

Immersion freezing in particle-based cloud μ -physics models

S. Arabas¹, J.H. Curtis², I. Silber³, A. Fridlind⁴, D.A. Knopf⁵, M. West² & N. Riemer²

1:



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Atmospheric
System Research

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University of California Davis, Sep 3rd 2025

JAMES | Journal of Advances in
Modeling Earth Systems

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EARTH AND
SPACE SCIENCES

Immersion Freezing in Particle-Based Aerosol-Cloud Microphysics: A Probabilistic Perspective on Singular and Time-Dependent Models

Sylwester Arabas¹ , Jeffrey H. Curtis² , Israel Silber^{3,4} , Ann M. Fridlind⁵ ,
Daniel A. Knopf⁶ , Matthew West⁷ , and Nicole Riemer² 

10.1029/2024MS004770

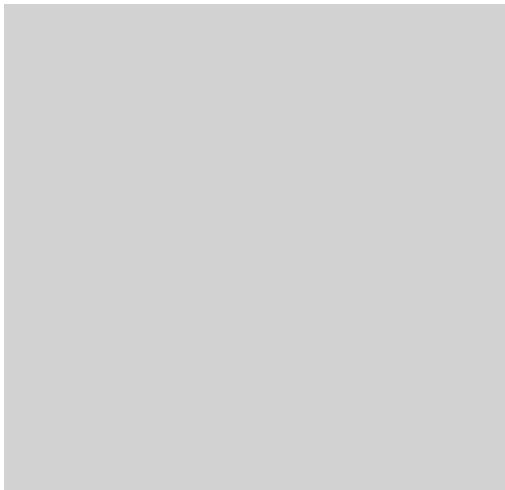
Aerosol-cloud interactions: a conceptual picture

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background image: vitsly / Hokusai

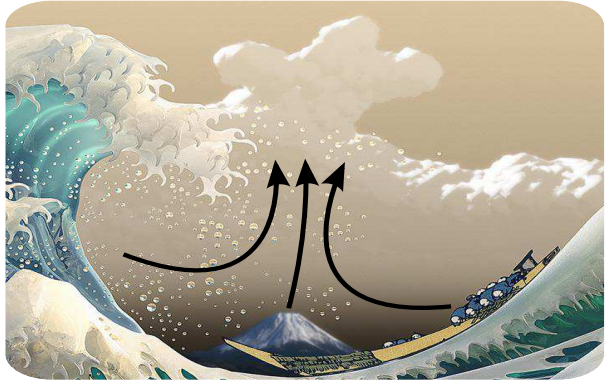
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background image: vitsly / Hokusai

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- ▶ aerosol particles of natural and anthropogenic origin act as condensation/crystallisation nuclei



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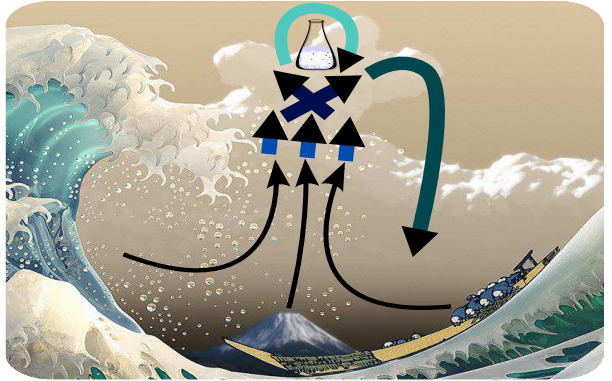
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two-way interactions:

- aerosol characteristics influence cloud microstructure
- cloud processes influence aerosol size and composition

background image: vitsly / Hokusai

Confronting the Challenge of Modeling Cloud and Precipitation Microphysics

Key Points:

- Microphysics is an important component of weather and climate models, but its representation in current models is highly uncertain

Hugh Morrison¹ , Marcus van Lier-Walqui² , Ann M. Fridlind³ ,
Wojciech W. Grabowski¹ , Jerry Y. Harrington⁴, Corinna Hoose⁵ , Alexei Korolev⁶ ,
Matthew R. Kumjian⁴ , Jason A. Milbrandt⁷, Hanna Pawlowska⁸ , Derek J. Posselt⁹,
Olivier P. Prat¹⁰, Karly J. Reimel⁴, Shin-Ichiro Shima¹¹ , Bastiaan van Dierenhoven² ,
and Lulin Xue¹ 

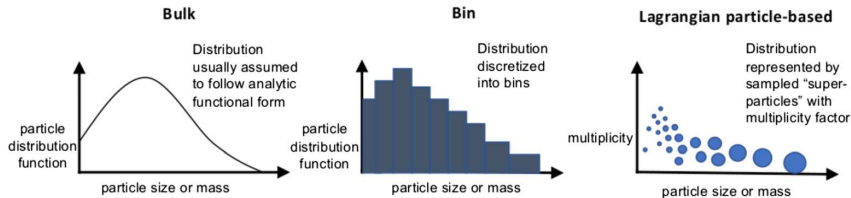


Figure 3. Representation of cloud and precipitation particle distributions in the three main types of microphysics

Aerosol-cloud interactions: μ -physics models

well suited for two-way aerosol-cloud interactions!

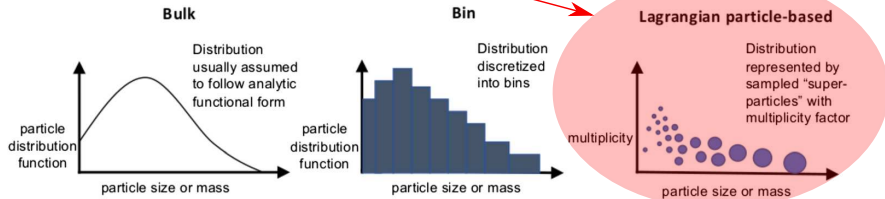
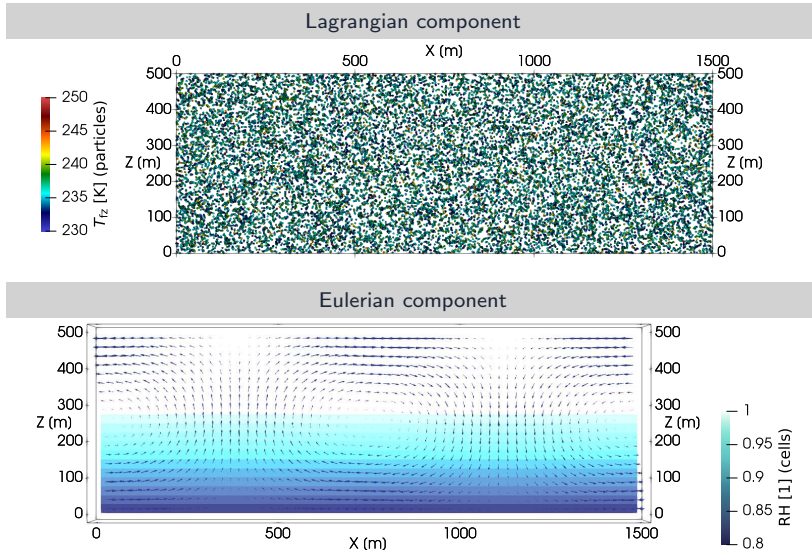


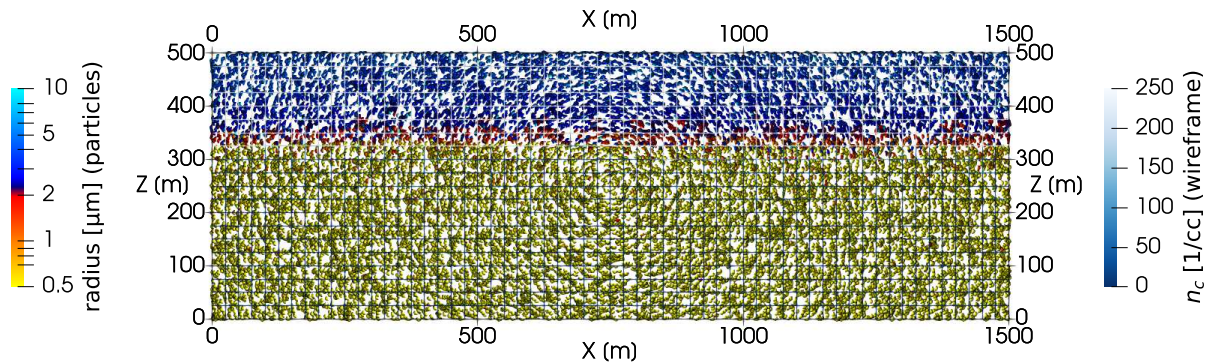
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Particle-based μ -physics + prescribed-flow: model state



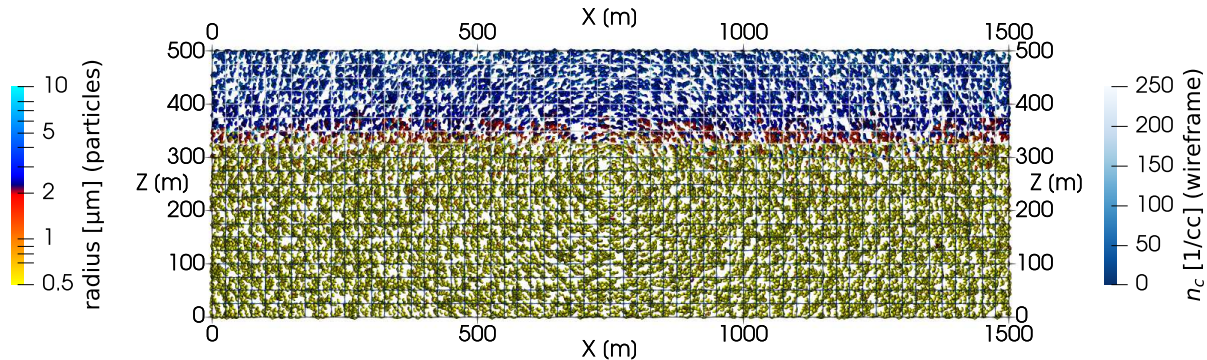
Particle-based μ -physics + prescribed-flow: spin-up

Time: 30 s (spin-up till 600.0 s)



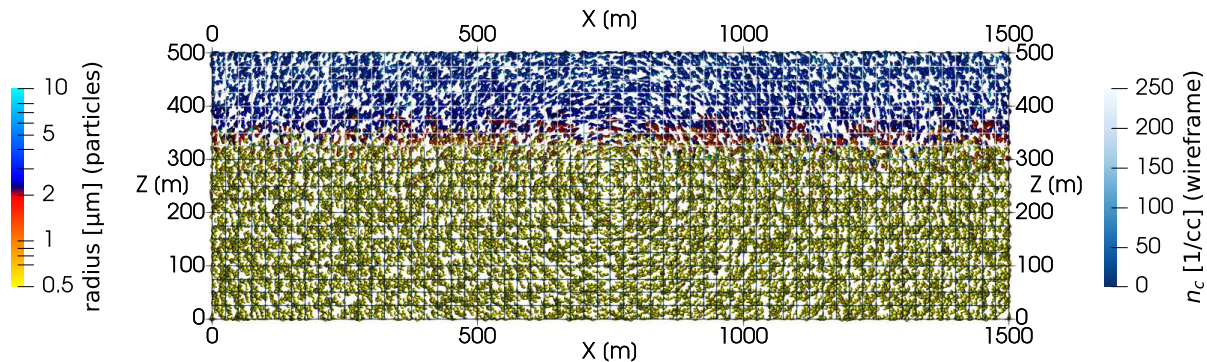
Particle-based μ -physics + prescribed-flow: spin-up

Time: 60 s (spin-up till 600.0 s)



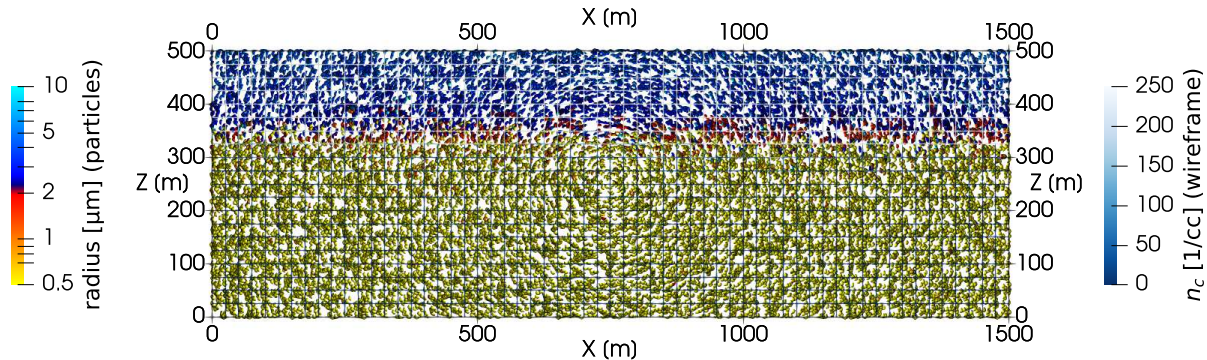
Particle-based μ -physics + prescribed-flow: spin-up

Time: 90 s (spin-up till 600.0 s)



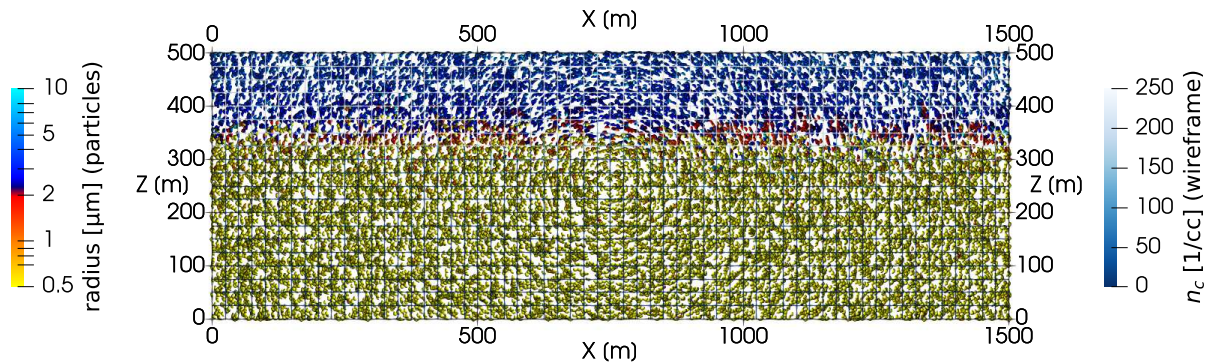
Particle-based μ -physics + prescribed-flow: spin-up

