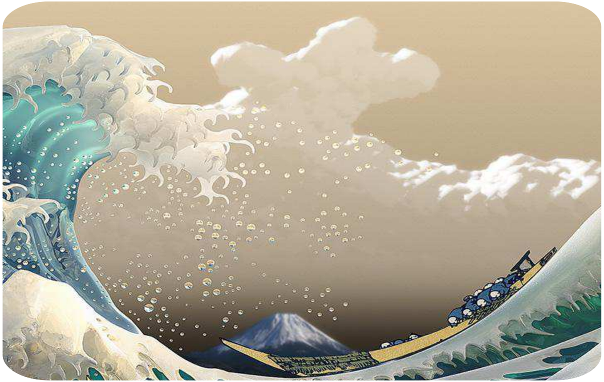

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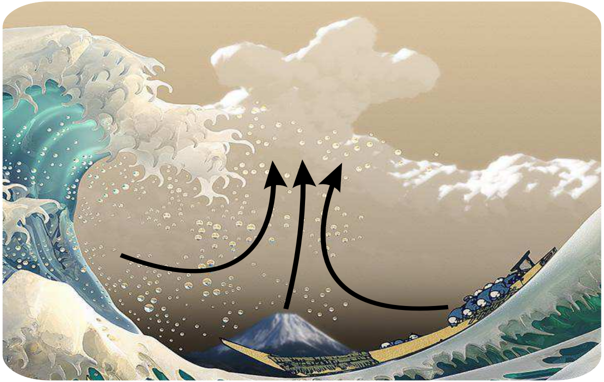
aerosol-cloud interactions: a conceptual picture

- aerosol particles of natural and anthropogenic origin act as condensation/crystallisation nuclei



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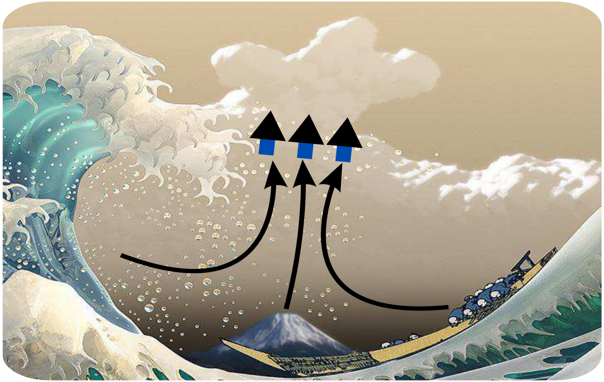

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two-way interactions:
- aerosol characteristics influence cloud microstructure
- cloud processes influence aerosol size and composition

background image: `vitsly.ru` / Hokusai

super-particles as a probabilistic alternative to bulk or bin μ-physics

JAMES | Journal of Advances in Modeling Earth Systems

COMMISSIONED MANUSCRIPT
10.1029/2019MS001689

Key Points:
- Microphysics is an important component of weather and climate models, but its representation in current models is highly uncertain

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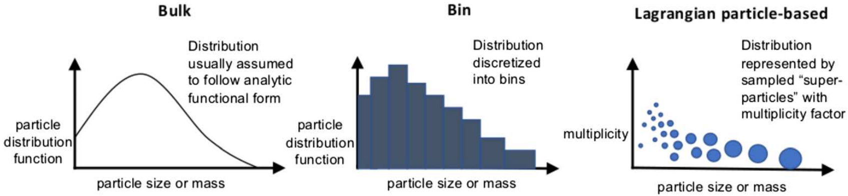

Figure 3. Representation of cloud and precipitation particle distributions in the three main types of microphysics

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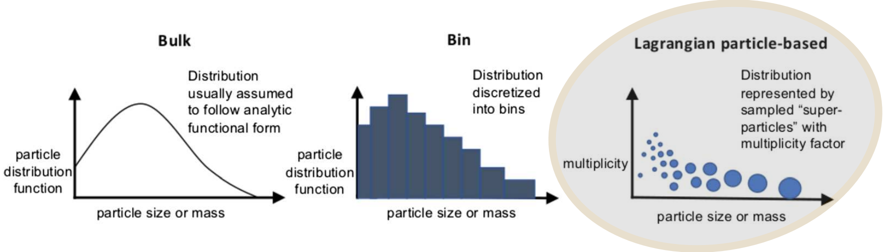

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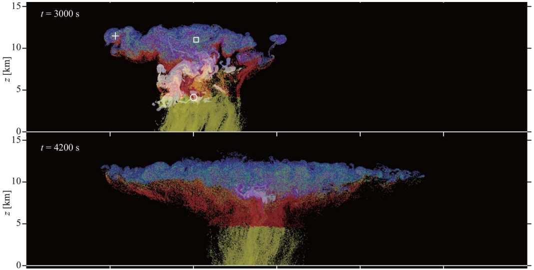

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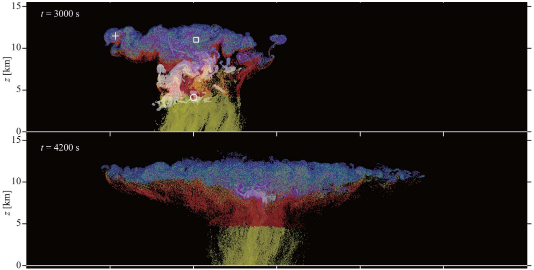

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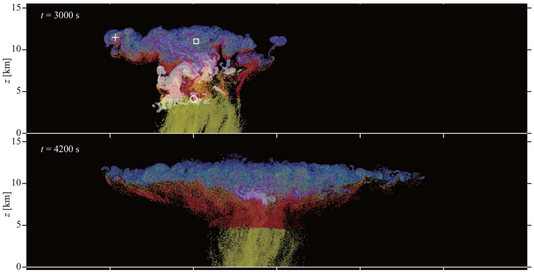

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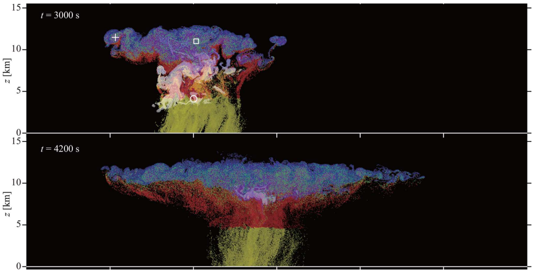

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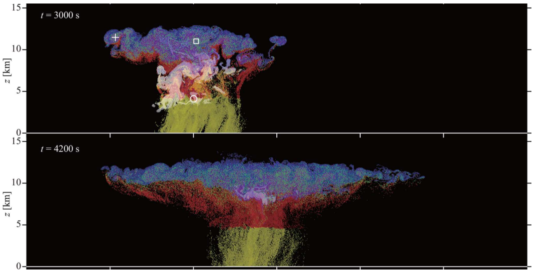

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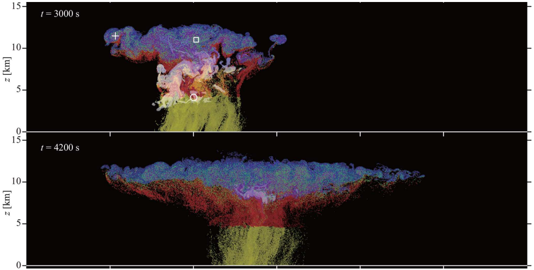

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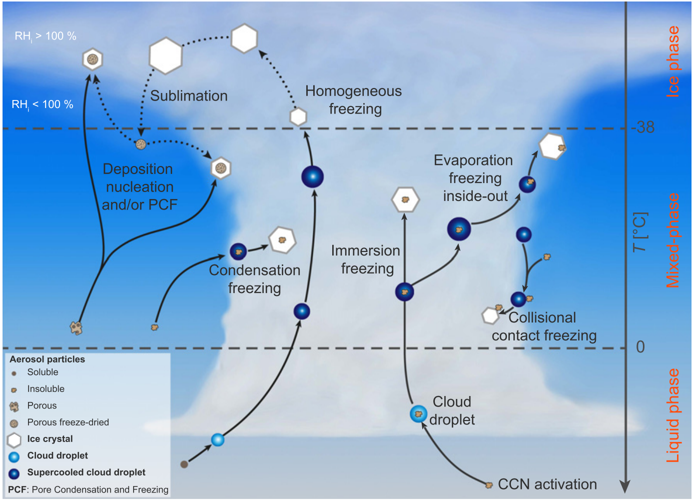
immersion freezing and other ice crystal formation pathways in clouds



Kanji et al. 2017, graphics F. Mahrt, https://doi.org/10.1175/AMSMONOGRAPHS-D-16-0006.1

immersion freezing: bacteria and the Olympics

Journal of Geophysical Research: Atmospheres

RESEARCH ARTICLE
10.1002/2016JD025251

Key Points:
- Very ice active Snomax protein aggregates are fragile and their ice nucleation ability decreases over months of freezer storage
- Partitioning of ice active protein aggregates into the immersion oil reduces the droplet's measured freezing temperature

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https://www.reuters.com/markets/commodities/making-snow-stick-wind-challenges-winter-games-slope-makers-2021-11-29/

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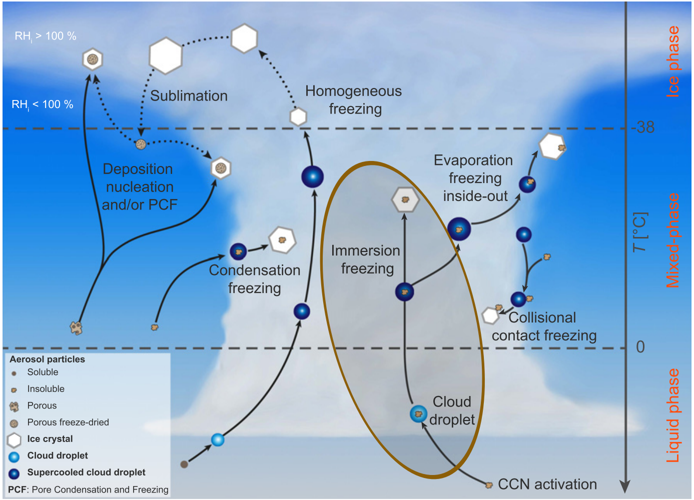

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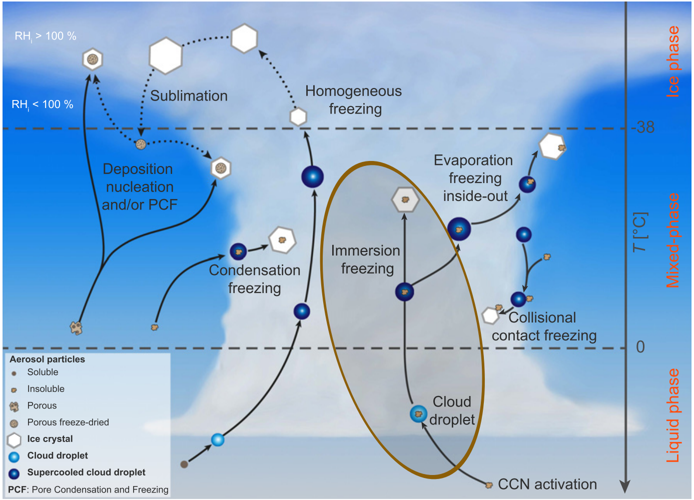

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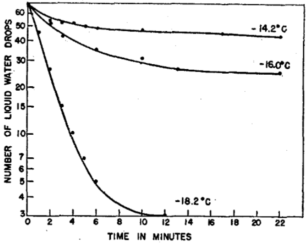

Fraction of water drops remaining unfrozen as a function of time.

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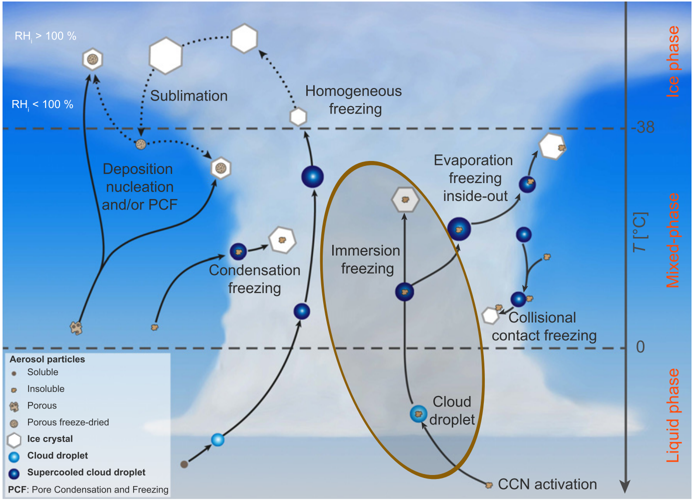

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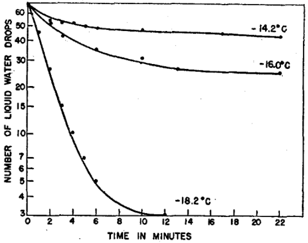

Fraction of water drops remaining unfrozen as a function of time.