Time: 120 s (spin-up till 600.0 s)



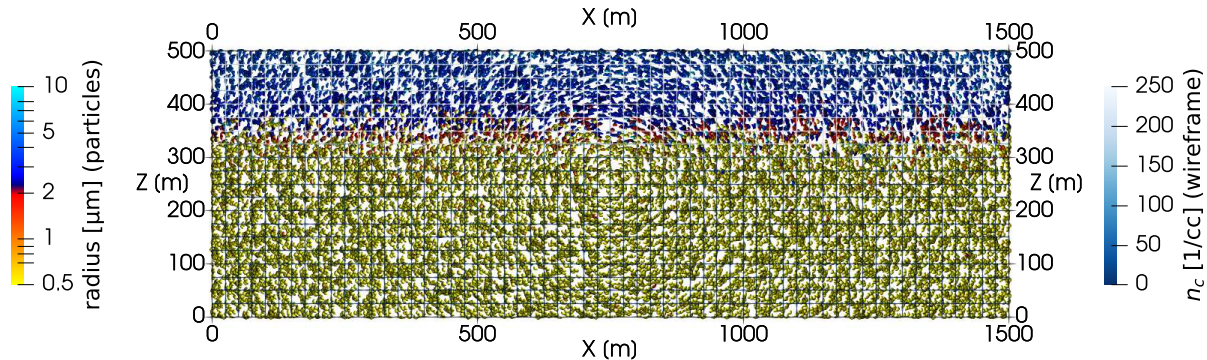
Particle-based μ -physics + prescribed-flow: spin-up

Time: 150 s (spin-up till 600.0 s)



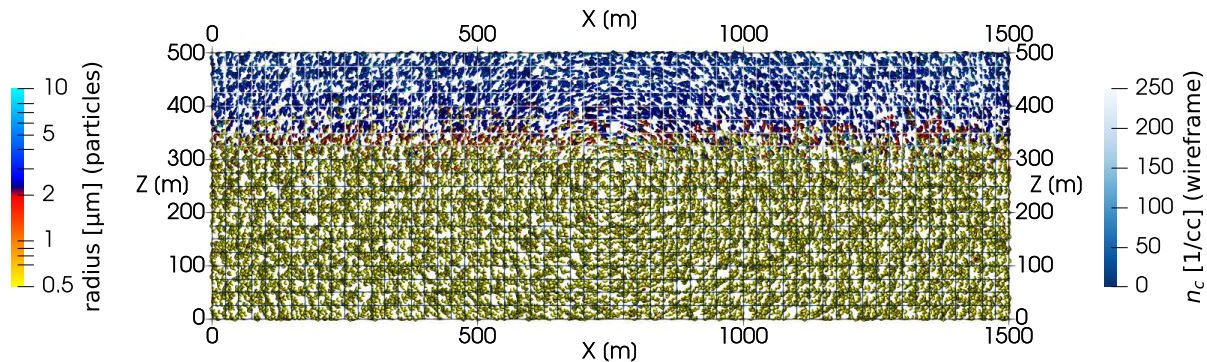
Particle-based μ -physics + prescribed-flow: spin-up

Time: 180 s (spin-up till 600.0 s)



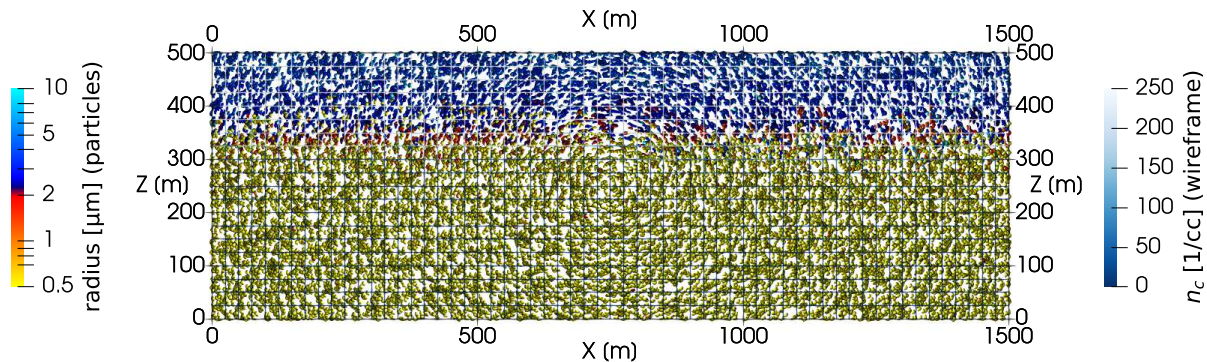
Particle-based μ -physics + prescribed-flow: spin-up

Time: 210 s (spin-up till 600.0 s)



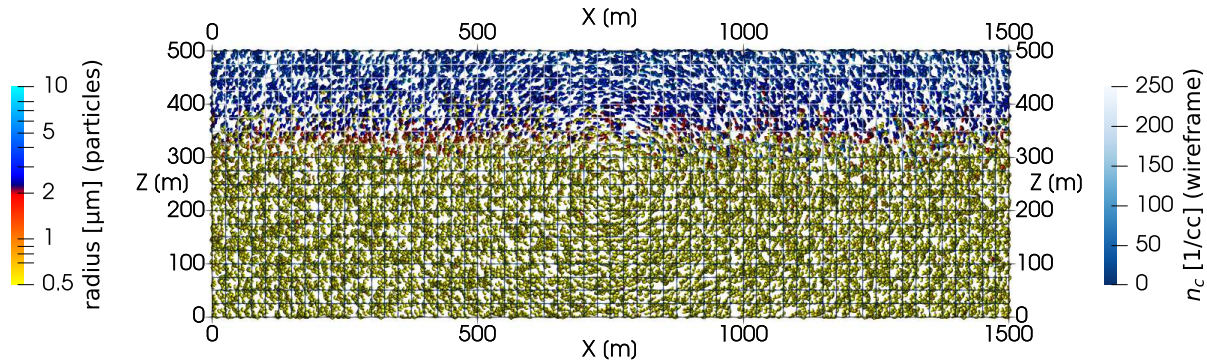
Particle-based μ -physics + prescribed-flow: spin-up

Time: 240 s (spin-up till 600.0 s)



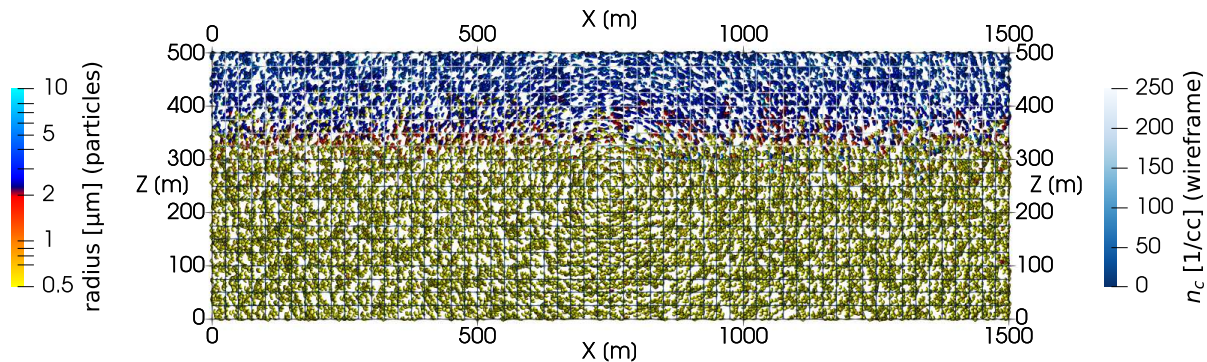
Particle-based μ -physics + prescribed-flow: spin-up

Time: 270 s (spin-up till 600.0 s)



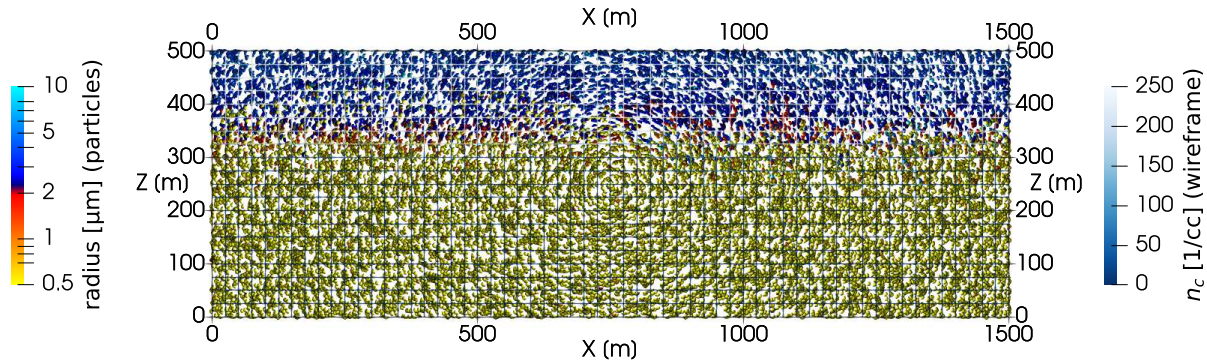
Particle-based μ -physics + prescribed-flow: spin-up

Time: 300 s (spin-up till 600.0 s)



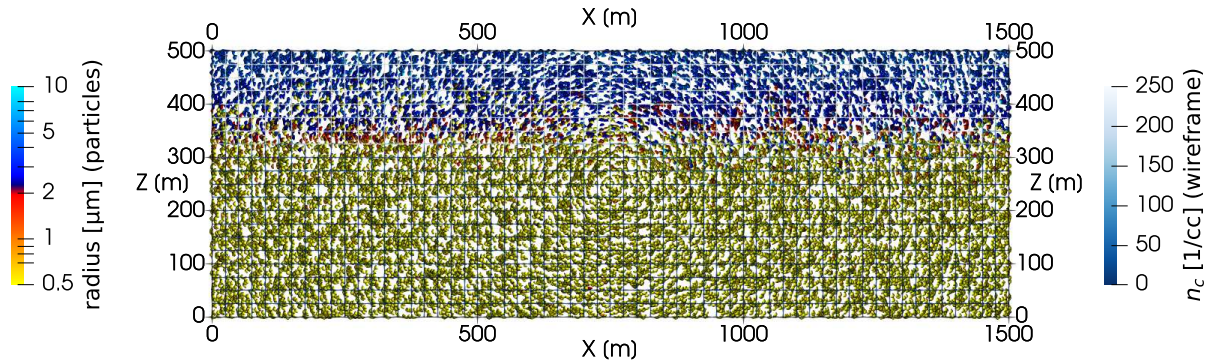
Particle-based μ -physics + prescribed-flow: spin-up

Time: 330 s (spin-up till 600.0 s)



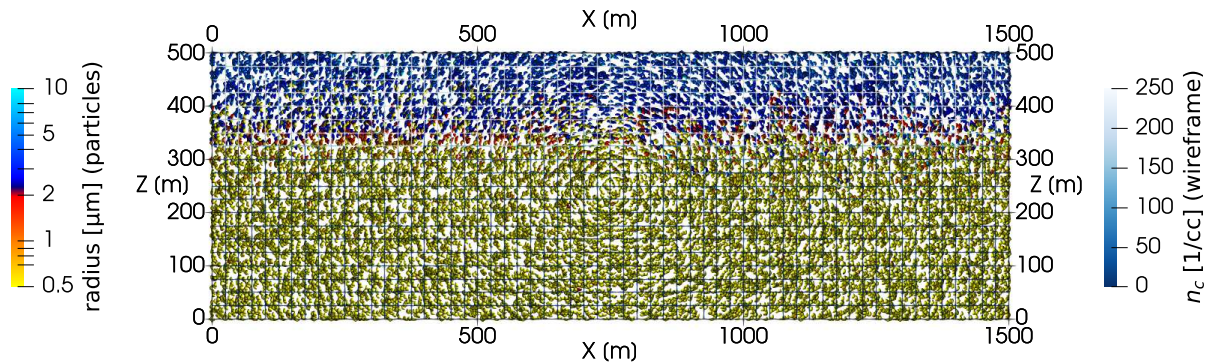
Particle-based μ -physics + prescribed-flow: spin-up

Time: 360 s (spin-up till 600.0 s)



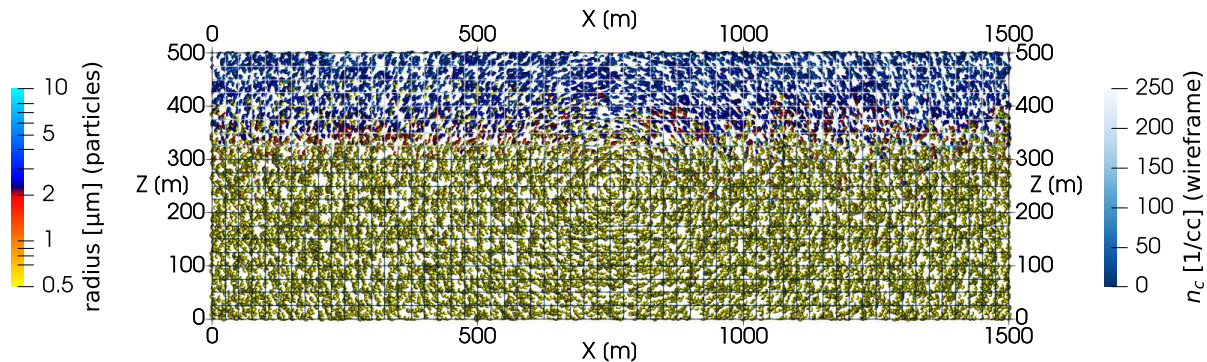
Particle-based μ -physics + prescribed-flow: spin-up

Time: 390 s (spin-up till 600.0 s)



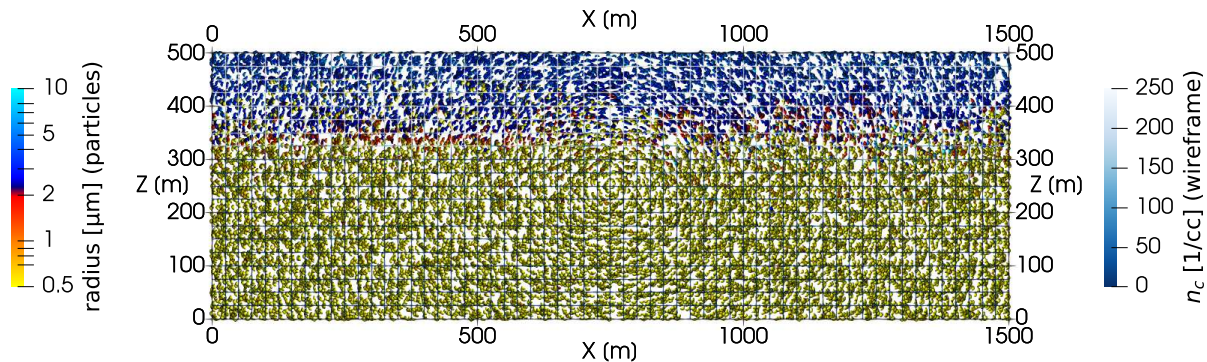
Particle-based μ -physics + prescribed-flow: spin-up

Time: 420 s (spin-up till 600.0 s)



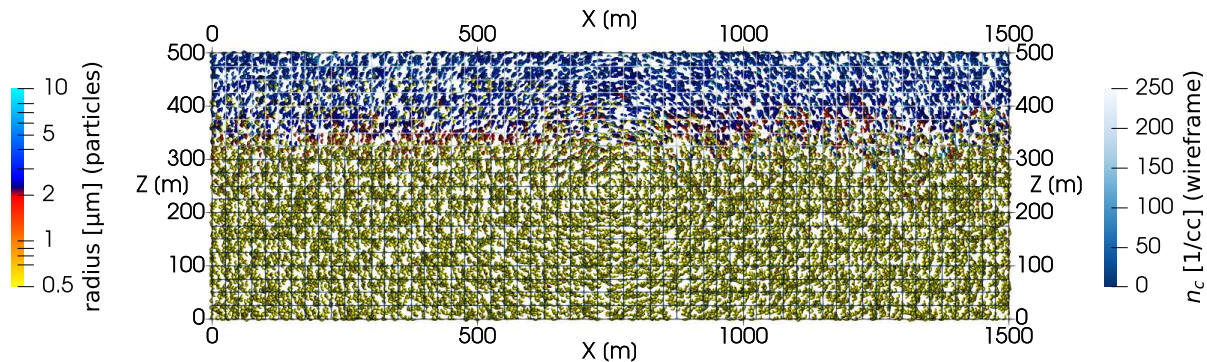
Particle-based μ -physics + prescribed-flow: spin-up

Time: 450 s (spin-up till 600.0 s)



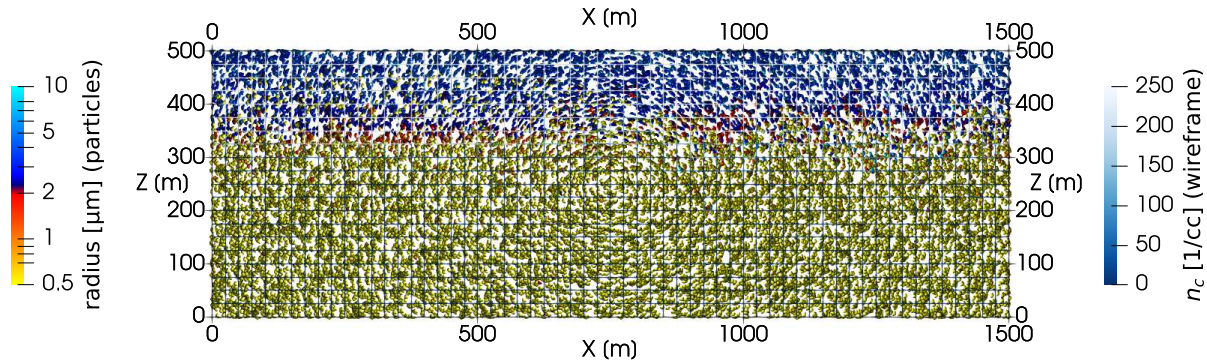
Particle-based μ -physics + prescribed-flow: spin-up

Time: 480 s (spin-up till 600.0 s)



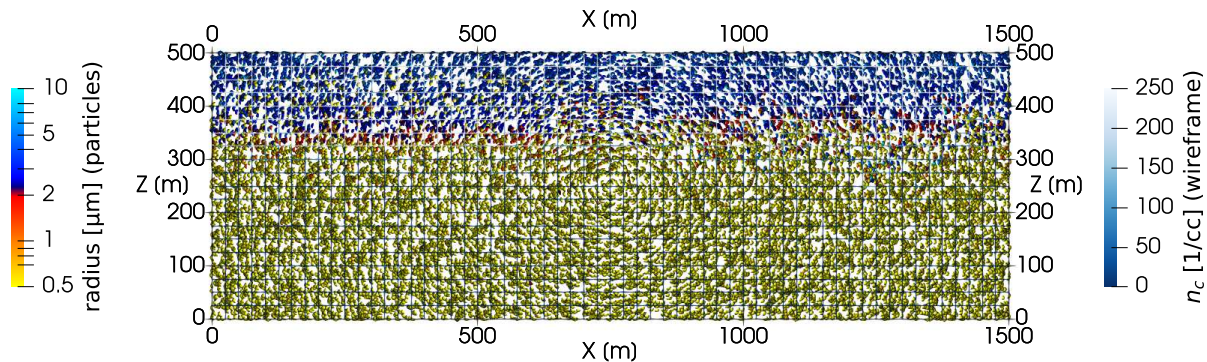
Particle-based μ -physics + prescribed-flow: spin-up

Time: 510 s (spin-up till 600.0 s)



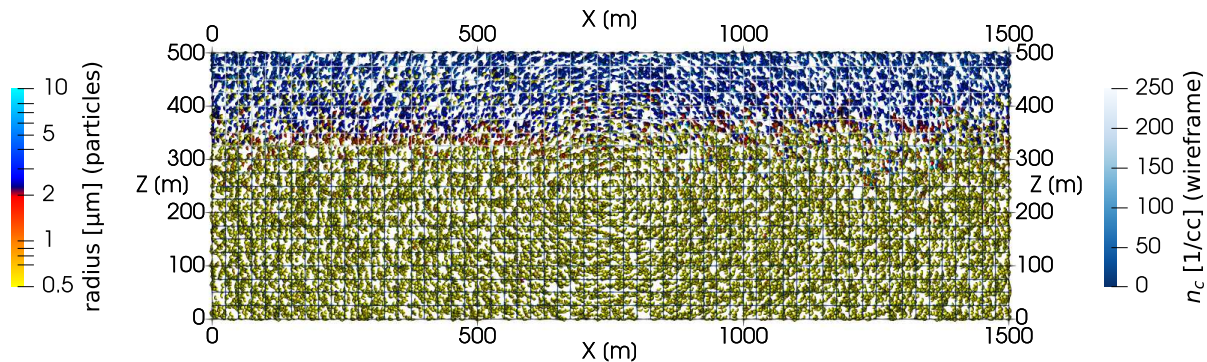
Particle-based μ -physics + prescribed-flow: spin-up

Time: 540 s (spin-up till 600.0 s)



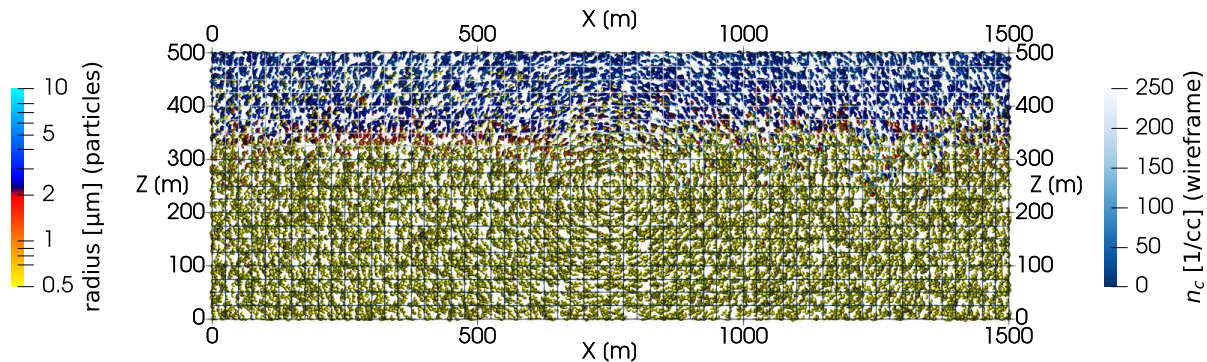
Particle-based μ -physics + prescribed-flow: spin-up

Time: 570 s (spin-up till 600.0 s)



Particle-based μ -physics + prescribed-flow: spin-up

Time: 600 s (spin-up till 600.0 s)



Shima, Sato, Hashimoto & Misumi 2020 (GMD):

Predicting the morphology of ice particles in deep convection using the super-droplet method

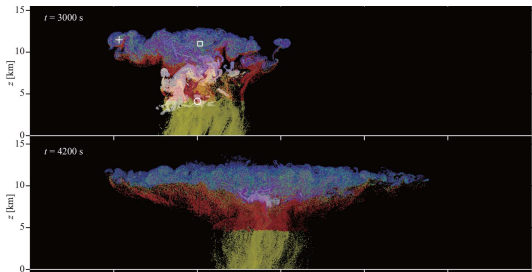


Figure 1. Typical realization of CTRL cloud spatial structures at $t = 2040, 2460, 3000, 4200$, and 5400 s. The mixing ratio of cloud water, rainwater, cloud ice, graupel, and snow aggregates are plotted in fading white, yellow, blue, red, and green, respectively. The symbols indicate examples of unrealistic predicted ice particles (Sects. 7.3 and 9.1). See also Movie 1 in the video supplement.

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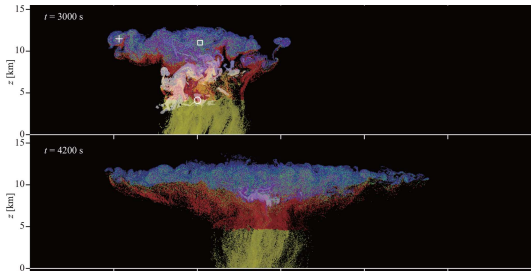


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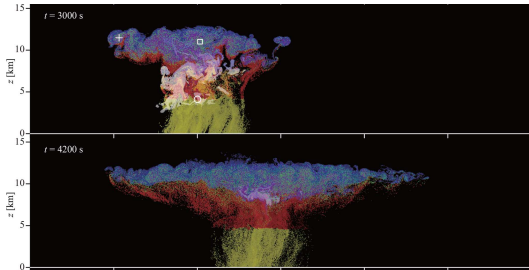


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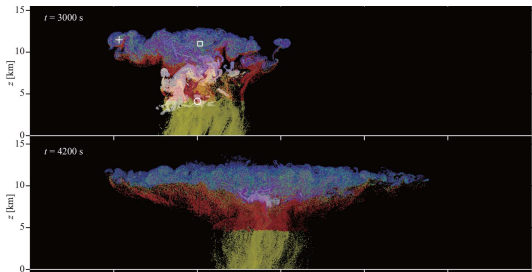


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- ▶ particle-resolved processes:
 - advection and sedimentation
 - homogeneous and immersion freezing (singular)
 - melting
 - condensation and evaporation (incl. CCN [de]activation)
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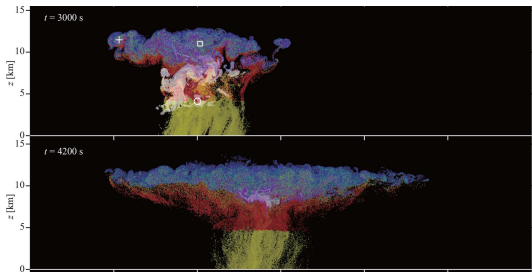


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- ▶ 2D Cb test case

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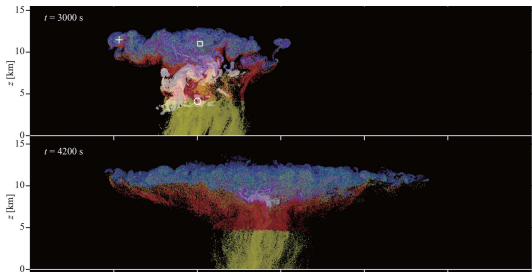
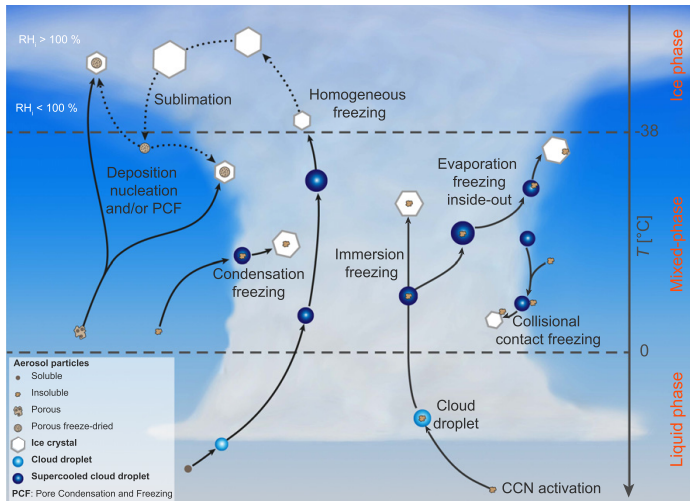