Vali 2014 (ACP)

"Interpretations of the experimental results face considerable difficulties … two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities … dichotomy – the stochastic and singular models"

Marshall et al. 1961 (Nubila 4(1))

Heterogeneous Nucleations

is a Stochastic Process

by

J. S. MARSHALL

McGill University, Montreal, Canad.

Presented at the International Congress on the Physics of Clouds (Hailstorms) at Verona 9-13 August 1960.

`http://cma.entecra.it/Astro2_sito/doc/Nubila_1_1961.pdf`

Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

theory (in modern notation)
(Bigg '53, Langham & Mason '58, Carte '59, Marshall '61)

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Poisson counting process with rate r:

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$$\ln(1 - P) = -rt$$

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introducing $J_{het}(T)$, $T(t)$ and INP surface A:

$$\ln(1-P(A,t)) = -A\underbrace{\int_0^t J_{het}(T(t'))\,dt'}_{I(T)}$$

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experimental $n_s(T)$ fits: e.g., Niemand et al. 2012

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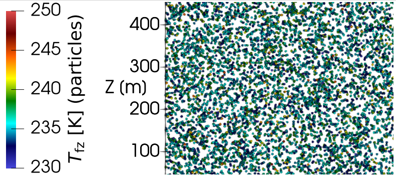
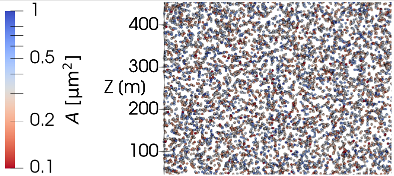
singular (INAS)



time-dependent (ABIFM)



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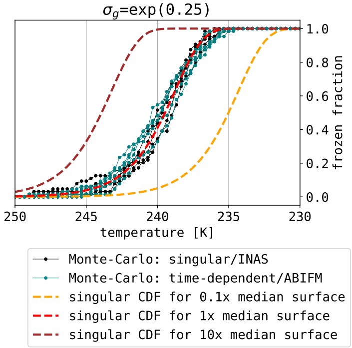

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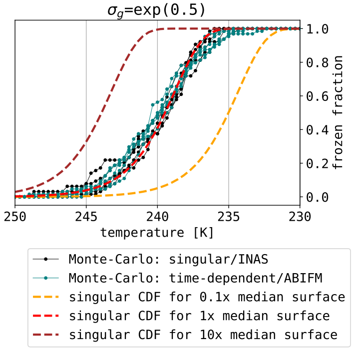

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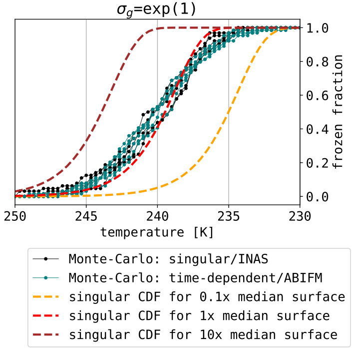

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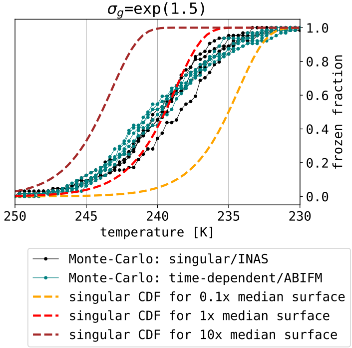

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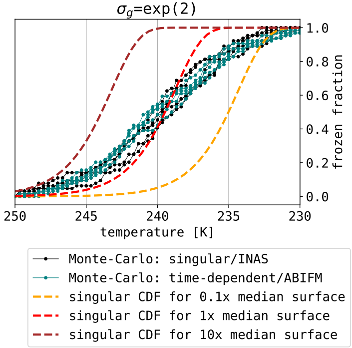

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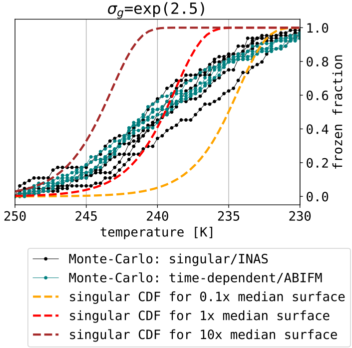

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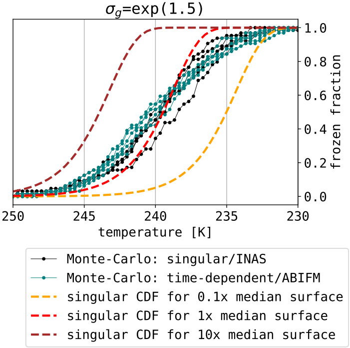
AIDA cooling rate: $0.5\ ^{K}/_{min}$



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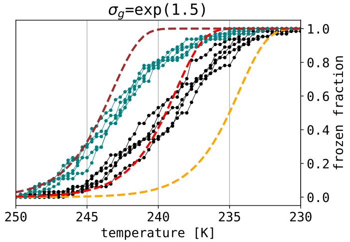

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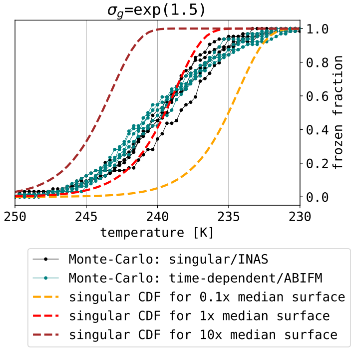

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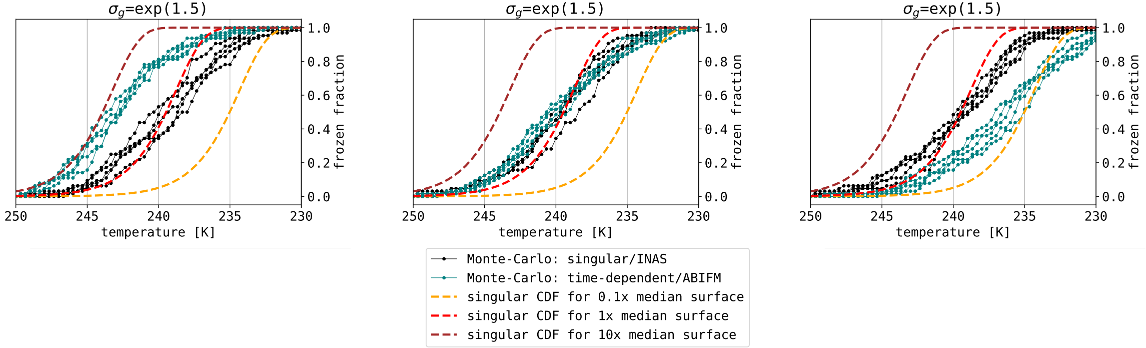
Monte-Carlo freezing trigger using $P(J_{het}(T(t)))$

cooling rate: $0.1\ K/min$	AIDA cooling rate: $0.5\ K/min$	cooling rate: $2.5\ K/min$



Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

theory (in modern notation)
(Bigg '53, Langham & Mason '58, Carte '59, Marshall '61)

Poisson counting process with rate r:

$$P^*(\text{k events in time t}) = \frac{(rt)^k \exp(-rt)}{k!}$$

$$P(\text{one or more events in time t}) = 1 - P^*(k=0, t)$$

$$\ln(1-P) = -rt$$

introducing $J_{\text{het}}(T)$, $T(t)$ and INP surface A:

$$\ln(1-P(A,t)) = -A \underbrace{\int_0^t J_{\text{het}}(T(t'))\, dt'}_{I(T)}$$

INAS: $I(T) = n_s(T) = \exp(a \cdot (T - T_{0^\circ C}) + b)$

experimental $n_s(T)$ fits: e.g., Niemand et al. 2012

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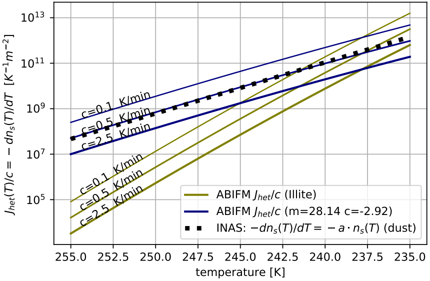

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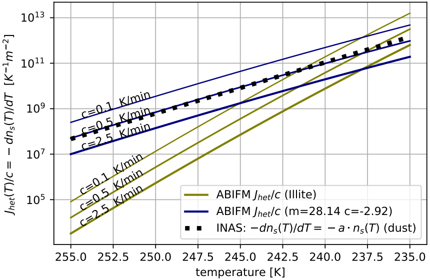

cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11)
but the singular ansatz limitation of sampling T_{fz} at t=0 remains

Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

Is it a problem?

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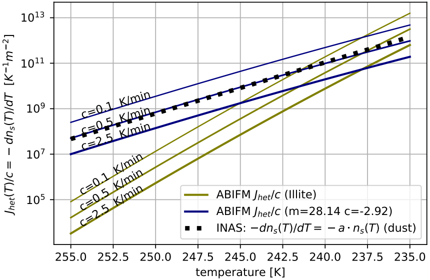

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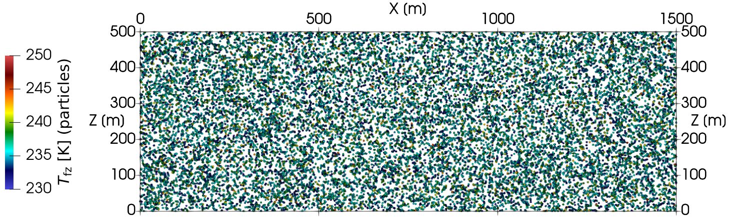
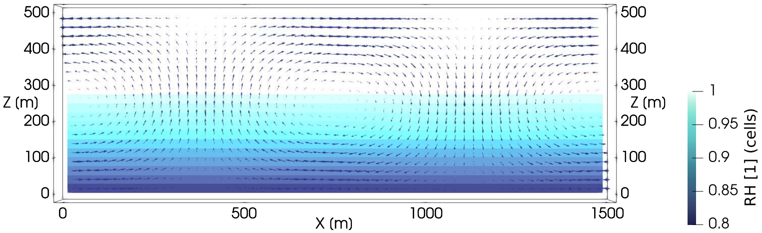
particle-based μ-physics + prescribed-flow test (aka KiD-2D)[a,b,c,d,e,f]

Lagrangian component (PySDM)



Eulerian component (PyMPDATA)