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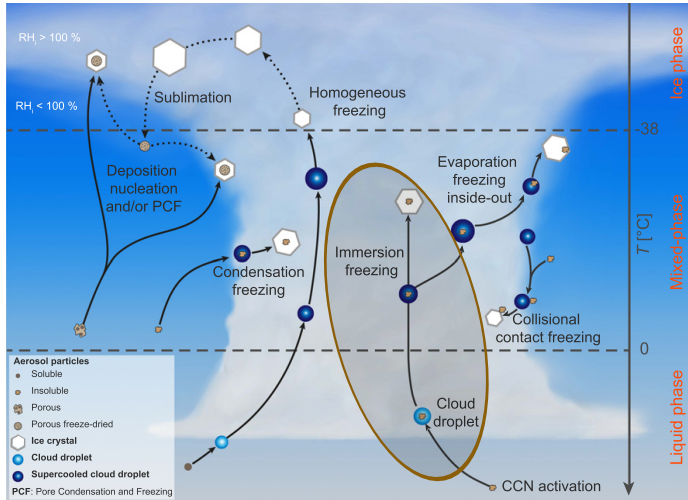
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Immersion freezing and other ice crystal formation pathways in clouds



Kanji et al. 2017, graphics F. Mahrt, <https://doi.org/10.1175/AMSMONOGRAPHS-D-16-0006.1>

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Kanji et al. 2017, graphics F. Mahrt, <https://doi.org/10.1175/AMSMONOGRAPHS-D-16-0006.1>

Immersion freezing: bacteria and the Olympics

Journal of Geophysical Research: Atmospheres

RESEARCH ARTICLE

10.1002/2016JD025251

Key Points:

- Very ice active Snomax protein aggregates are fragile and their ice nucleation ability decreases over months of freezer storage
- Partitioning of ice active protein aggregates into the immersion oil reduces the droplet's measured freezing temperature
- Caution is warranted in the use of

The unstable ice nucleation properties of Snomax® bacterial particles

Michael Polen¹, Emily Lawlis¹, and Ryan C. Sullivan¹

¹Center for Atmospheric Particle Studies, Carnegie Mellon University, Pittsburgh, Pennsylvania, USA

Abstract Snomax® is often used as a surrogate for biological ice nucleating particles (INPs) and has recently been proposed as an INP standard for evaluating ice nucleation methods. We have found the immersion freezing properties of Snomax particles to be substantially unstable, observing a loss of ice nucleation ability

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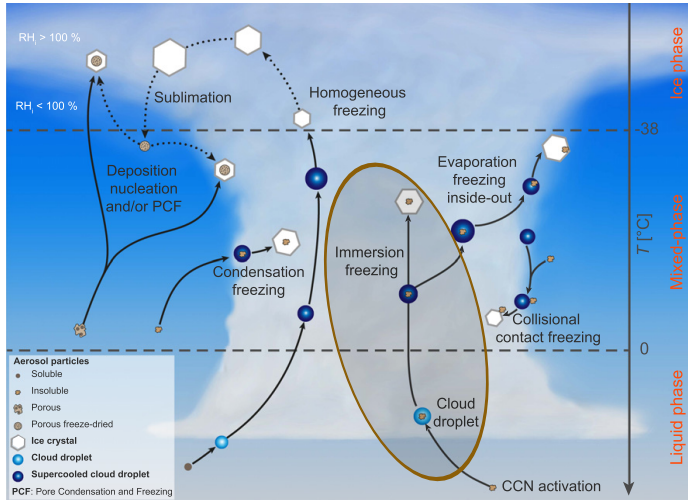
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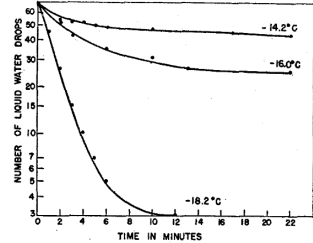


<https://www.reuters.com/markets/commodities/making-snow-stick-wind-challenges-winter-games-slope-makers-2021-11-29/>

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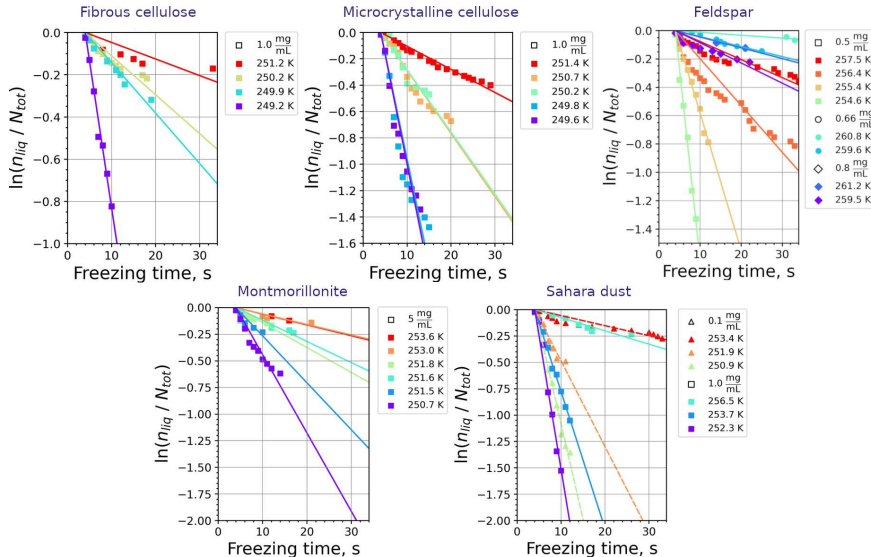


Vonnegut 1948 (J. Colloid Sci.)



Fraction of water drops remaining unfrozen as a function of time.

Szakáll et al. 2021, ACP 21: isothermal experiments



Heterogeneous Nucleations is a Stochastic Process

by

J. S. MARSHALL

McGill University, Montreal, Canada.

*Presented at the International Congress on the Physics of Clouds (Hailstorms)
at Verona 9-13 August 1960.*

Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

theory (in modern notation)

(Bigg '53, Langham & Mason '58, Carte '59, Marshall '61)

Poisson counting process with rate r :

$$P^*(k \text{ events in time } t) = \frac{(rt)^k \exp(-rt)}{k!}$$

$$P(\text{one or more events in time } t) = 1 - P^*(k = 0, t)$$

$$\ln(1 - P) = -rt$$

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introducing $J_{\text{het}}(T)$, $T(t)$ and INP surface A :

$$\ln(1 - P(A, t)) = -A \underbrace{\int_0^t J_{\text{het}}(T(t')) dt'}_{\text{ }}$$

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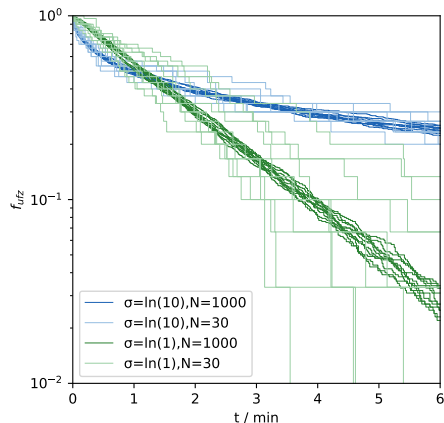
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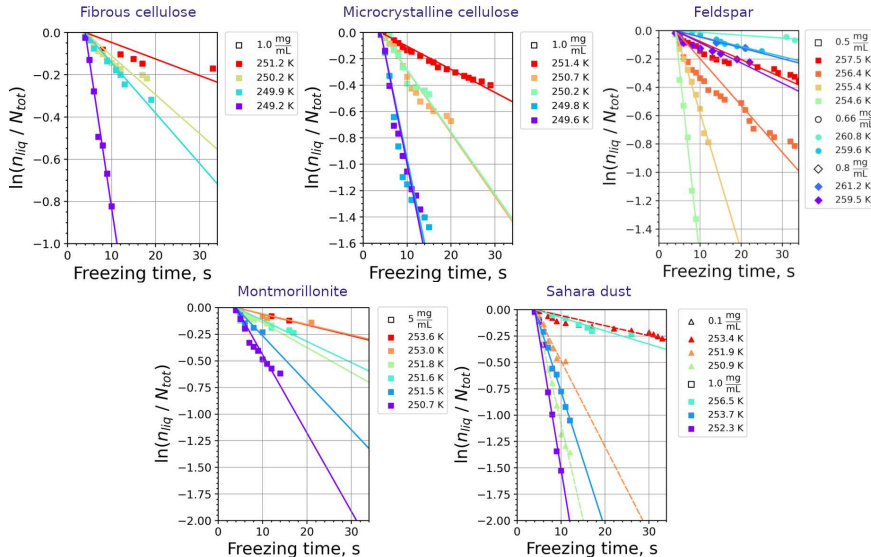
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Monte Carlo: const J_{het} , lognormal A



(as in Alpert & Knopf 2016, Fig. 1a)

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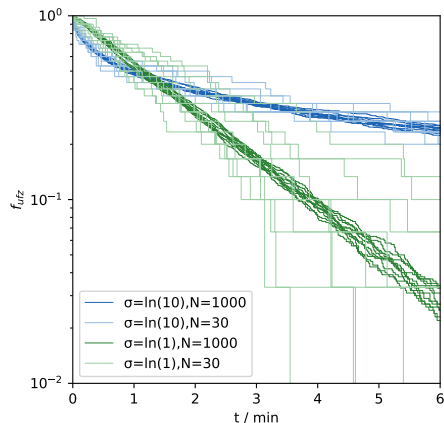
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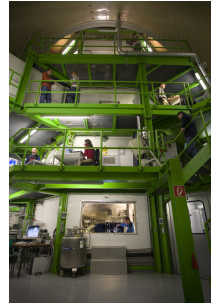
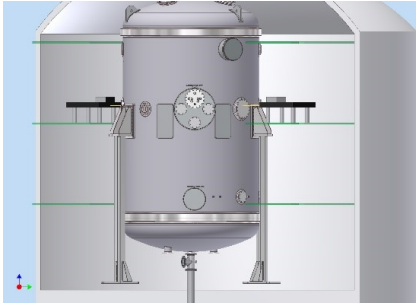
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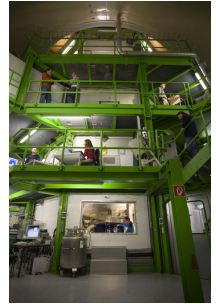
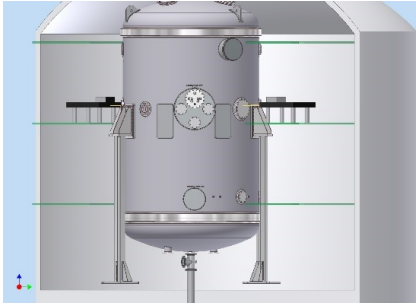
experimental $n_s(T)$ fits: e.g., Niemand et al. 2012

AIDA @ KIT



(<https://www.imk-aaf.kit.edu/>, photo: KIT/Ottmar Möhler)

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J_{het} or n_s ?

Vali 2014 (ACP)

*“Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... **dichotomy – the stochastic and singular models**”*

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Minimal cooling rate dependence of ice nuclei activity in the immersion mode
JOURNAL OF GEOPHYSICAL RESEARCH-ATMOSPHERES, VOL. 118
Timothy P. Wright,¹ Markus D. Petters,¹ John D. Hader,¹ Travis Morton,¹
and Amara L. Holder,¹
Received 12 April 2013; revised 4 September 2013; accepted 5 September 2013; published 23 September 2013.

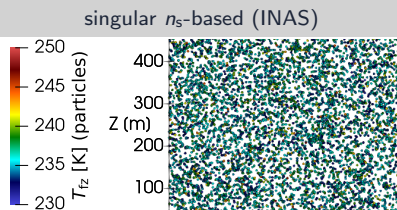
Stochastic nucleation processes and substrate abundance explain time-dependent freezing in supercooled droplets
npj | climate and atmospheric science
Article | [Open access](#) | Published: 17 January 2020
Daniel A. Knopf¹, Peter A. Albert, Assaf Zipori, Naama Reicher, & Ynon Rudich²

Heterogeneous ice nucleation: exploring the transition from stochastic to singular freezing behavior
Atmos. Chem. Phys., 11, 8767–8775, 2011
www.atmos-chem-phys.net/11/8767/2011/
doi:10.5194/acp-11-8767-2011
© Author(s) 2011. CC Attribution 3.0 License.
D. Niedermeier,¹ R. A. Shaw,² S. Hartmann,¹ H. Wex,¹ T. Claus,¹ J. Voigtländer,¹ and F. Stratmann,¹
¹Leibniz Institute for Tropospheric Research, 04318 Leipzig, Germany
²Dept. of Physics, Michigan Technological University, Houghton, Michigan 49931, USA
Received: 12 November 2010 – Published in Atmos. Chem. Phys. Discuss.: 28 January 2011
Revised: 24 June 2011 – Accepted: 19 August 2011 – Published: 30 August 2011

Atmospheric Chemistry and Physics

Particle-based freezing: singular (Shima et al.)