[a] concept: Gedzelman & Arnold '93

[b] stratiform: Morrison & Grabowski '07

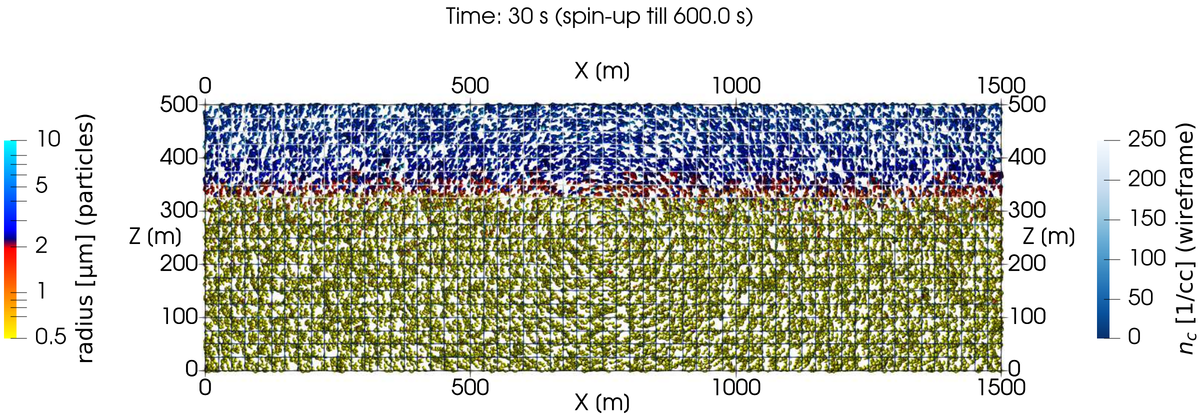
[c] ice phase: e.g., Yang et al. '15

[d] particle-based: e.g., Arabas et al. '15

[e] KiD-2D: github.com/BShipway/KiD

[f] here: SHEBA case (Fridlind et al. '12)

particle-based μ-physics + prescribed-flow test

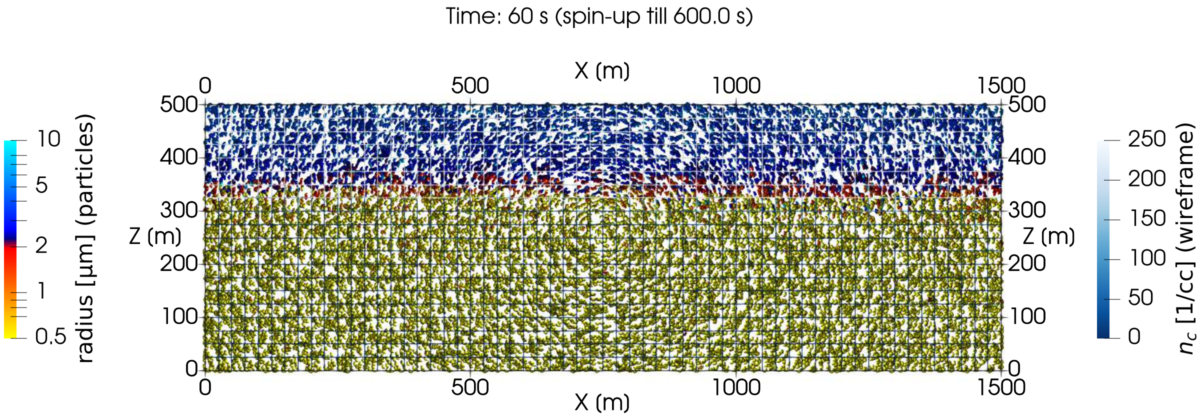


16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/cc$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74$ μm, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

particle-based µ-physics + prescribed-flow test



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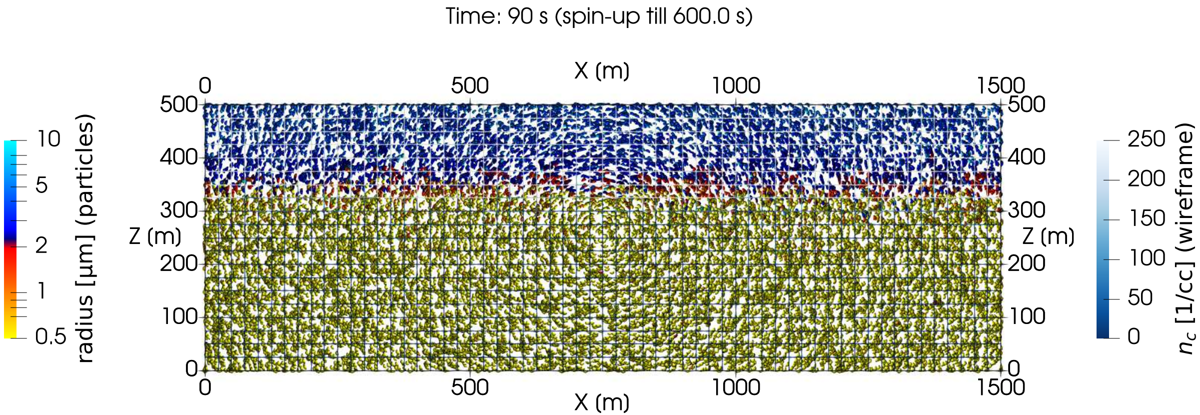

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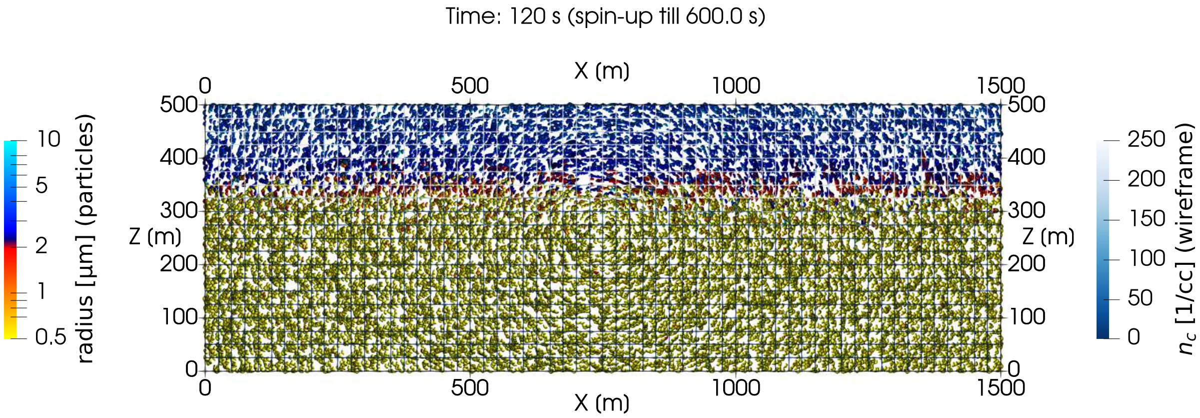

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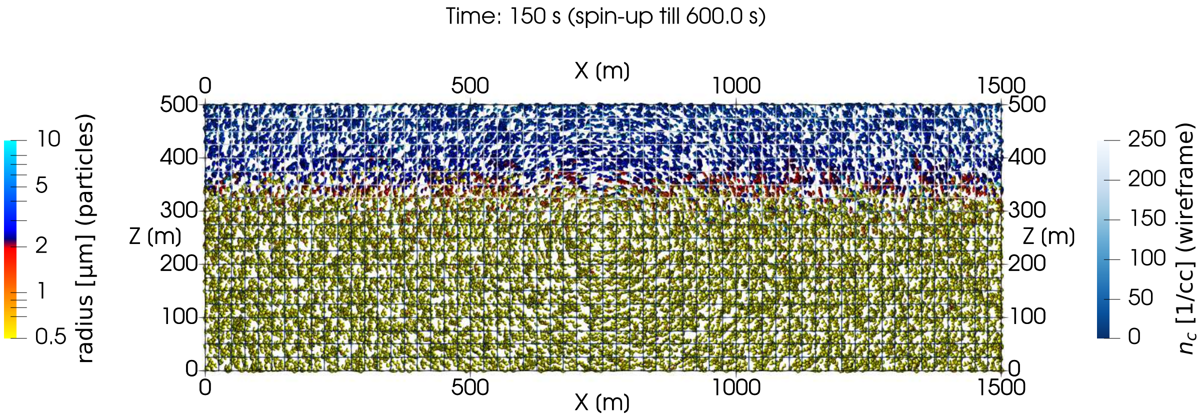

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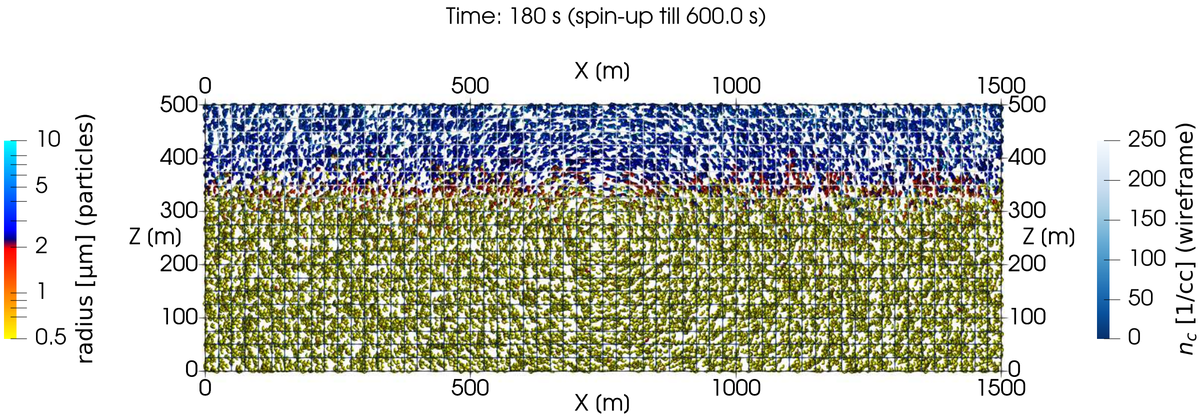

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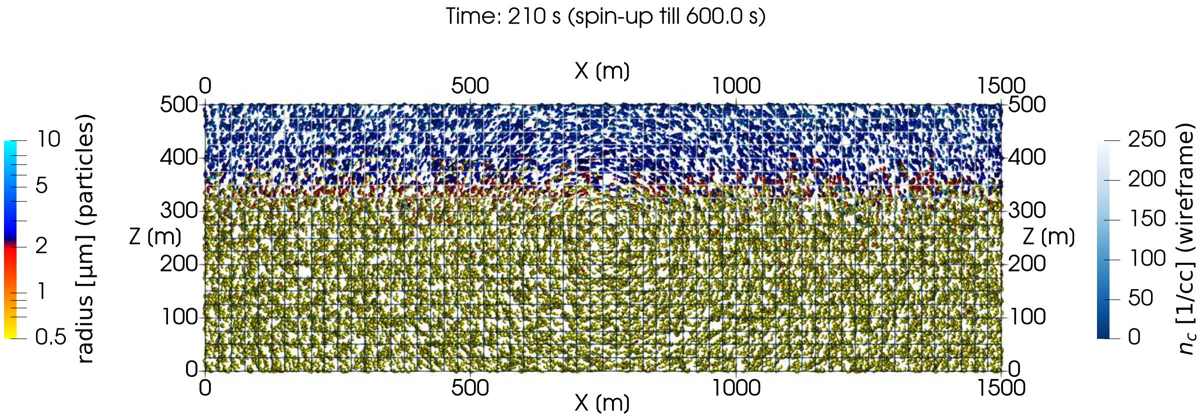

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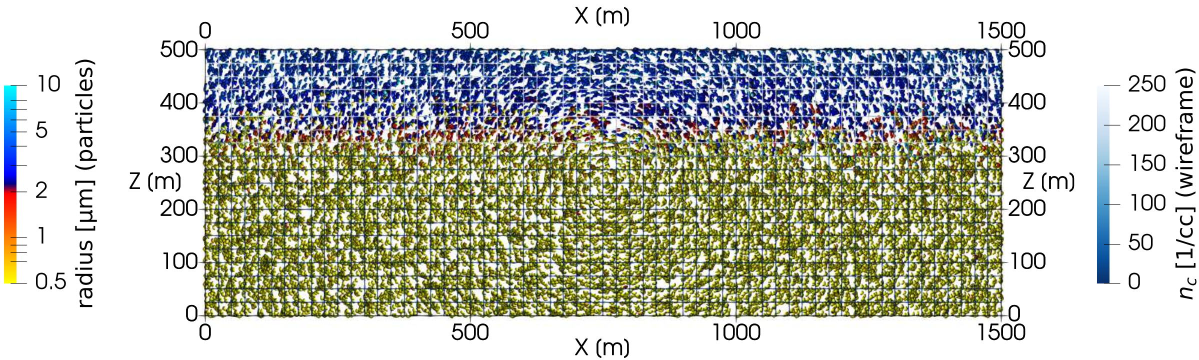
particle-based μ-physics + prescribed-flow test

Time: 240 s (spin-up till 600.0 s)



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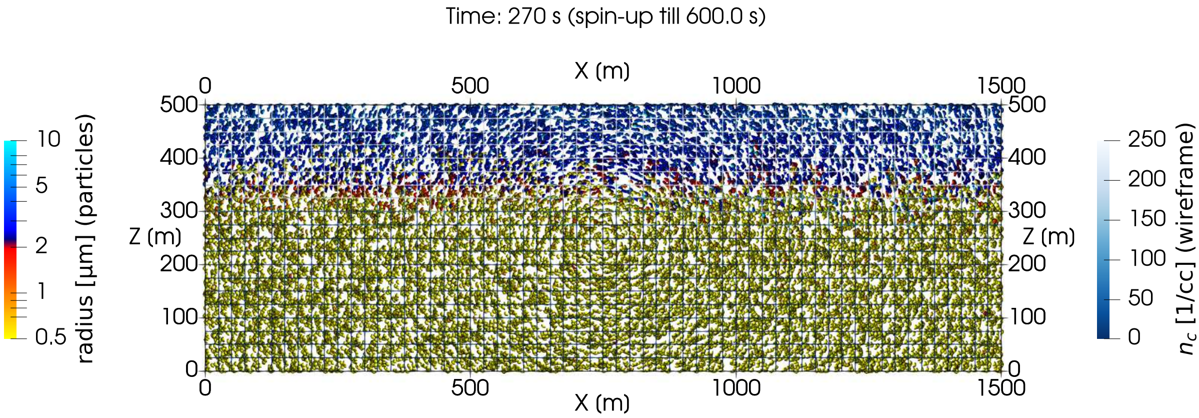