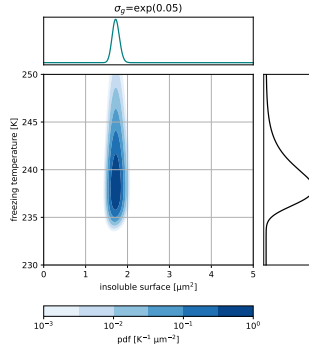
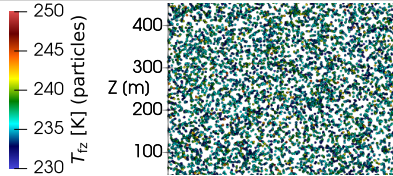
singular: INAS T_{fz} as **attribute**; initialisation by random sampling from $P(A, T_{fz})$ with lognormal A
freezing if $T_{ambient}(t) < T_{fz}|_{\text{sampled at } t=0}$



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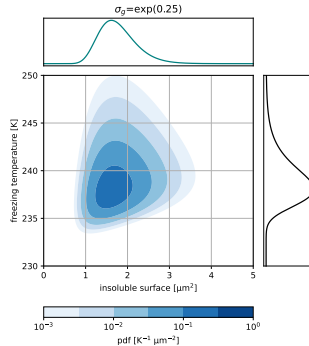
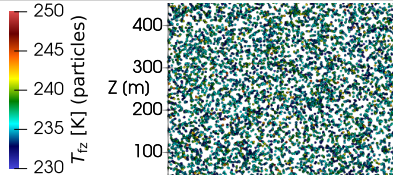
singular n_s -based (INAS)



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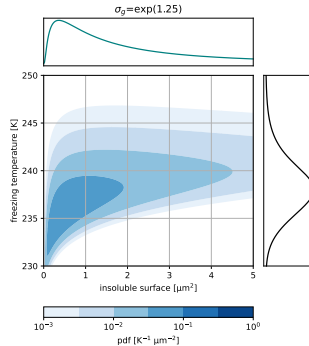
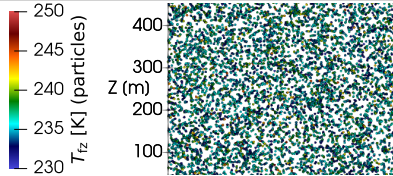
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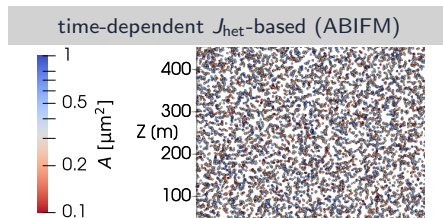
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Particle-based freezing: singular (Shima et al.) / time-dependent (this work)

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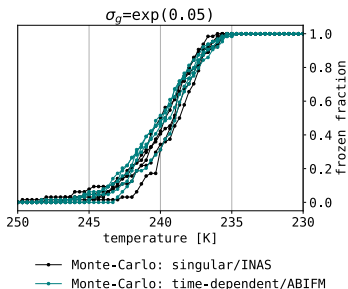


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AIDA cooling rate: 0.5 K/min

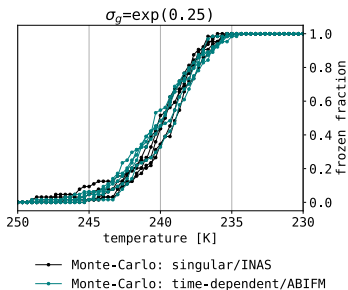


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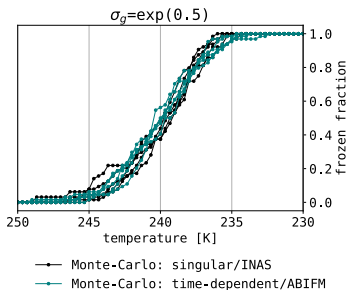


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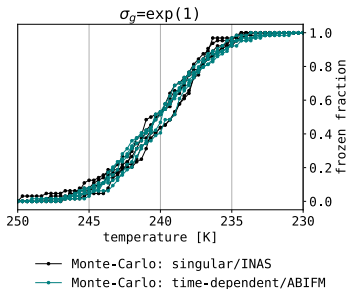


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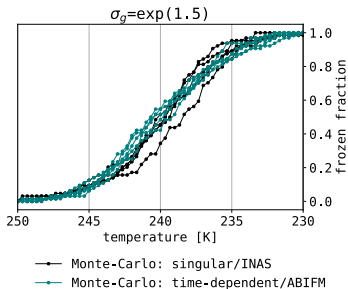


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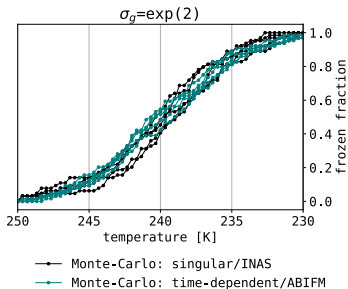


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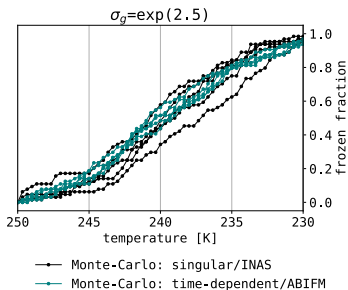


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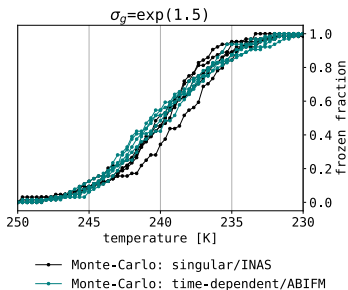


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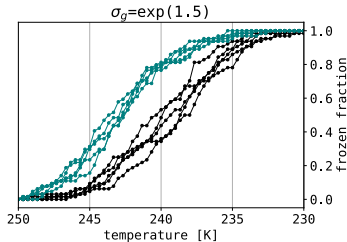


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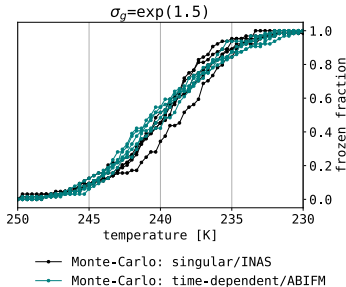
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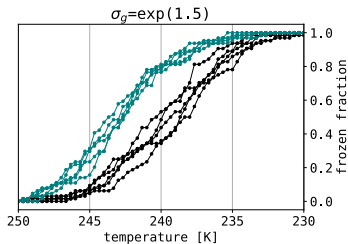


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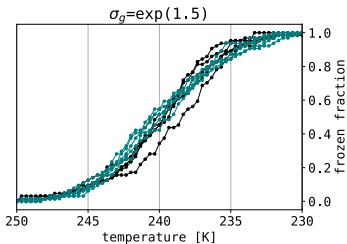
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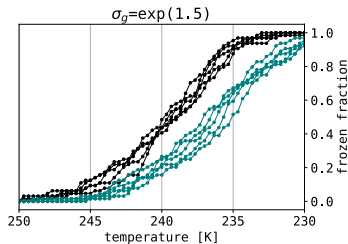
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AIDA cooling rate: 0.5 K/min



cooling rate: 2.5 K/min



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—●— Monte-Carlo: time-dependent/ABIFM

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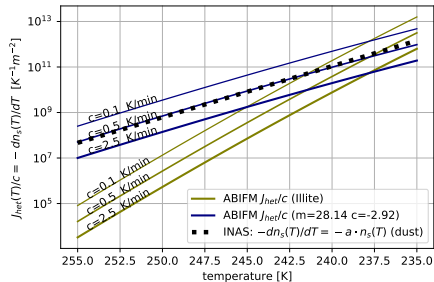
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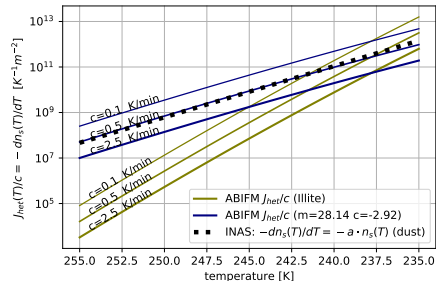
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cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11)
but the **singular ansatz limitation of sampling T_{fz} at $t=0$** remains

Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

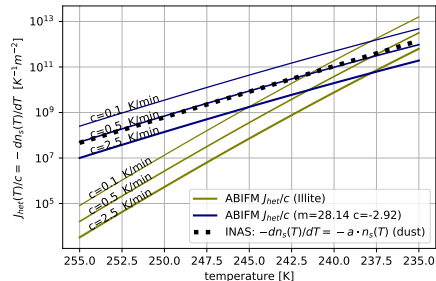
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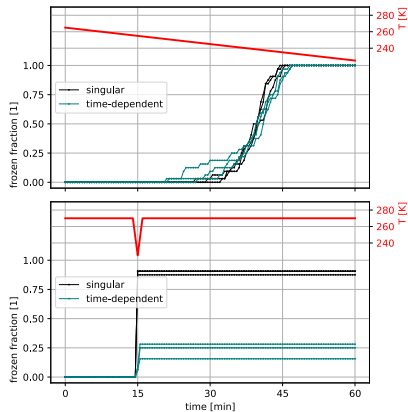
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ABIFM J_{het} (Knopf & Alpert '13)

Is it a problem?

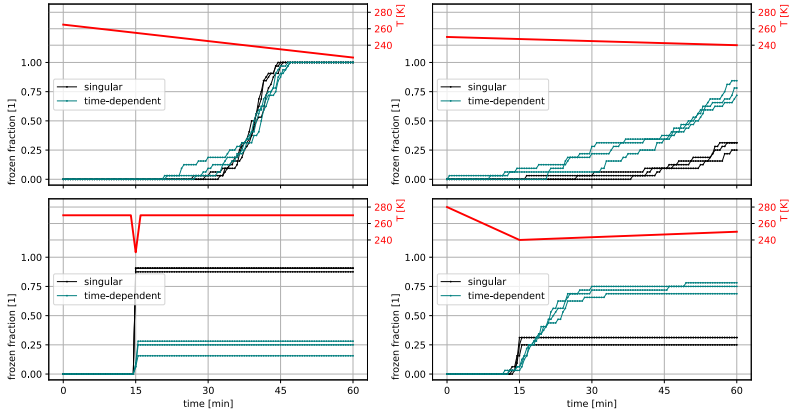


cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11)
but the **singular ansatz limitation of sampling T_{fz} at $t=0$** remains

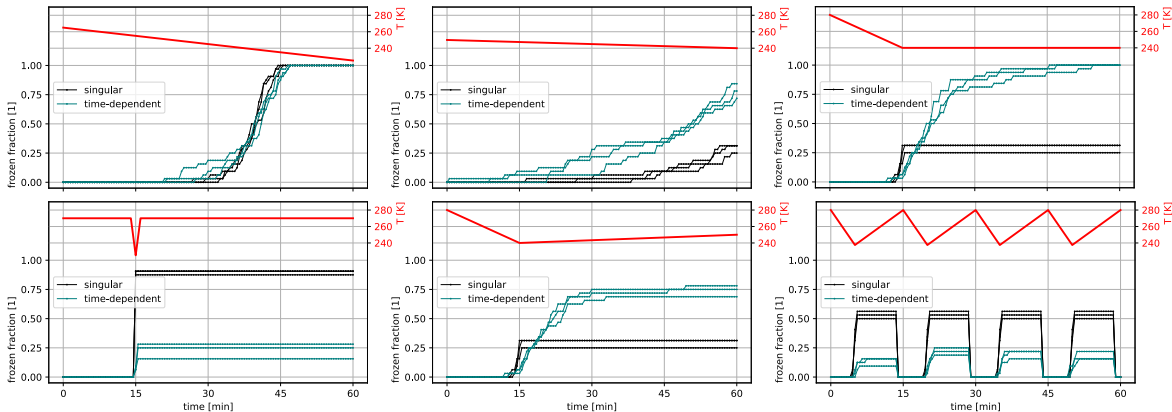
Testing different cooling-rate profiles in a box model



Testing different cooling-rate profiles in a box model

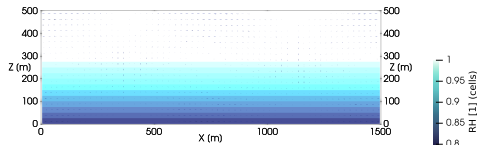


Testing different cooling-rate profiles in a box model



Testing three flow regimes and two immersion freezing representations

$w_{\max} \approx 1/3 \text{ m/s}$

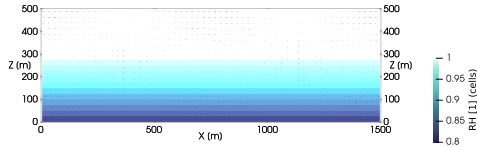


$w_{\max} \approx 1 \text{ m/s}$

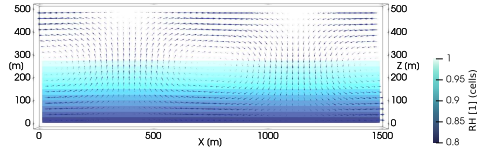
$w_{\max} \approx 3 \text{ m/s}$

Testing three flow regimes and two immersion freezing representations

$w_{\max} \approx 1/3 \text{ m/s}$



$w_{\max} \approx 1 \text{ m/s}$

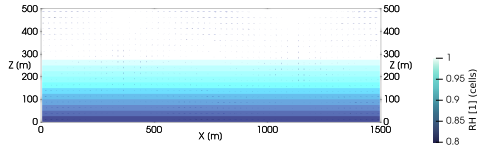


$w_{\max} \approx 3 \text{ m/s}$

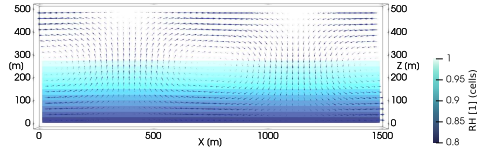


Testing three flow regimes and two immersion freezing representations

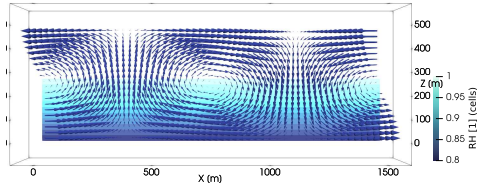
$w_{\max} \approx 1/3 \text{ m/s}$



$w_{\max} \approx 1 \text{ m/s}$

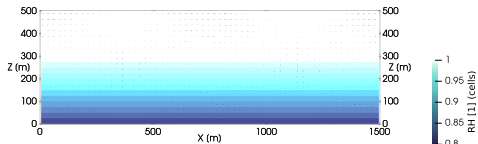


$w_{\max} \approx 3 \text{ m/s}$

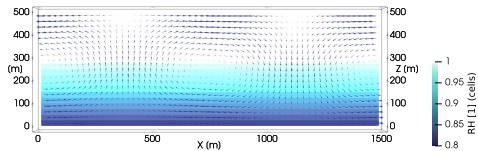


Testing three flow regimes and two immersion freezing representations

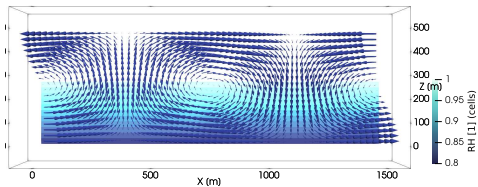
$w_{\max} \approx 1/3 \text{ m/s}$



$w_{\max} \approx 1 \text{ m/s}$

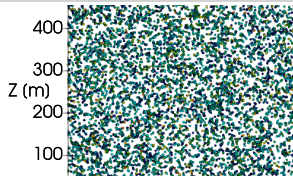


$w_{\max} \approx 3 \text{ m/s}$



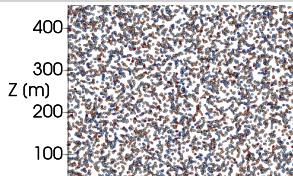
singular (INAS)