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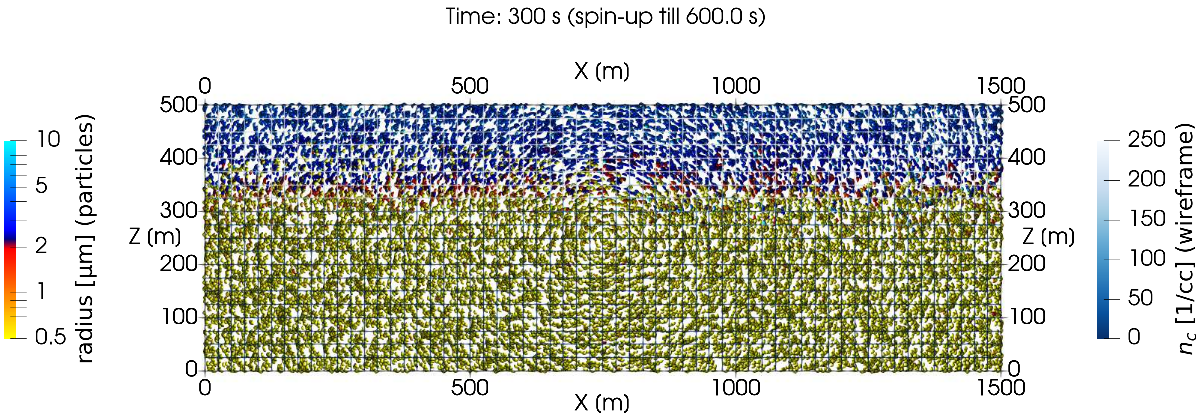

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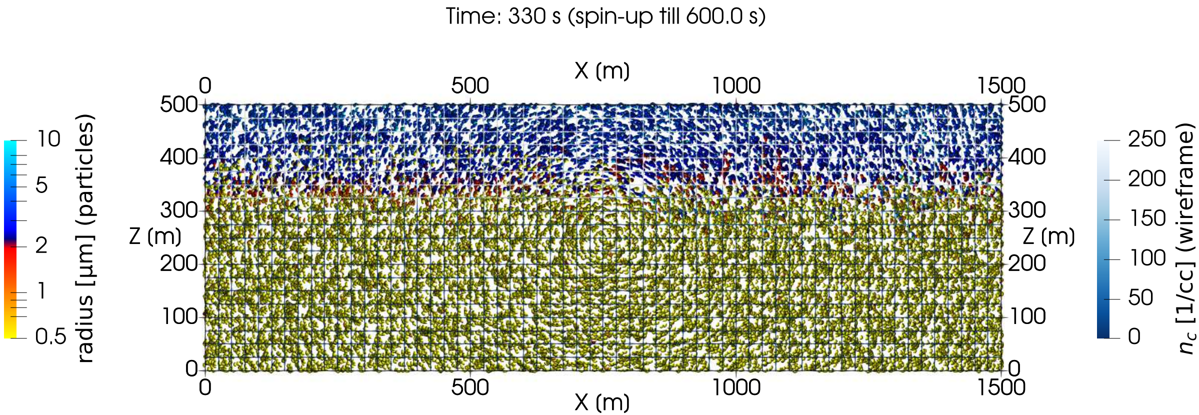

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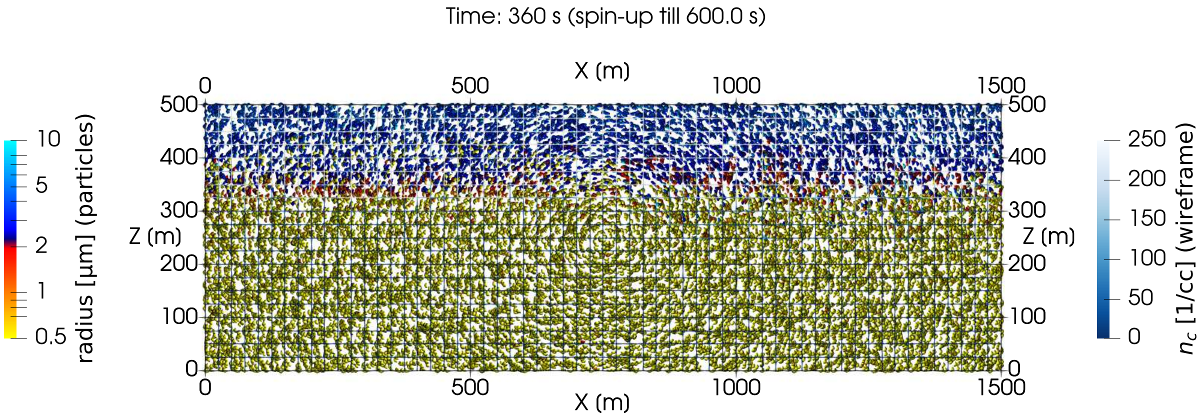

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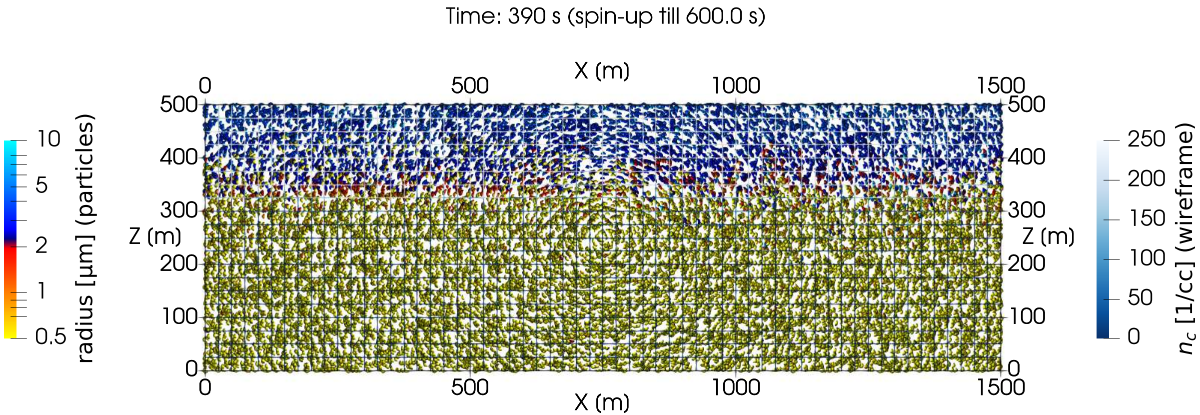

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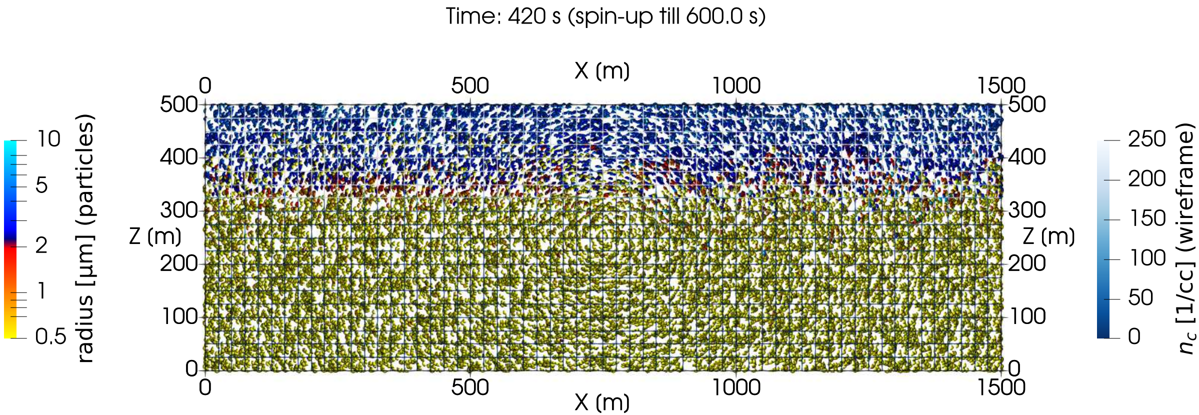

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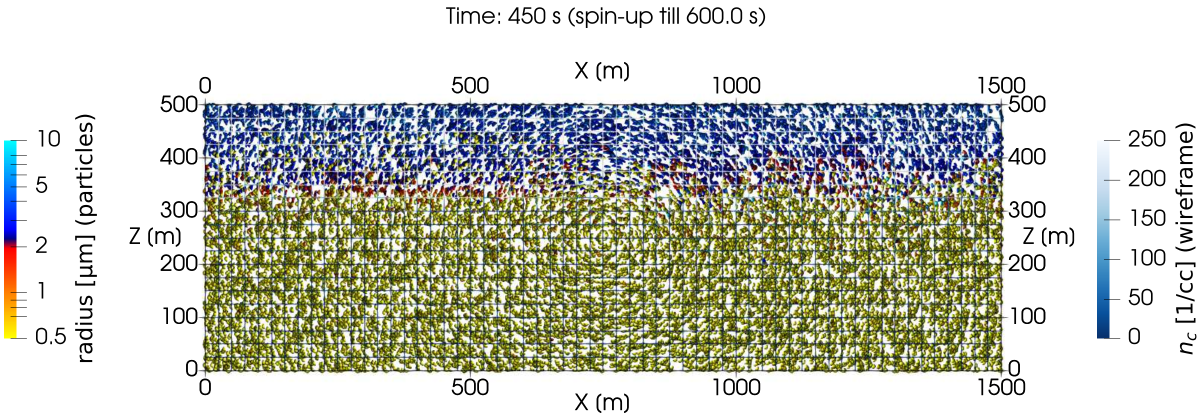

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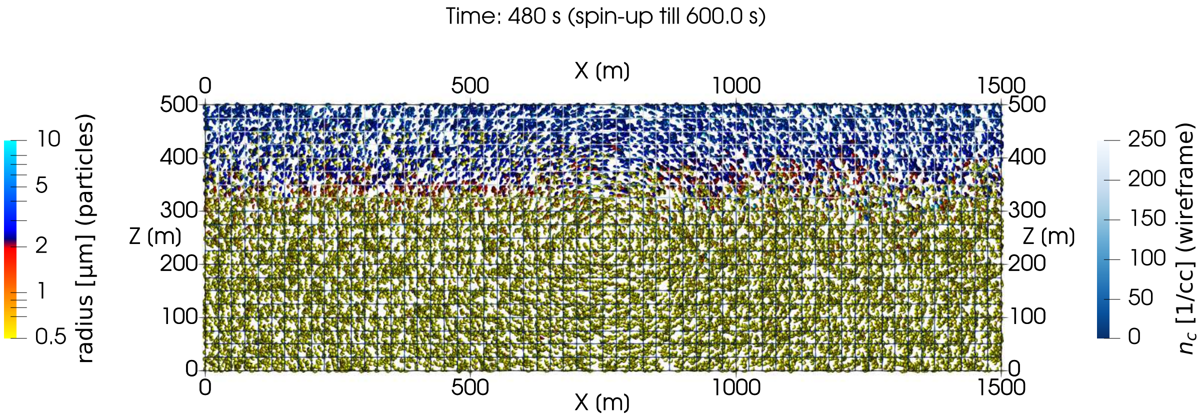

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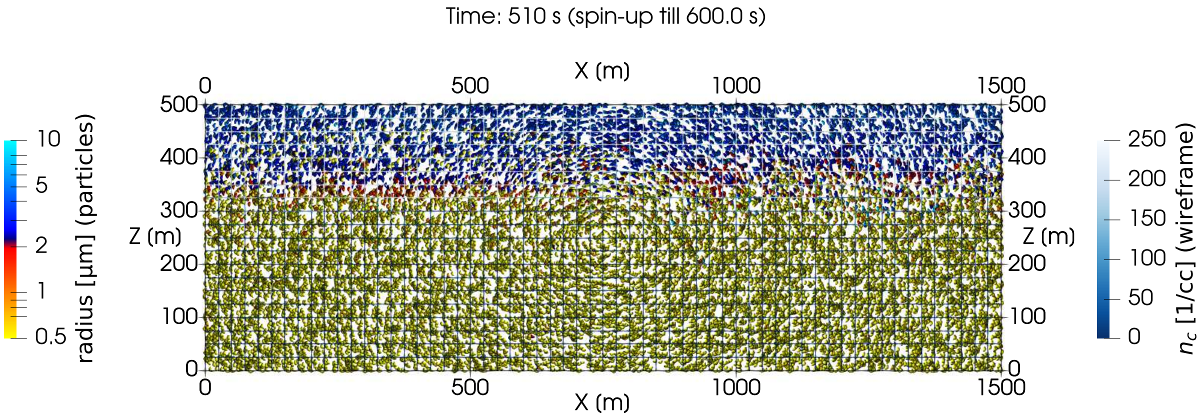

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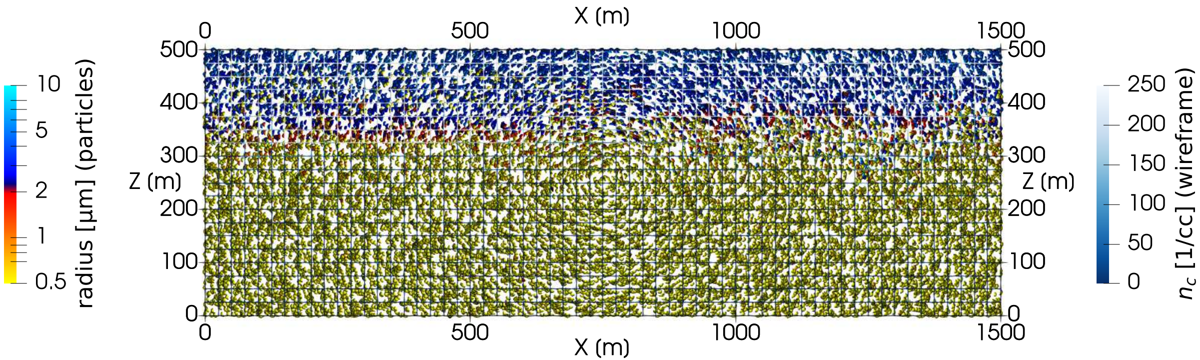
Time: 540 s (spin-up till 600.0 s)



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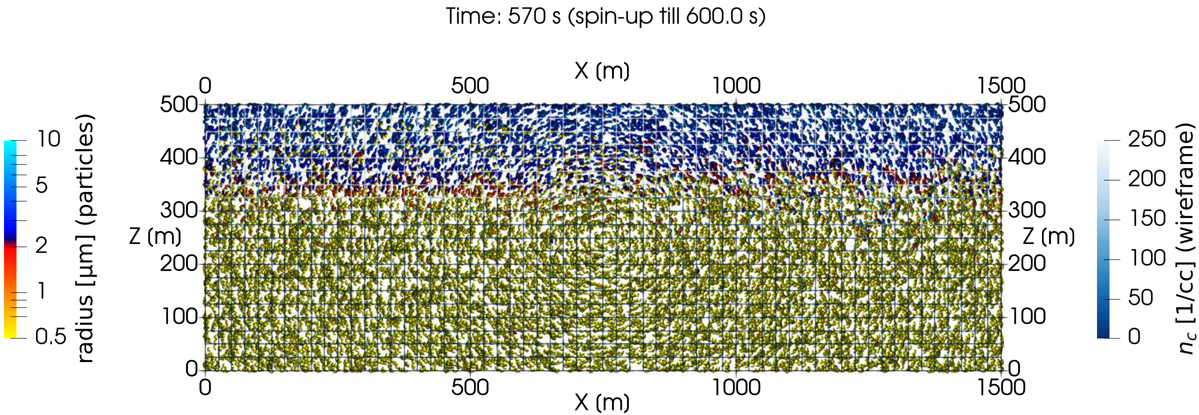

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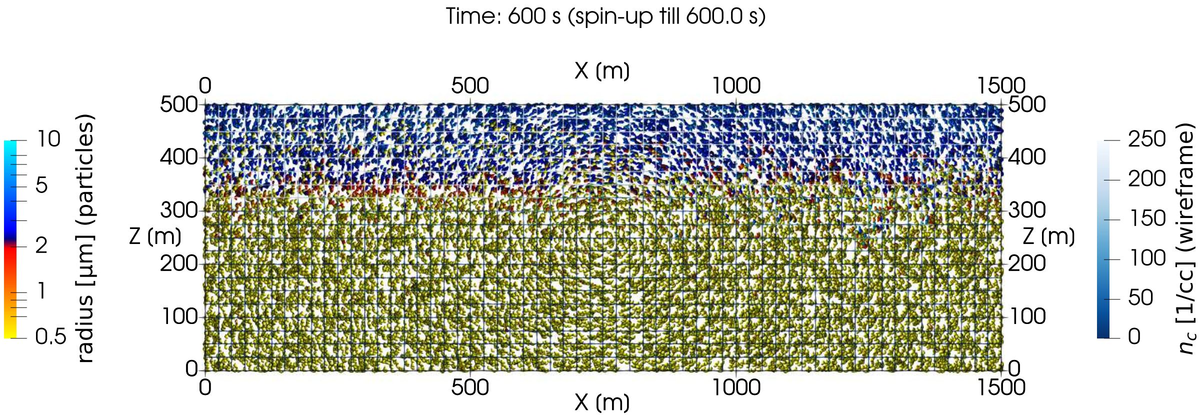

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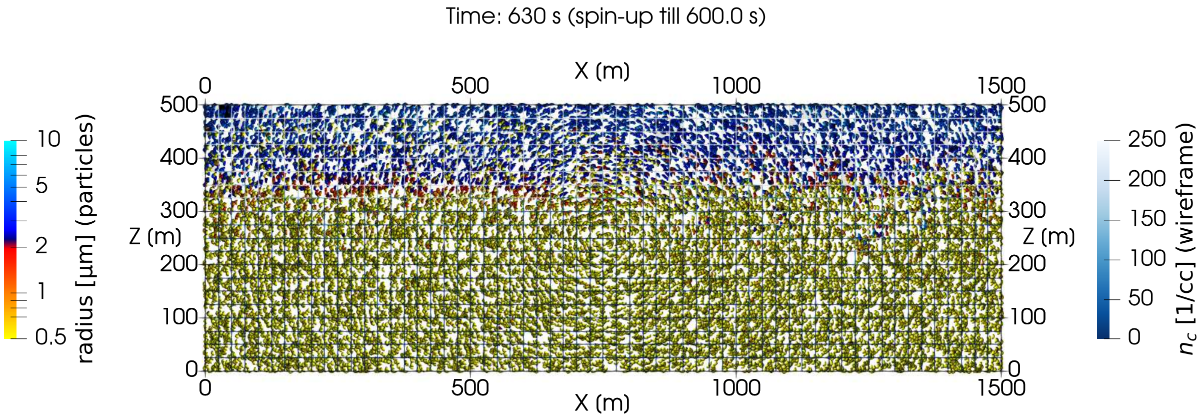

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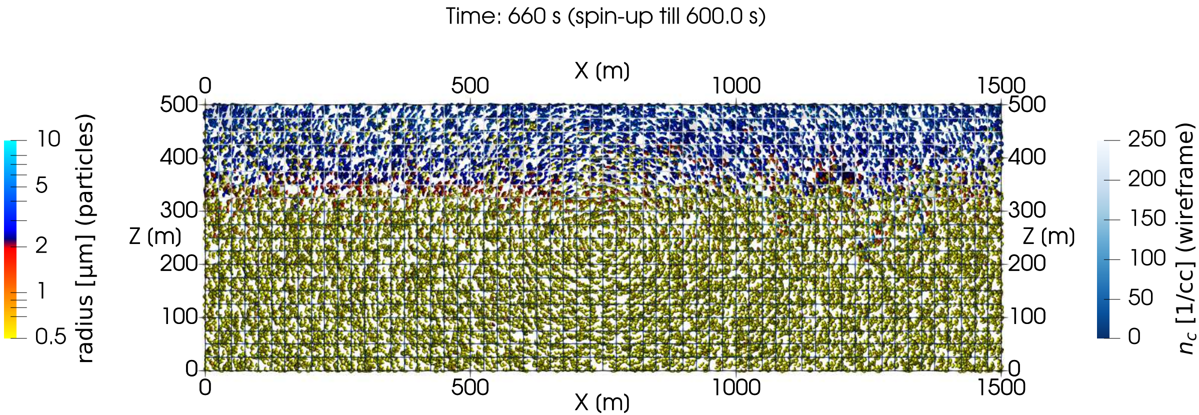

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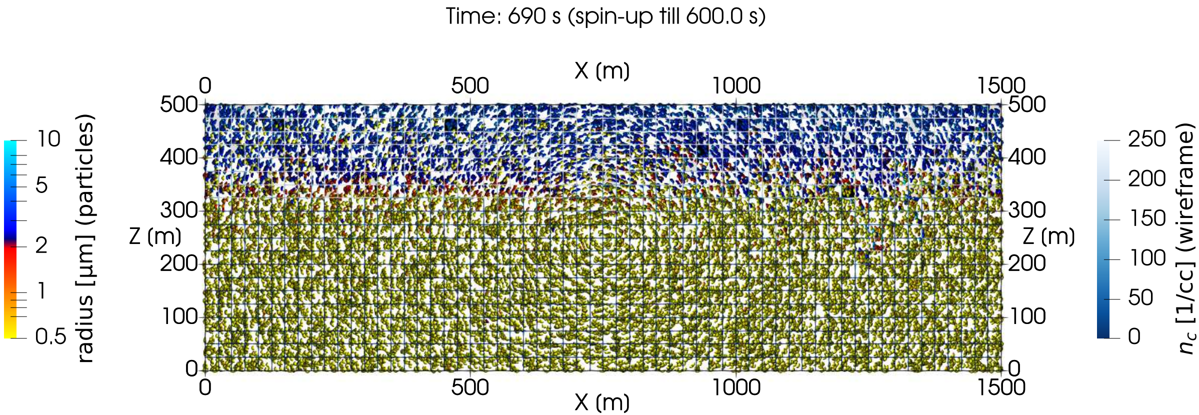
particle-based μ-physics + prescribed-flow test



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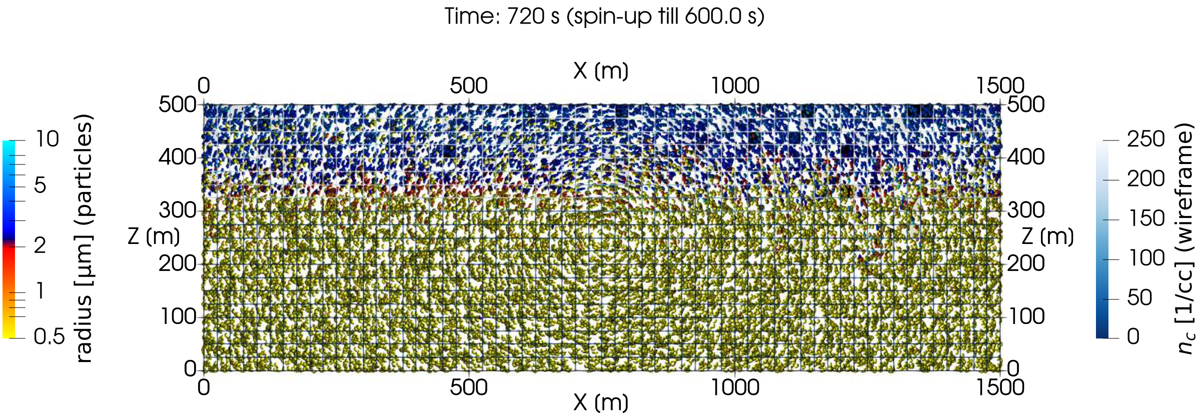