$T_{fz} \text{ [K (particles)]}$



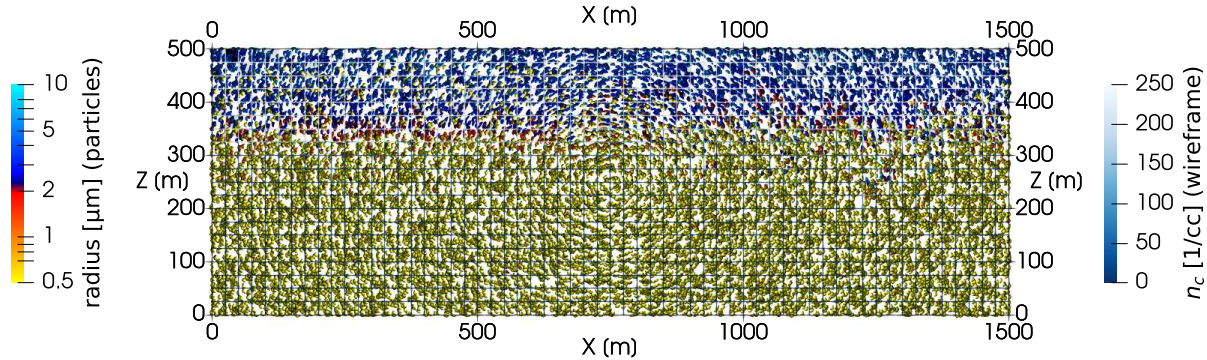
time-dependent (J_{het})

$A \text{ [}\mu\text{m}^2\text{]}$



Particle-based μ -physics + prescribed-flow: glaciation

Time: 630 s (spin-up till 600.0 s)



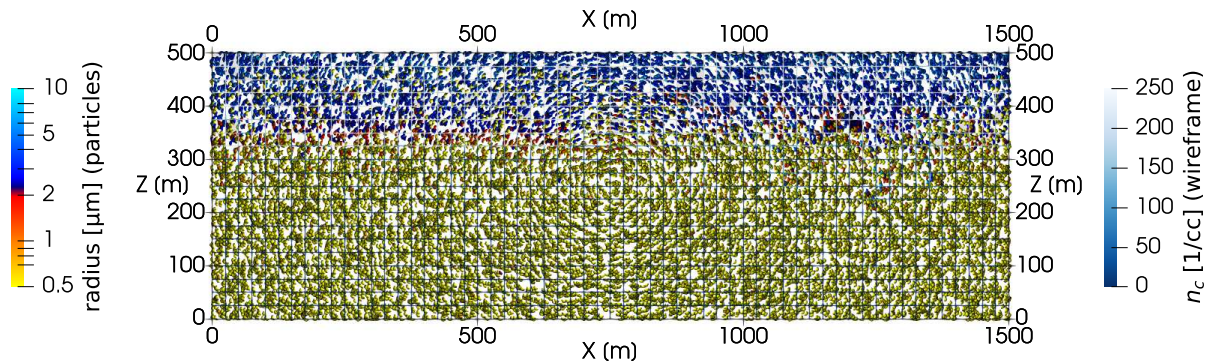
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 660 s (spin-up till 600.0 s)



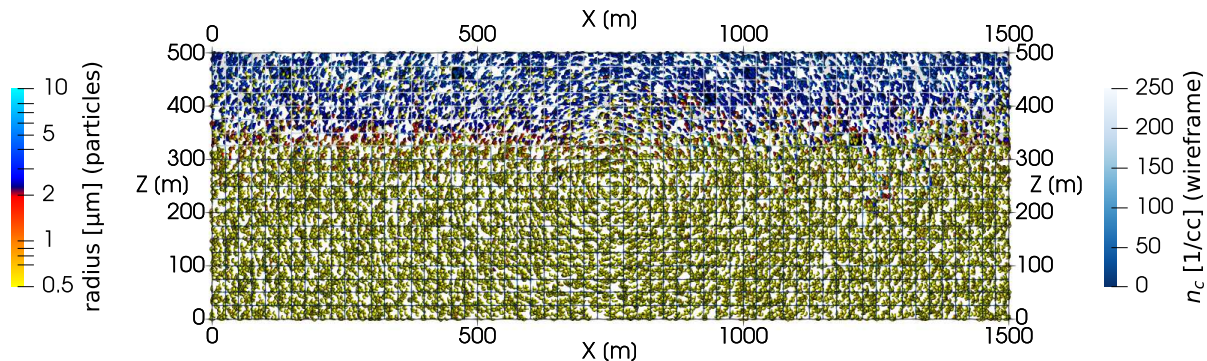
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 690 s (spin-up till 600.0 s)



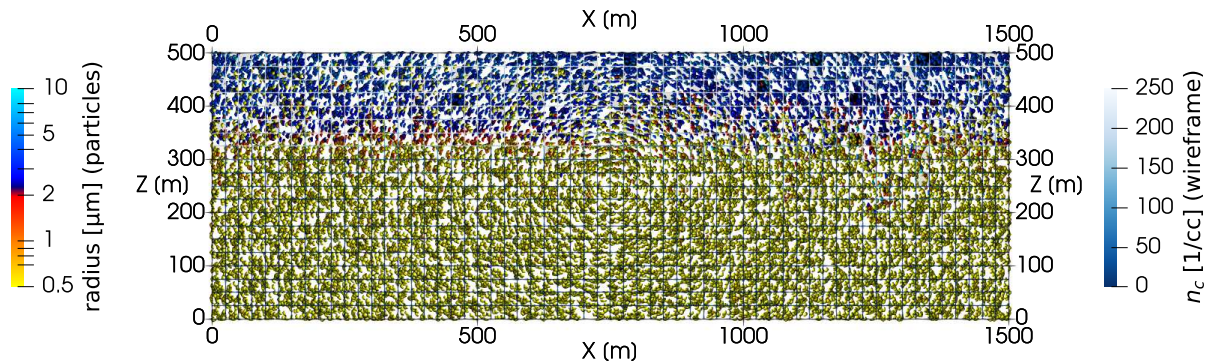
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 720 s (spin-up till 600.0 s)



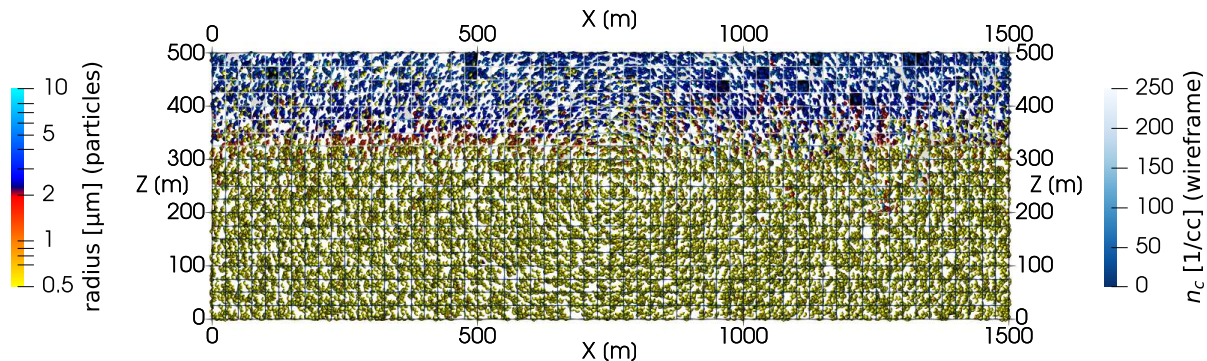
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 750 s (spin-up till 600.0 s)



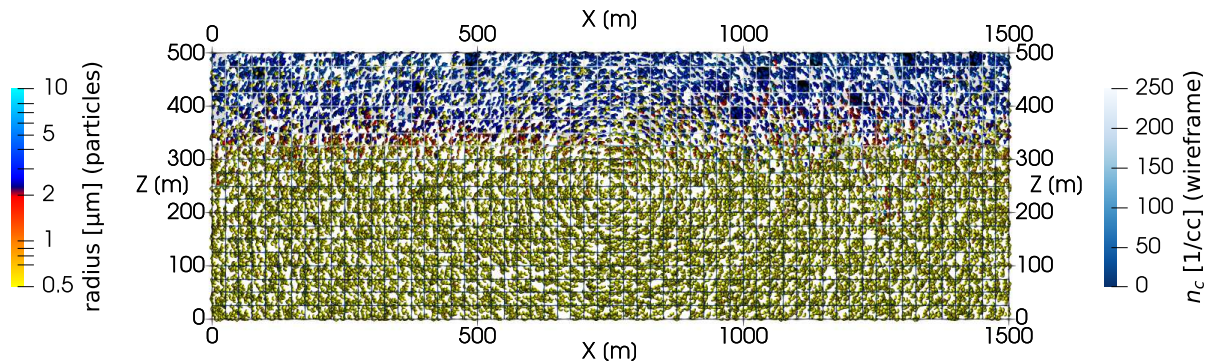
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 780 s (spin-up till 600.0 s)



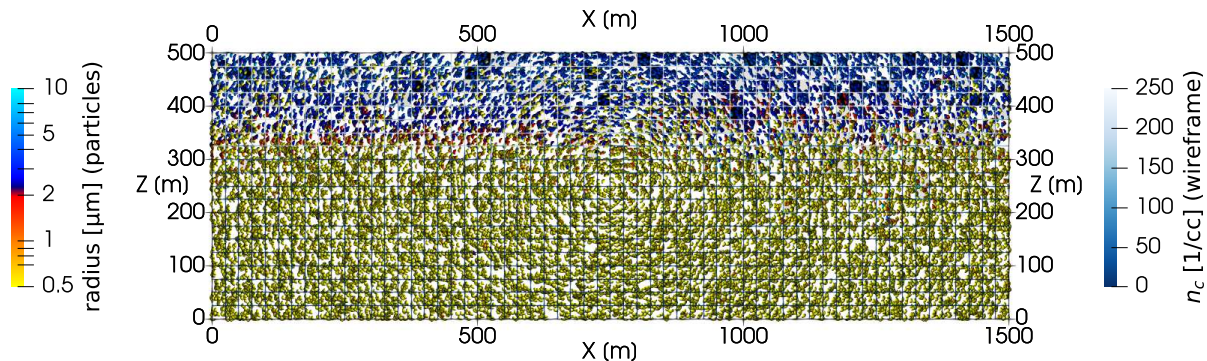
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 810 s (spin-up till 600.0 s)



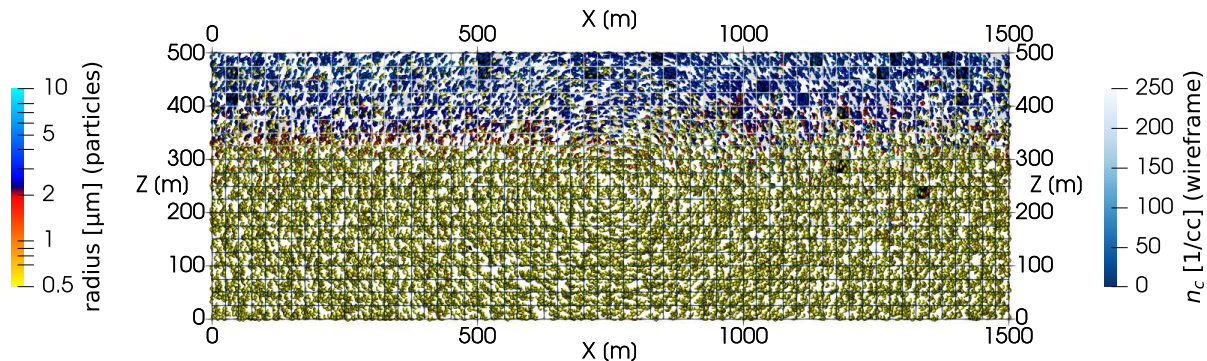
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 840 s (spin-up till 600.0 s)



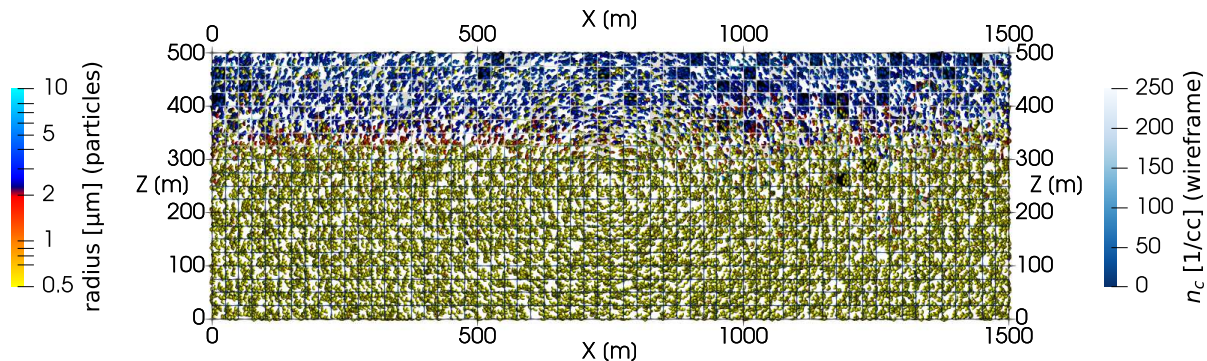
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 870 s (spin-up till 600.0 s)