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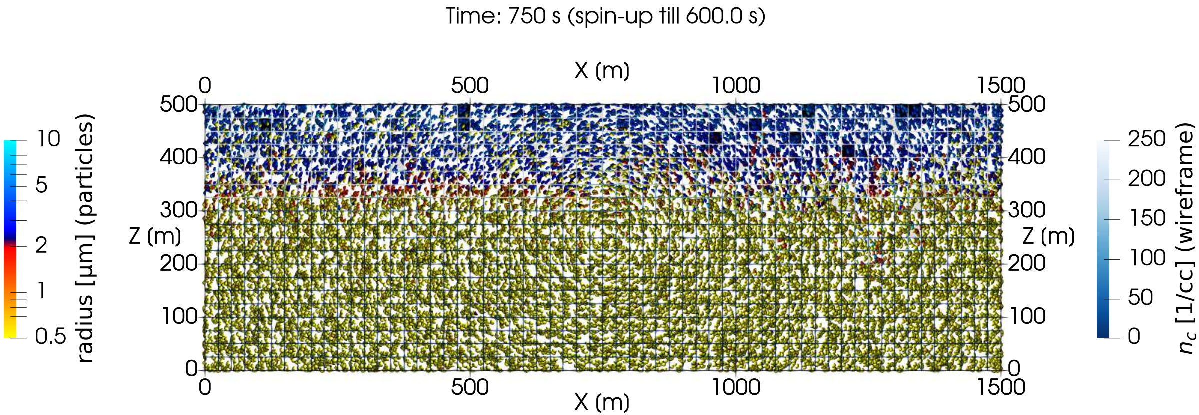

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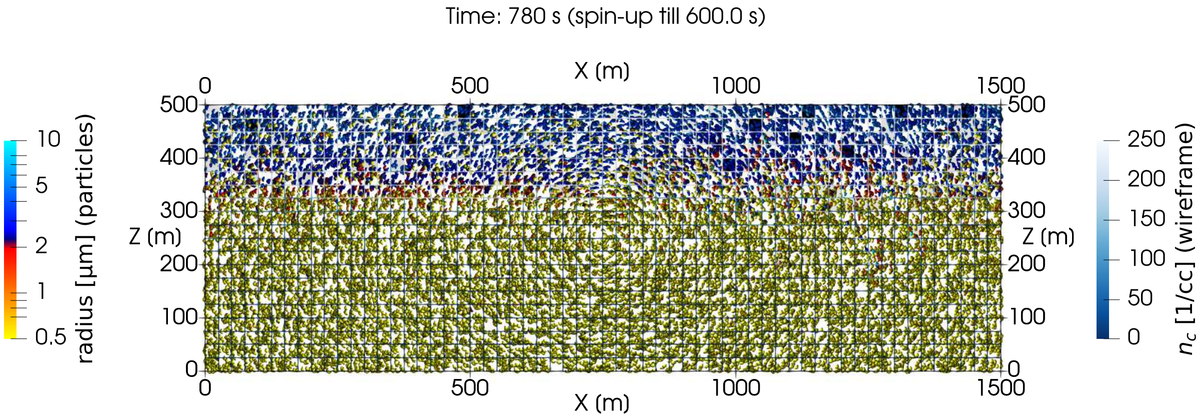

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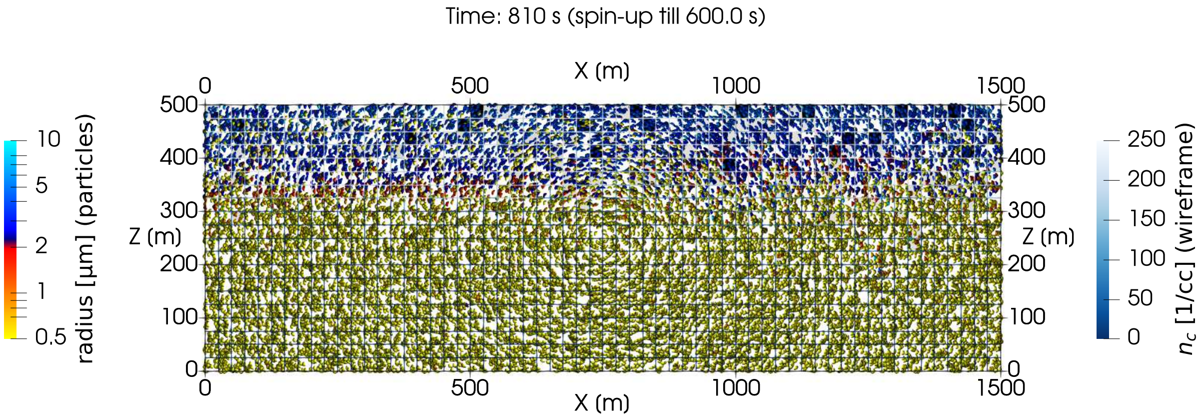

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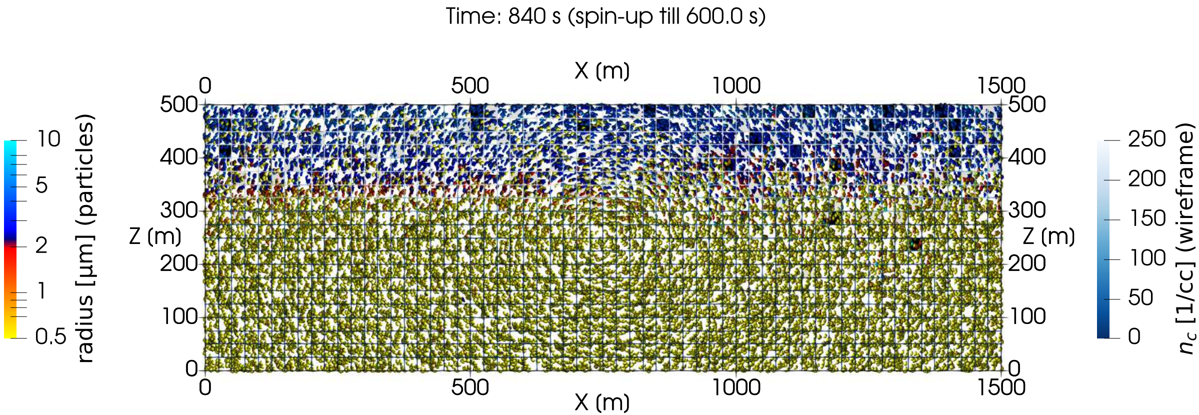

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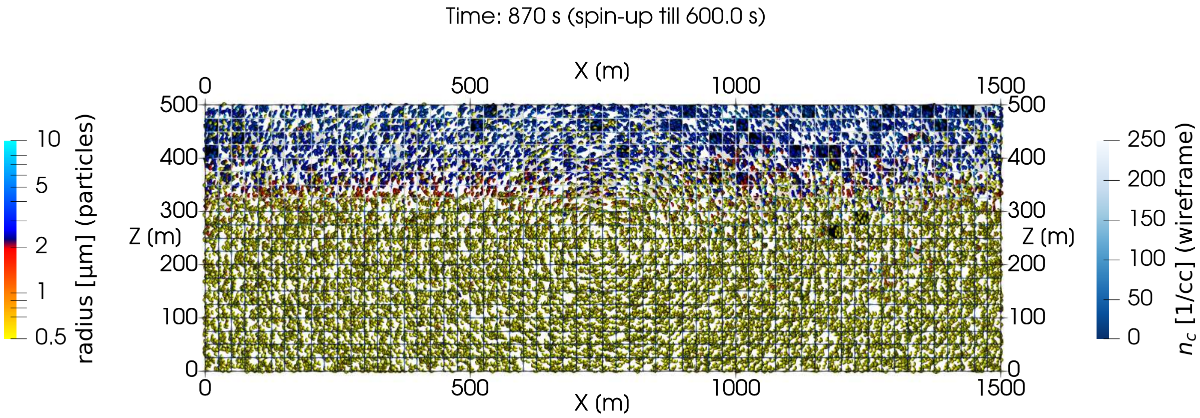

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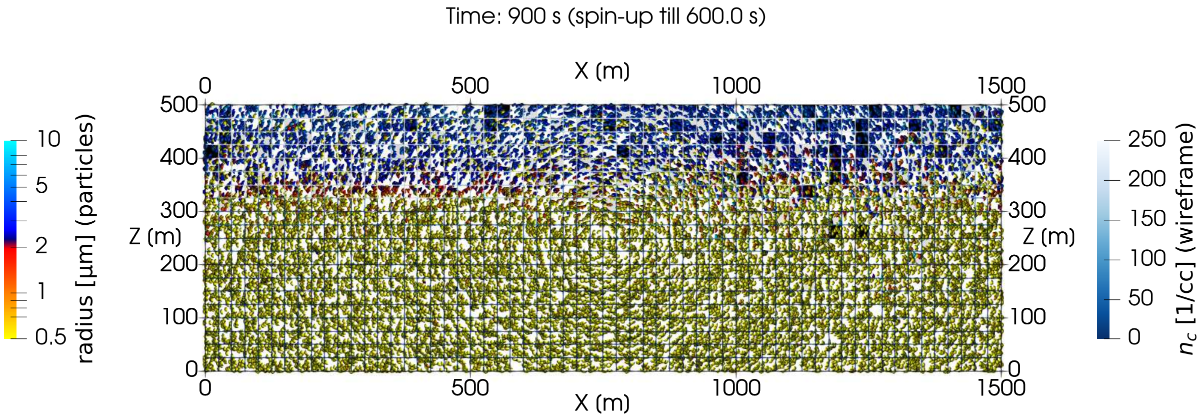

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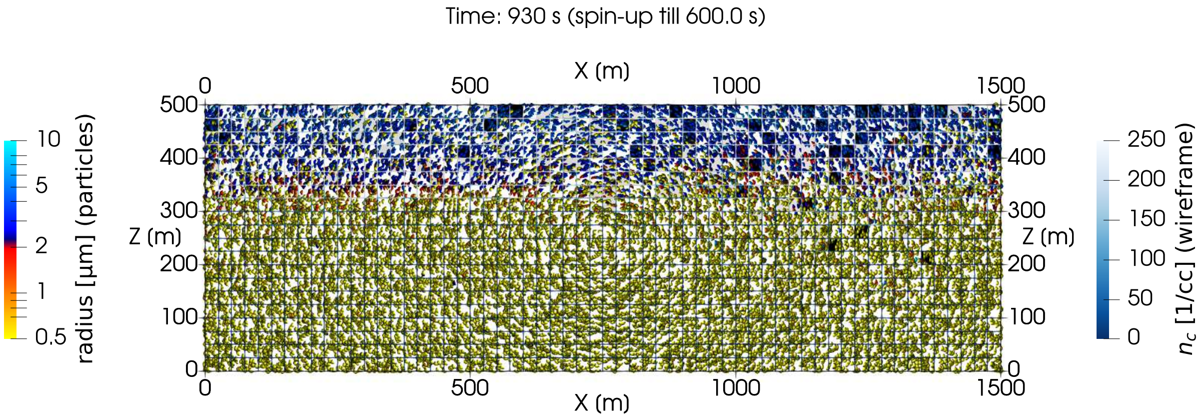

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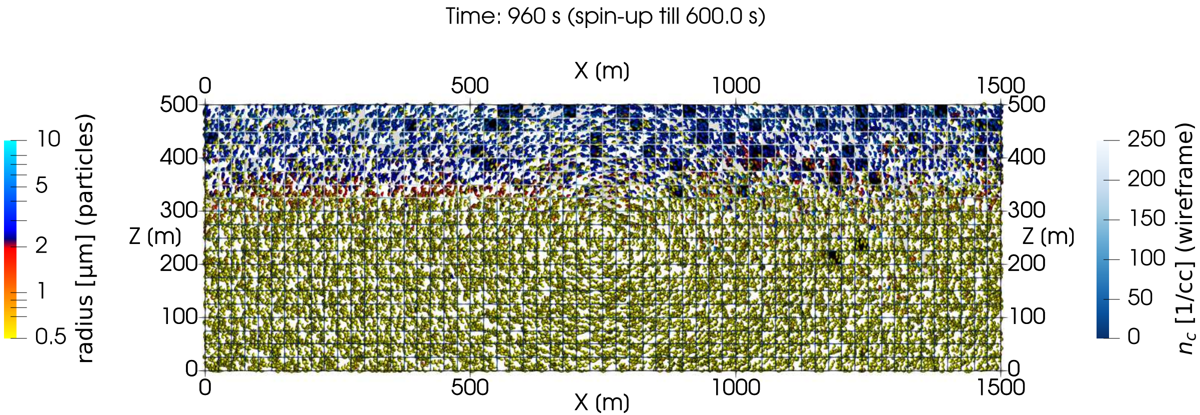

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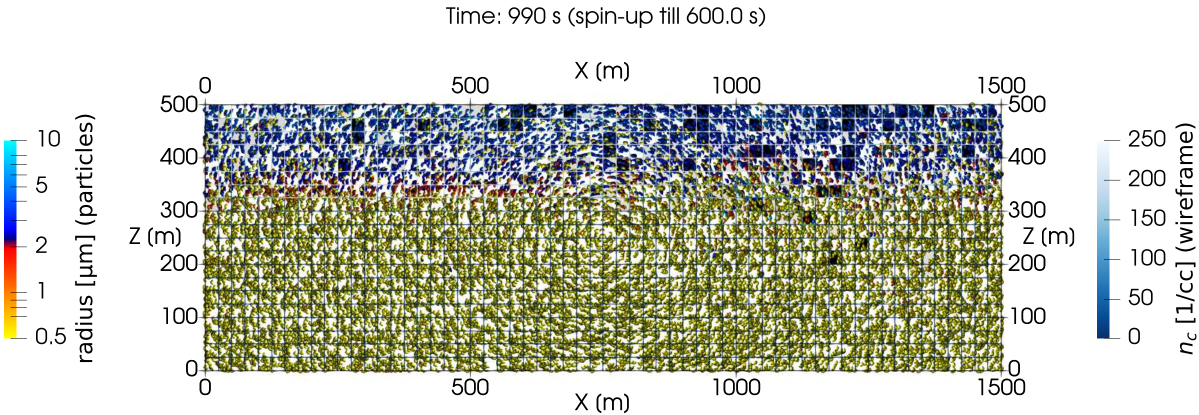

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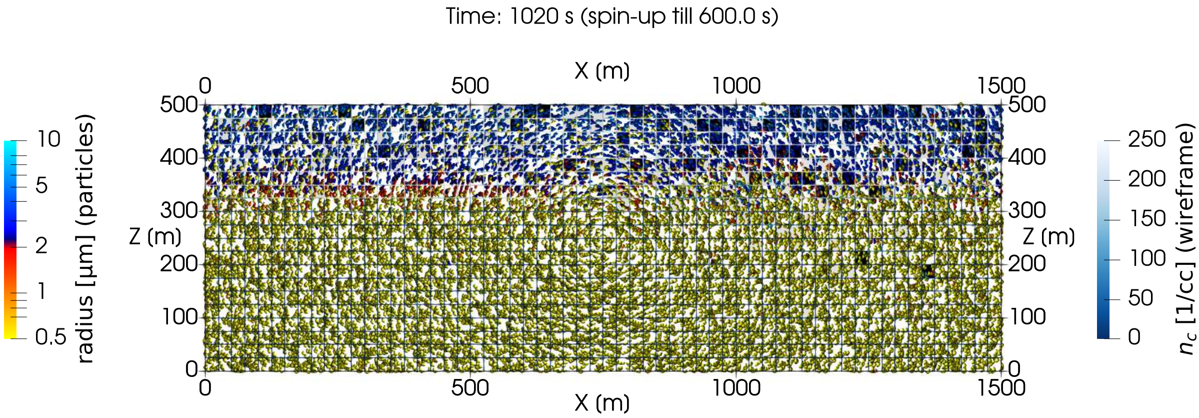

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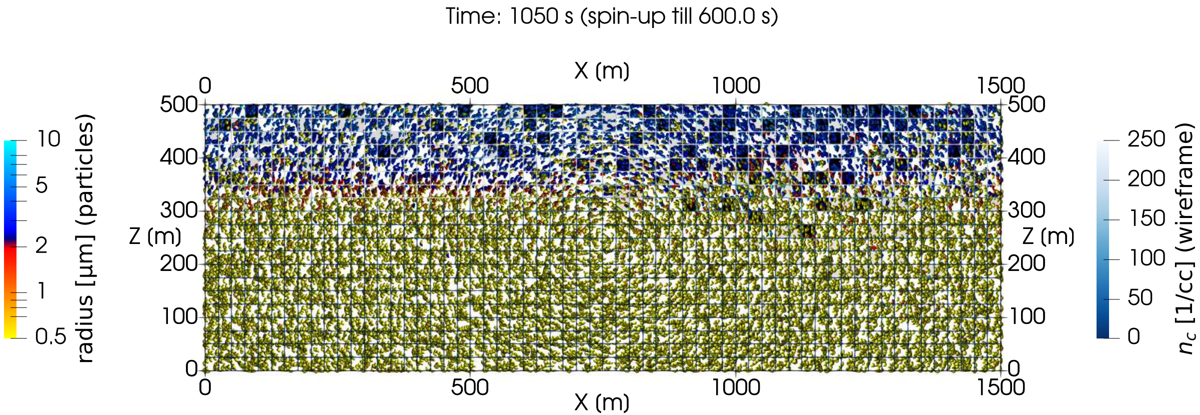

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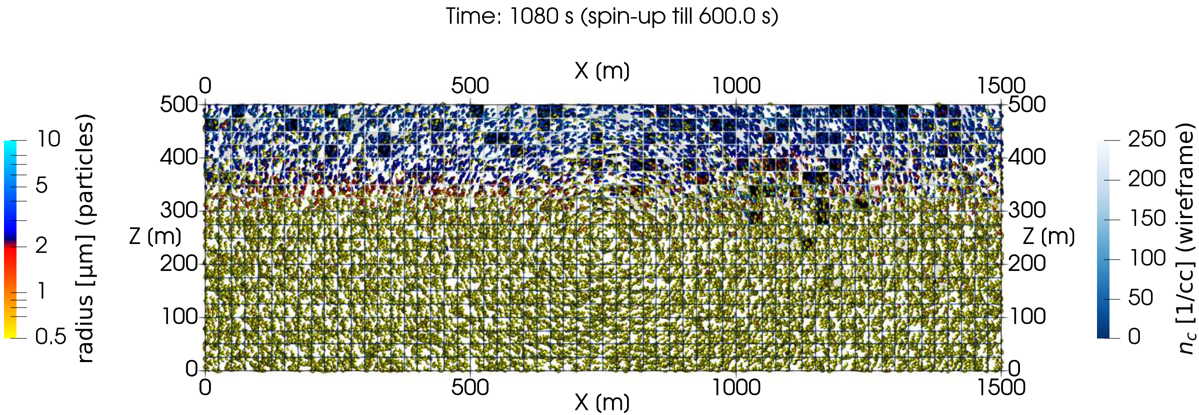

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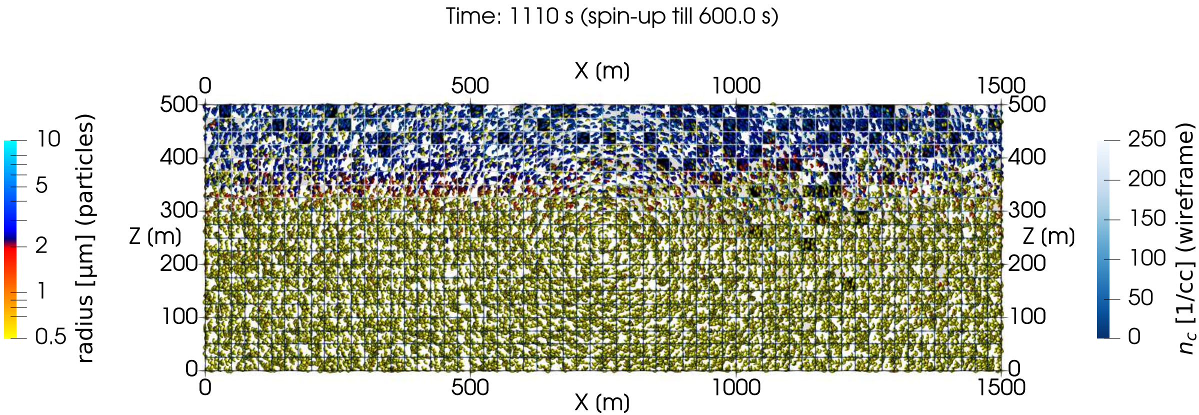

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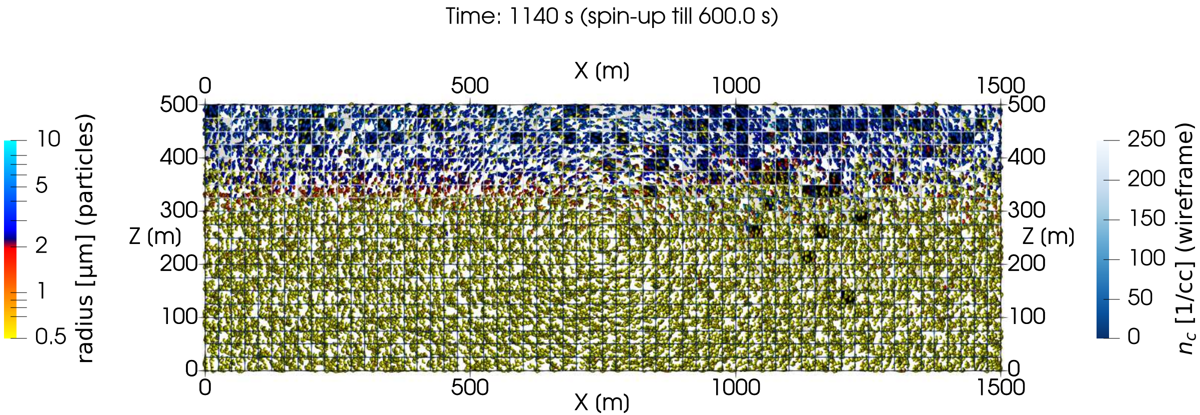

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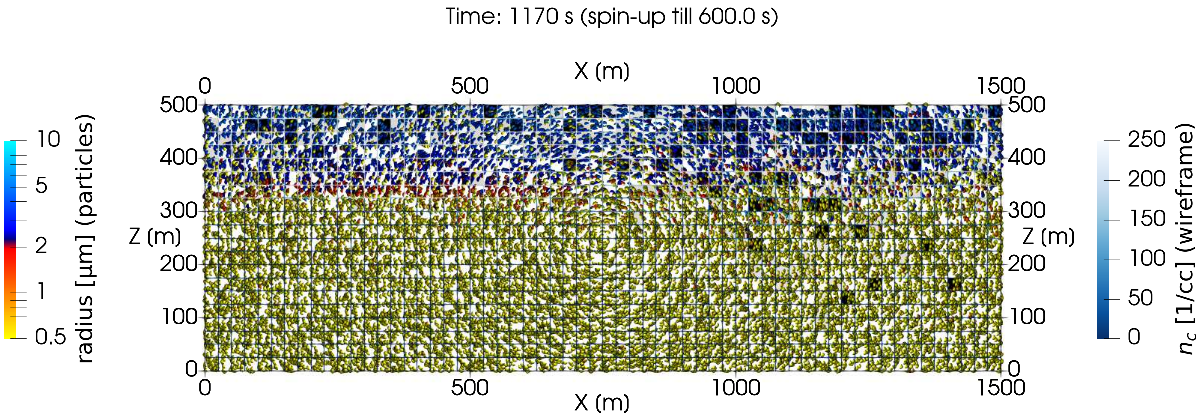

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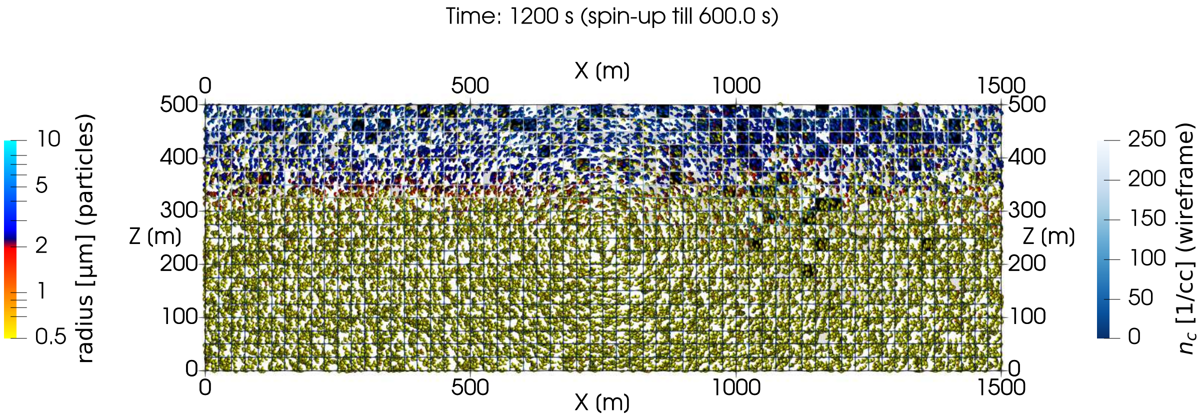