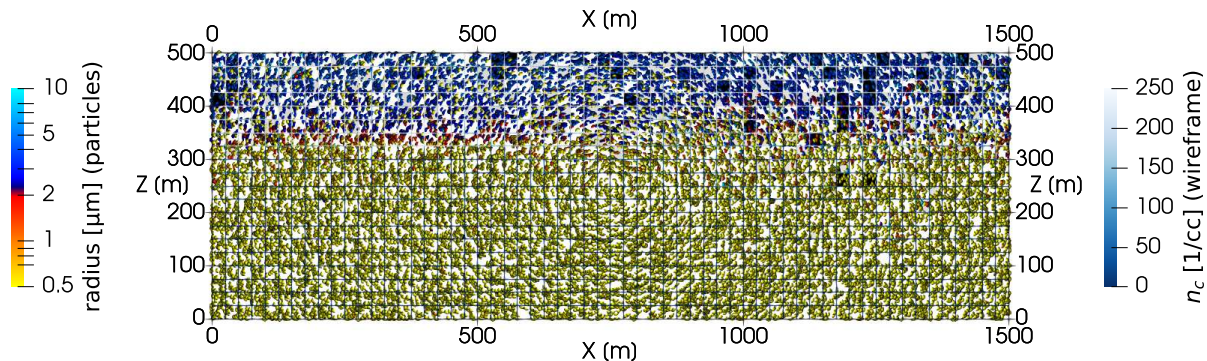
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 900 s (spin-up till 600.0 s)



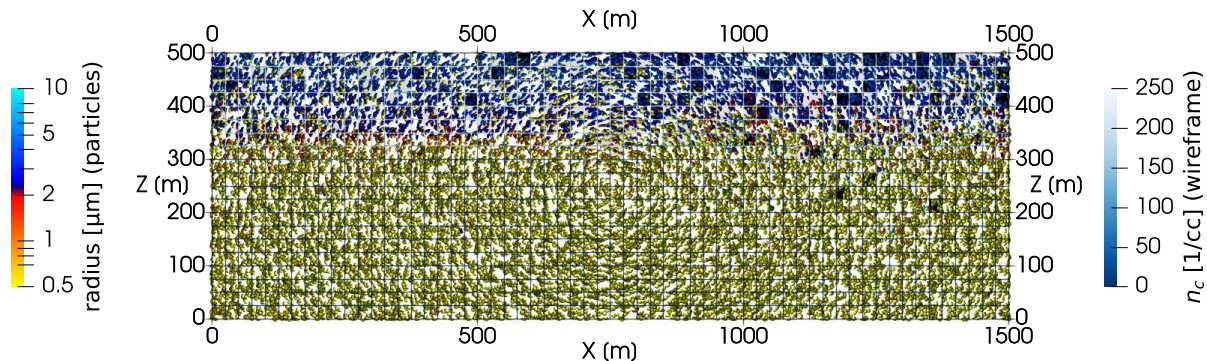
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 930 s (spin-up till 600.0 s)



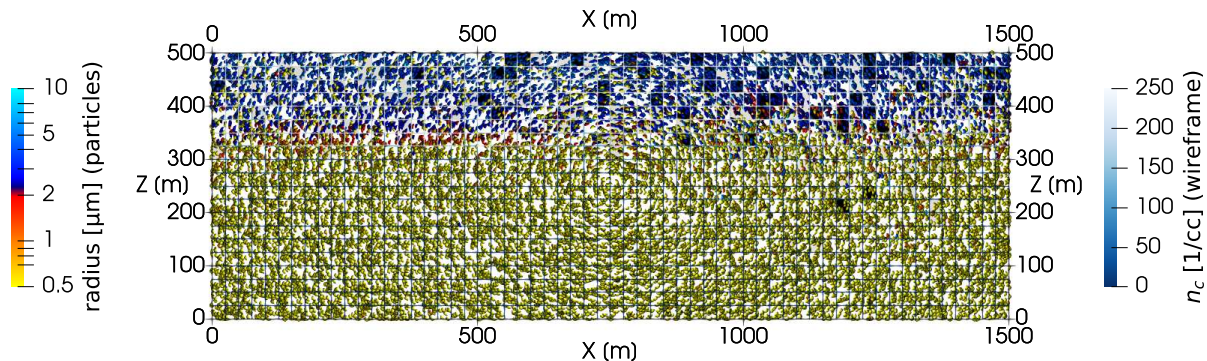
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 960 s (spin-up till 600.0 s)



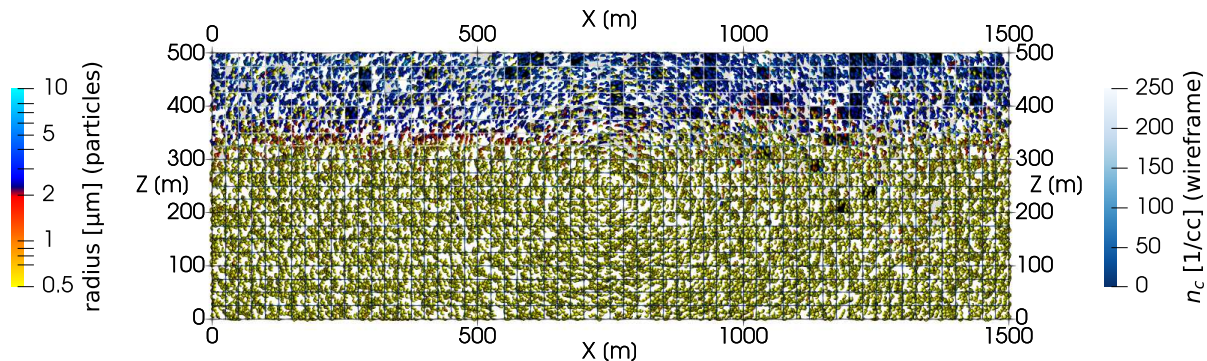
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 990 s (spin-up till 600.0 s)



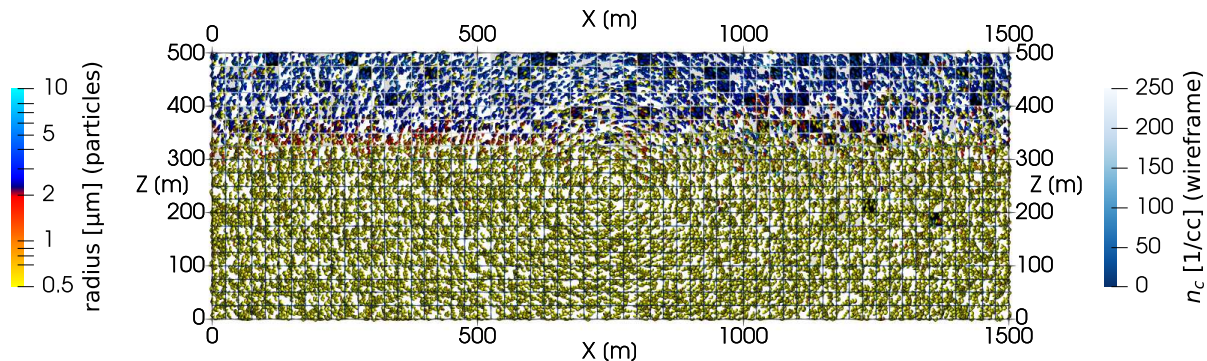
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1020 s (spin-up till 600.0 s)



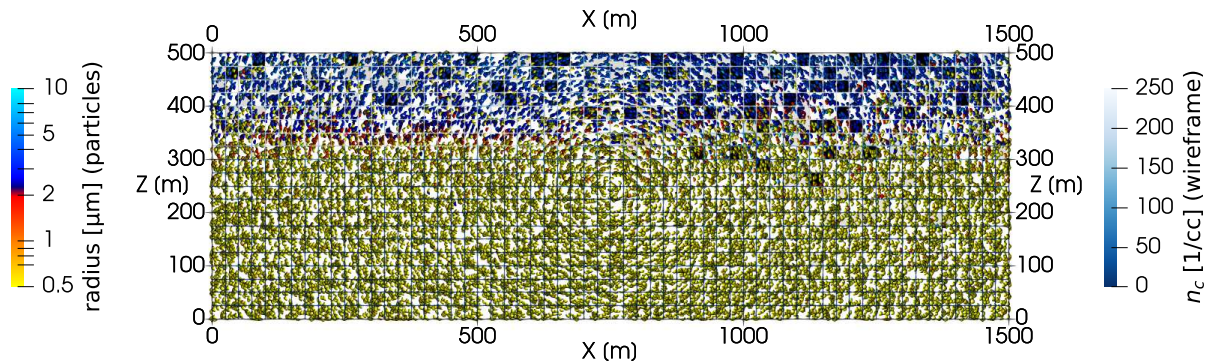
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1050 s (spin-up till 600.0 s)



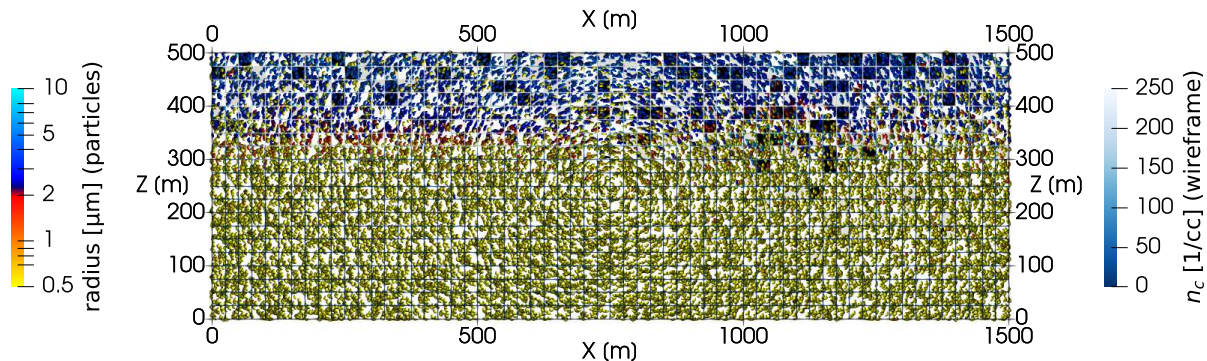
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1080 s (spin-up till 600.0 s)



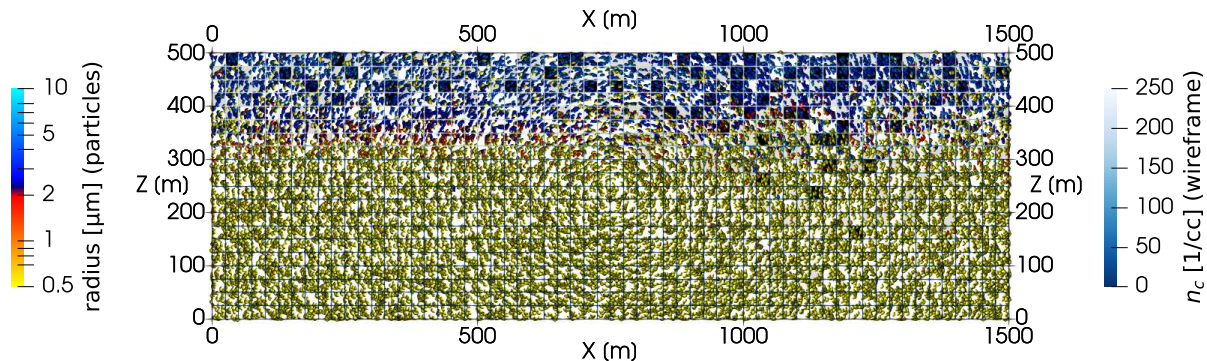
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1110 s (spin-up till 600.0 s)



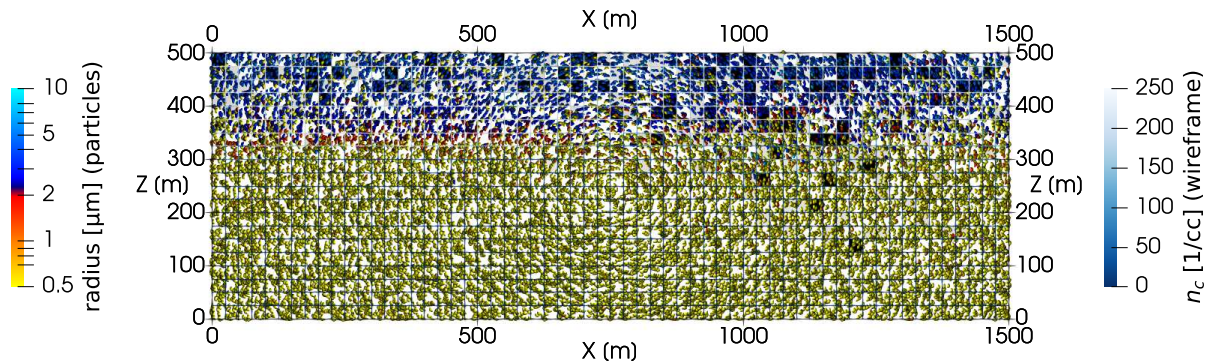
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1140 s (spin-up till 600.0 s)



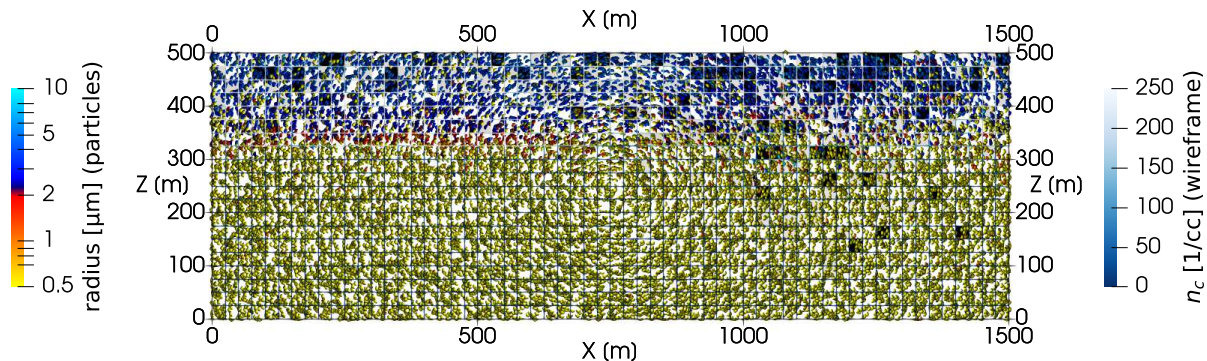
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1170 s (spin-up till 600.0 s)