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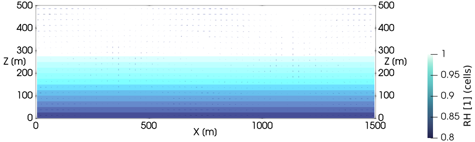
testing three flow regimes and two immersion freezing representations

$w_{max} \approx 1/3$ m/s



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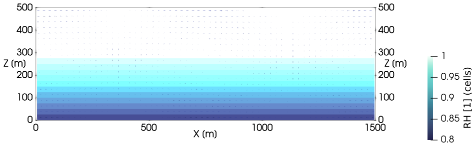

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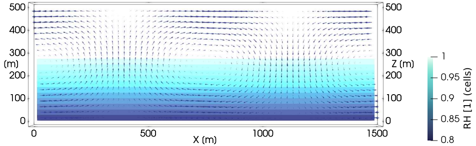

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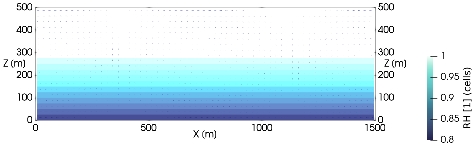

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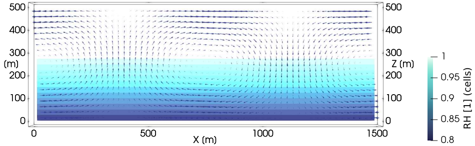

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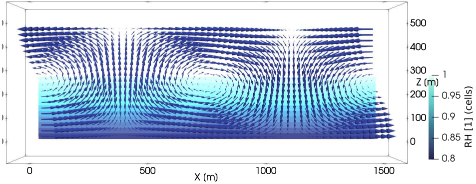

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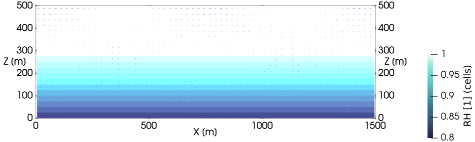

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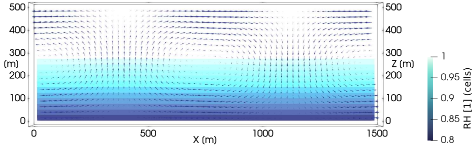

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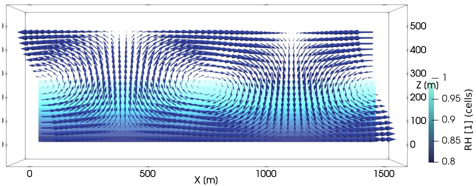
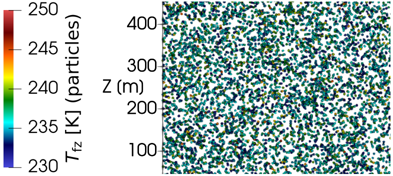


singular (INAS)



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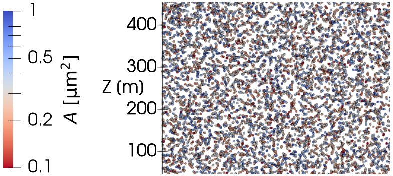

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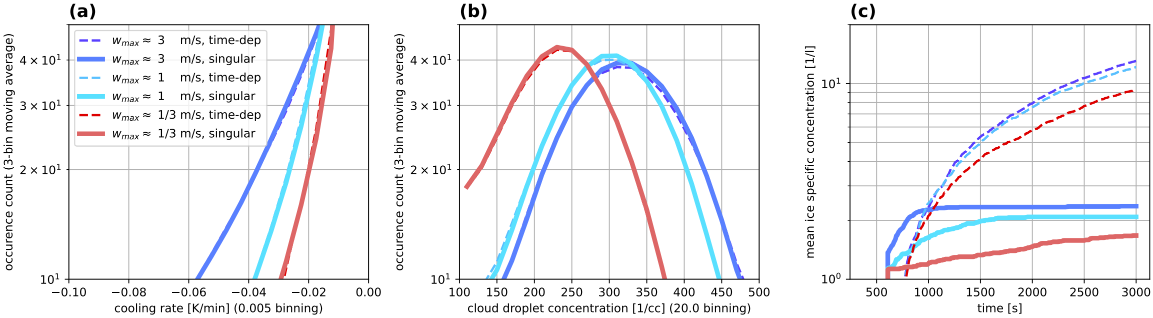

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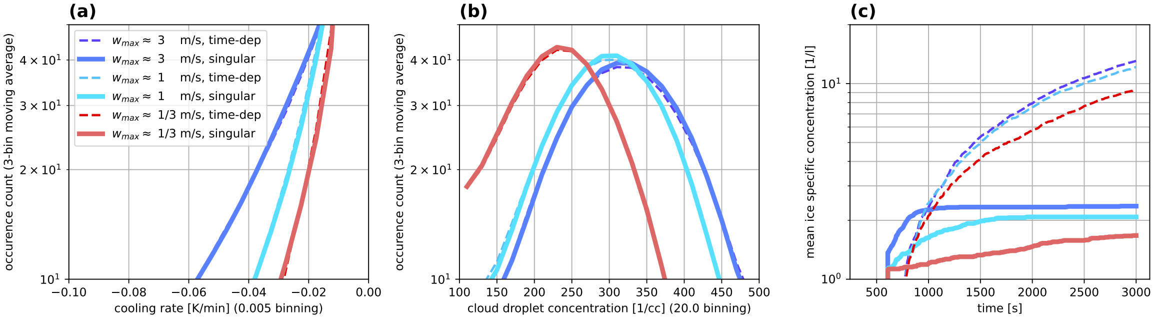

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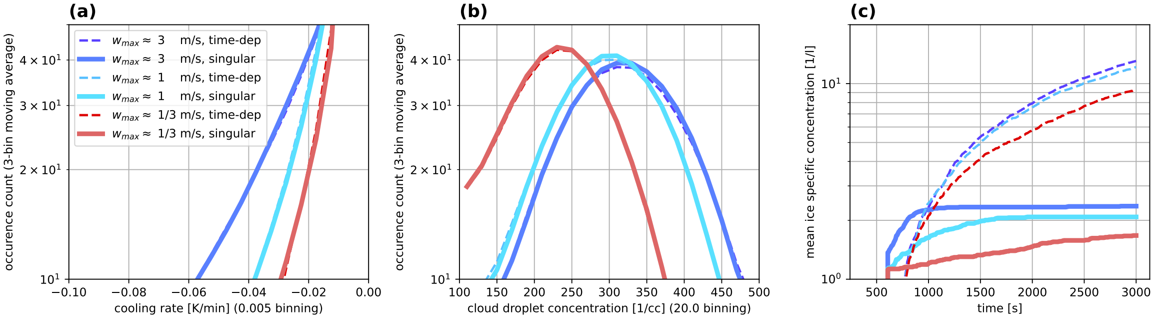

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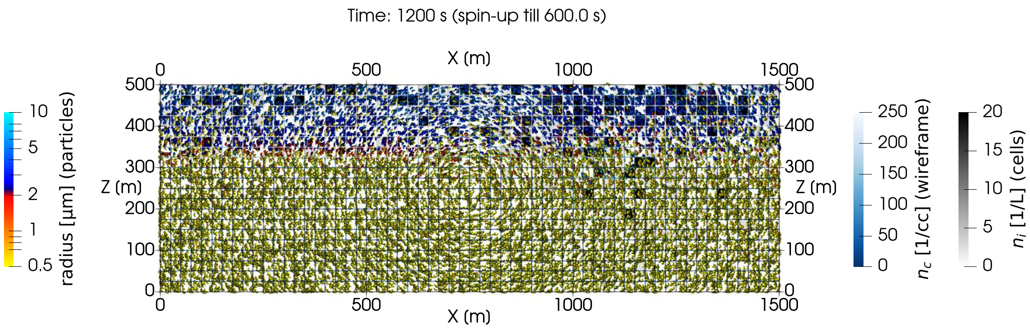

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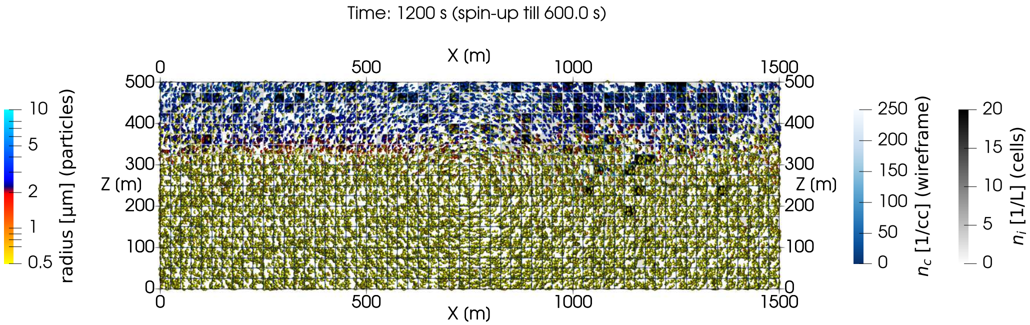
– cooling rate embedded in INAS fits ⇝ limited robustness to different flow regimes



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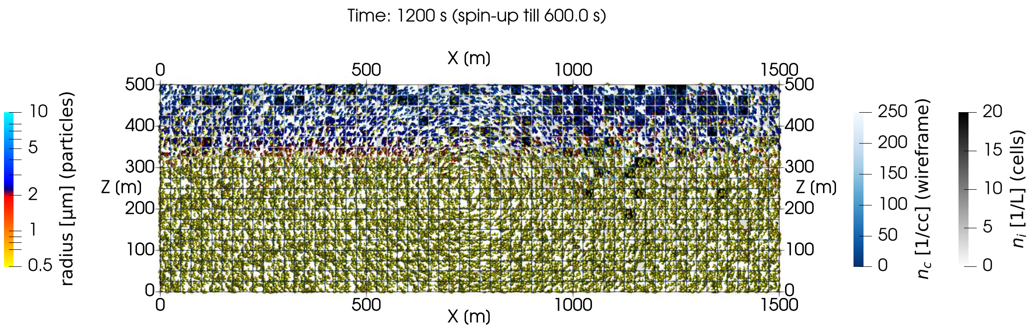

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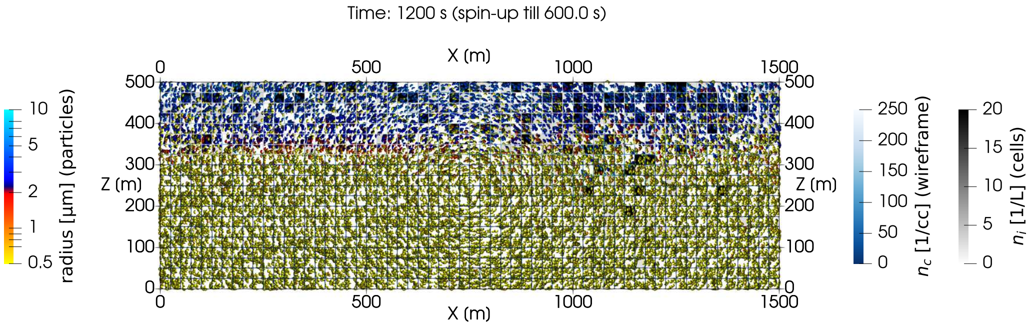

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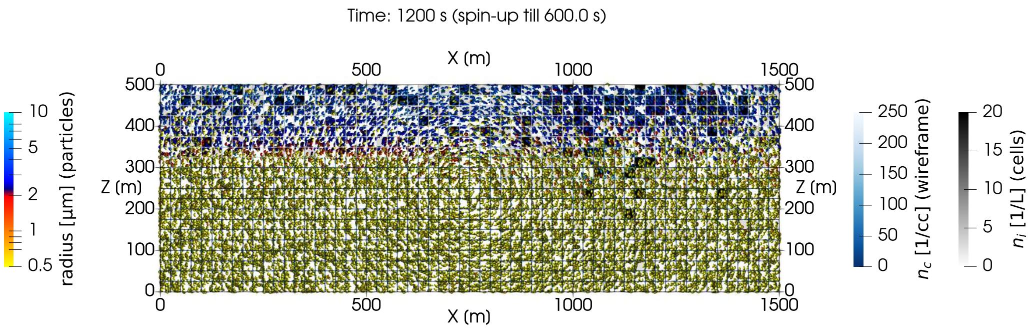

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