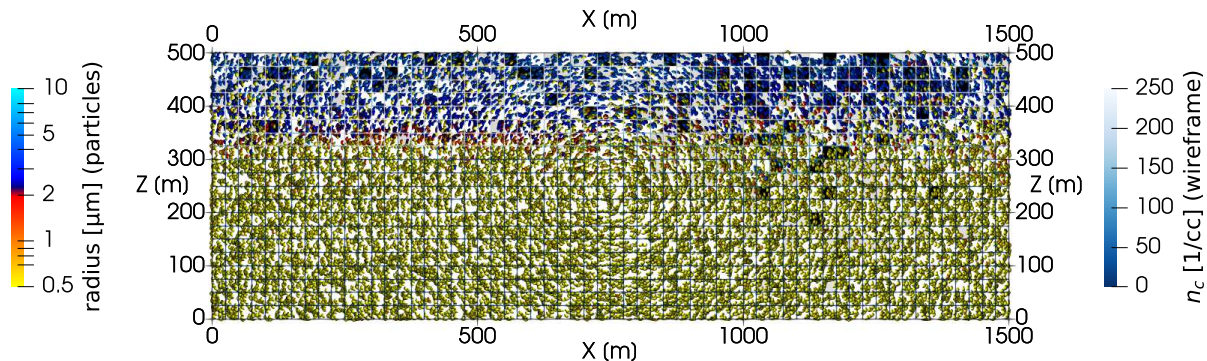
16+16 super-particles/cell for INP-rich + INP-free particles

$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

Particle-based μ -physics + prescribed-flow: glaciation

Time: 1200 s (spin-up till 600.0 s)

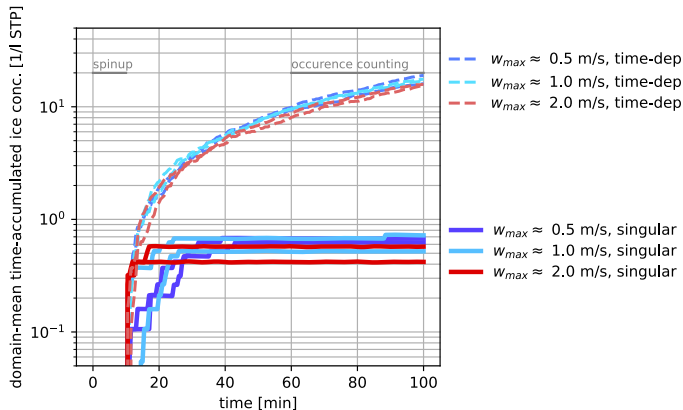


16+16 super-particles/cell for INP-rich + INP-free particles

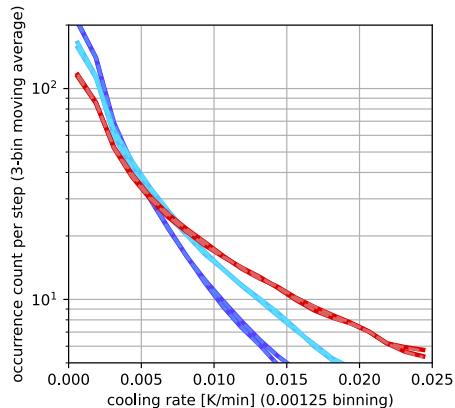
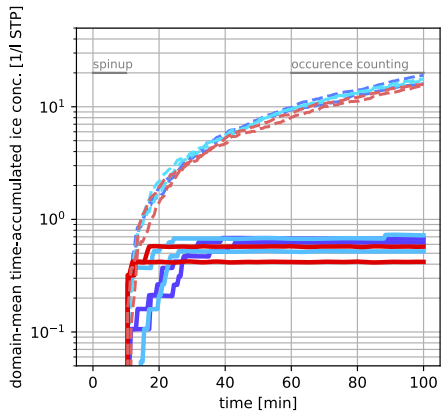
$N_{\text{aer}} = 300/\text{cc}$ (two-mode lognormal) $N_{\text{INP}} = 150/L$ (lognormal, $D_g = 0.74 \mu\text{m}$, $\sigma_g = 2.55$)

spin-up = freezing off; subsequently frozen particles act as tracers

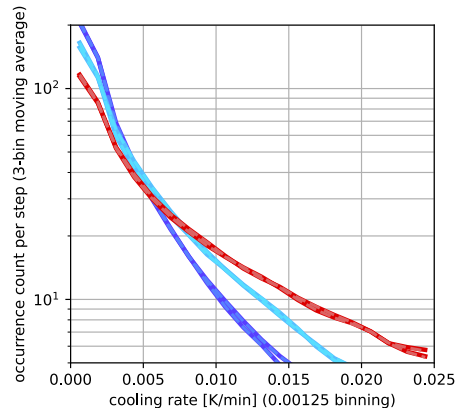
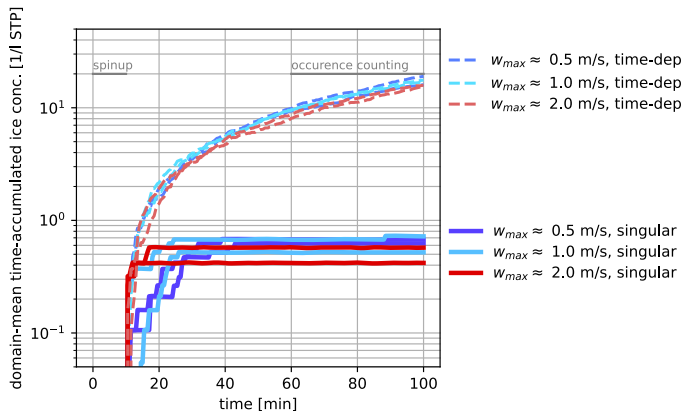
Testing three flow regimes and two immersion freezing representations



Testing three flow regimes and two immersion freezing representations

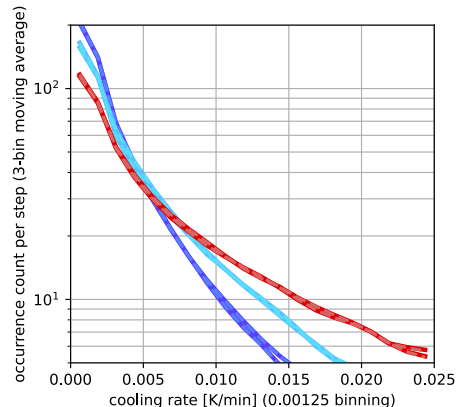
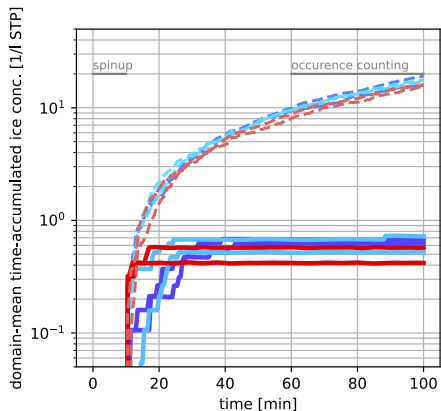


Testing three flow regimes and two immersion freezing representations



► range of cooling rates in simple flow (far from 0.5 K/min for AIDA as in Niemand et al. 2012)

Testing three flow regimes and two immersion freezing representations



- ▶ range of cooling rates in simple flow (far from 0.5 K/min for AIDA as in Niemand et al. 2012)
- ▶ **only time-dependent scheme robust across flow regimes** (consistent with box model & theory)

100%  python™ open-source code:

/ OPEN**ATMOS** /

PySDM

J_{het} or n_s ?

Vali 2014 (ACP)

*“Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... **dichotomy – the stochastic and singular models**”*

Minimal cooling rate dependence of ice nuclei activity in the immersion mode
JOURNAL OF GEOPHYSICAL RESEARCH-ATMOSPHERES, VOL. 118
Timothy P. Wright,¹ Markus D. Petters,¹ John D. Hader,¹ Travis Morton,¹
and Amara L. Holder,¹
Received 12 April 2013; revised 4 September 2013; accepted 5 September 2013; published 23 September 2013.

Stochastic nucleation processes and substrate abundance explain time-dependent freezing in supercooled droplets
npj | climate and atmospheric science
Article | [Open access](#) | Published: 17 January 2020
Daniel A. Knopf¹, Peter A. Albert, Assaf Zipori, Naama Reicher, & Ynon Rudich²

Heterogeneous ice nucleation: exploring the transition from stochastic to singular freezing behavior
Atmos. Chem. Phys., 11, 8767–8775, 2011
www.atmos-chem-phys.net/11/8767/2011/
doi:10.5194/acp-11-8767-2011
© Author(s) 2011. CC Attribution 3.0 License.
D. Niedermeier,¹ R. A. Shaw,² S. Hartmann,¹ H. Wex,¹ T. Claus,¹ J. Voigtlander,¹ and F. Stratmann,¹
¹Leibniz Institute for Tropospheric Research, 04318 Leipzig, Germany
²Dept. of Physics, Michigan Technological University, Houghton, Michigan 49931, USA
Received: 12 November 2010 – Published in Atmos. Chem. Phys. Discuss.: 28 January 2011
Revised: 24 June 2011 – Accepted: 19 August 2011 – Published: 30 August 2011

Atmospheric Chemistry and Physics

Knopf & Alpert '13

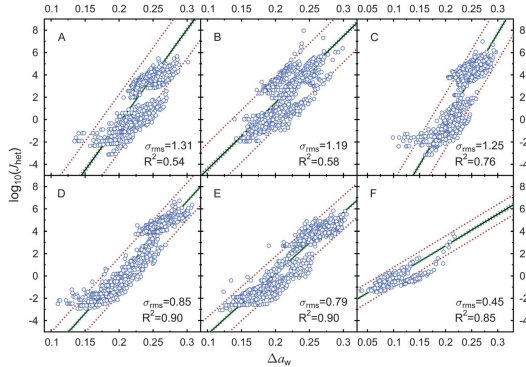


Fig. 3 The decadal log of the heterogeneous ice nucleation rate coefficients, $\log_{10}(J_{\text{het}})$, are shown as a function of Δa_w for individually analysed freezing events, initiated by the different IN types investigated in this study and previous work.^{41,43,57,66,73,75,78,95} $\log_{10}(J_{\text{het}})$ are shown for (A) *Nannochloris atomus*, (B) *Thalassiosira pseudonana*, (C) Pahokee Peat, (D) Leonardite, (E) Illite, and (F) 1-nonadecanol. The solid black line is a linear fit where dashed green and red lines represent confidence intervals and prediction bands at 95% level. The root mean square error, σ_{rms} , and the adjusted coefficient of determination, R^2 , are given in each panel.

Kanji et al. '17

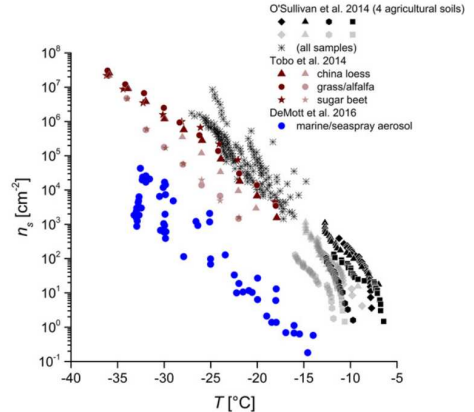


FIG. 1-6. Ice nucleation active site densities n_s , as a function of temperature for H_2O_2 (hydrogen peroxide) treated (lighter-shaded symbols) and untreated (dark symbols) agricultural soil dusts in comparison to the n_s of marine aerosol. Differences between various black symbols are for organic content (OC). High OC (12.7 wt%)

J_{het} or n_s ?

Vali 2014 (ACP)

"Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... dichotomy – the stochastic and singular models"

stochastic or deterministic?

J_{het} or n_s ?

Vali 2014 (ACP)

"Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... dichotomy – the stochastic and singular models"

stochastic or deterministic?

DeMott 1990 (J. Appl. Meteorol.)

"If one takes the standard definition of the "threshold temperature" for ice formation: 1 particle in 10^4 producing an ice crystal, then this temperature (assuming all particles are immersed in drops) can be predicted from [a power law versus temperature]"

J_{het} or n_s ?

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"Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... dichotomy – the stochastic and singular models"

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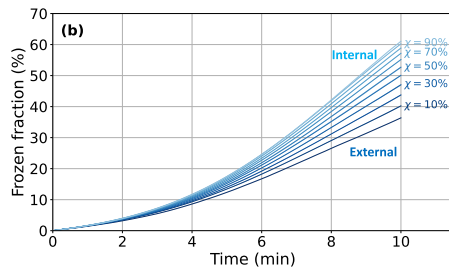
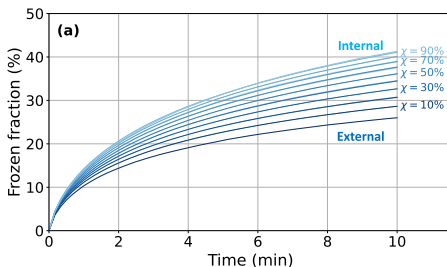
"If one takes the standard definition of the "threshold temperature" for ice fomatation: 1 particle in 10^4 producing an ice crystal, then this temperature (assuming all particles are immersed in drops) can be predicted from [a power law versus temperature]"

common underlying Poissonian model

The impact of aerosol mixing state on immersion freezing: Insights from classical nucleation theory and particle-resolved simulations

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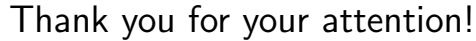
Abstract. Immersion freezing, initiated by ice-nucleating particles (INPs) in supercooled aqueous droplets, plays an important role in the formation of ice crystals within clouds. The efficiency of immersion freezing depends strongly on INP composition and, crucially, on the mixing state—how chemical species are distributed across the particle population. Here, we quantify the impact of aerosol mixing state on immersion freezing using a combined theoretical and particle-resolved modeling approach.



(a): isothermal freezing conditions with $-20\text{ }^{\circ}\text{C}$

(b): constant cooling rate from $-10\text{ }^{\circ}\text{C}$ to $-30\text{ }^{\circ}\text{C}$ within 10 minutes





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