Euler-Lagrangian cloud model with dynamic particle forces

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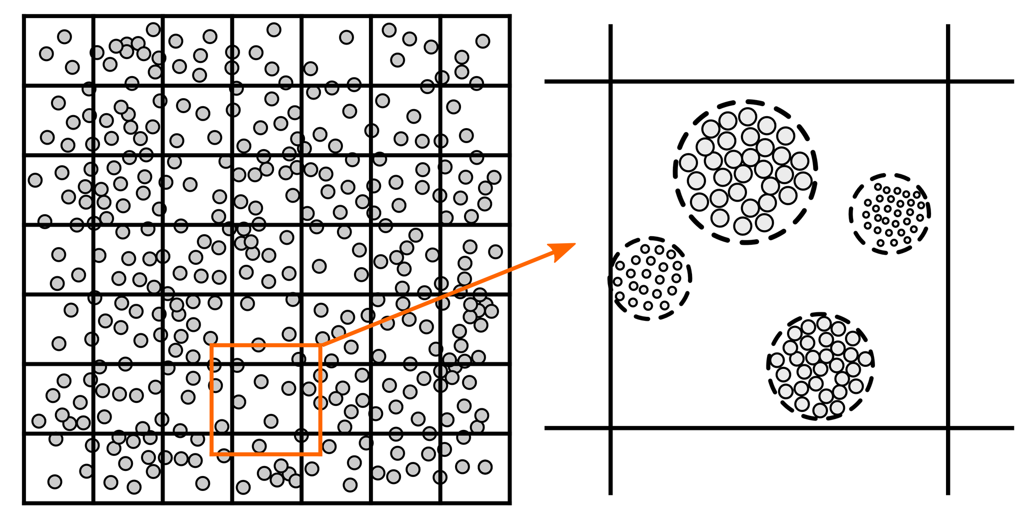
Introduction

We aim to extend an existing scalable solver for atmospheric applications (All Scale Atmospheric Model, ASAM) by a two-way coupled Euler-Lagrangian model for the description of cloud processes on a broad range of dimensional scales. By coupling Computational Fluid Dynamics (CFD) and Discrete Particle Methods (DPM), it is possible to obtain a sophisticated representation of the dynamic movement and continuous mass evolution of the droplets. While the surrounding moist air is described as continuous fluid, the aqueous solution cloud droplets are considered to be discrete particles of finite size, which interact with the carrier fluid phase by momentum, mass and heat exchange. The approach of distinct particles enables a detailed generation and analysis of droplet size distributions. Moreover, the sedimentation as well as the collision and break-up can be modeled close to natural processes. Since the number of cloud particles in atmospheric domains exceeds computational limitations by orders of magnitudes, we use the super-droplet model proposed by [Shi09], where one computational particle represents a multiplicity ξ of particles with identical physical properties.



Spatial grid with discretized atmospheric fields **Super-droplets in cell**

Euler-Lagrangian Model

Atmospheric advection with condensation source term

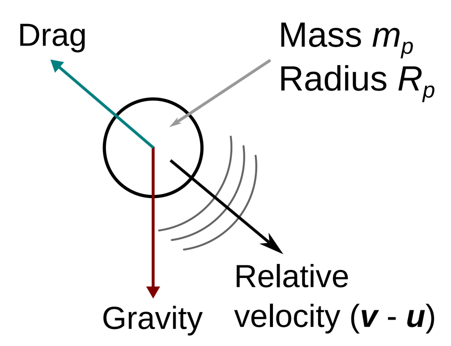
Water vapor ($\rho_{\mathrm{dry}} r_v$) and thermal energy ($\rho_{\mathrm{dry}} \Theta$) are exchanged by condensation with rate ϕ_{cond} (source term) and transported by atmospheric advection with wind velocity $\boldsymbol{u}(\boldsymbol{x}, t)$:

$$\partial_t(\rho_{\mathrm{dry}}\, r_v) = -\nabla \cdot (\rho_{\mathrm{dry}}\, r_v\, \boldsymbol{u}) - \phi_{\mathrm{cond}} \qquad \partial_t(\rho_{\mathrm{dry}}\, \Theta) = -\nabla \cdot (\rho_{\mathrm{dry}}\, \Theta\, \boldsymbol{u}) + \frac{L_v\, \Theta}{c_{p,\mathrm{dry}}\, T}\, \phi_{\mathrm{cond}}$$

Particle forces: Drag + gravity

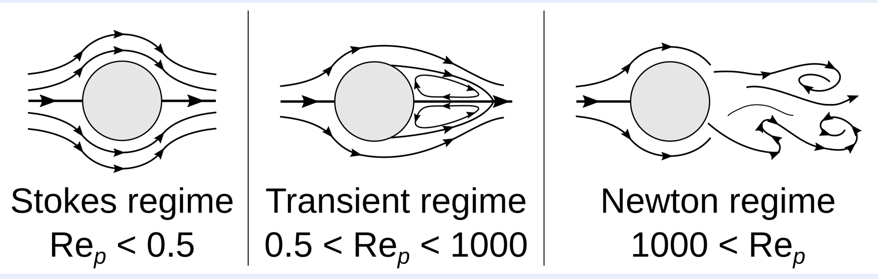
The total force acting on a spherical particle moving with velocity $\boldsymbol{v}$ through air with local velocity $\boldsymbol{u}$ is modeled by the sum of drag force and gravity:

$$m_p\, \ddot{\boldsymbol{x}}_p = \boldsymbol{F}_d + \boldsymbol{F}_g = m_p\, k_d\, (\boldsymbol{u}(\boldsymbol{x}, t) - \boldsymbol{v}) + m_p\, \boldsymbol{g}$$



The drag parameter $k_d(\mathrm{Re}_p)$ depends on the regime of the particle Reynolds number $\mathrm{Re}_p = 2\, \rho_f R_p\, |\boldsymbol{u} - \boldsymbol{v}| / \mu_f$ with $k_d^0 = 9\, \mu_f / (2\, \rho_p\, R_p^2)$ [Zhu07]:

$$k_d = \begin{cases} k_d^0 \\ k_d^0 \times \left[1 + 0.15\, (\mathrm{Re}_p)^{0.687}\right] \\ k_d^0 \times 0.0183\, \mathrm{Re}_p \end{cases}$$

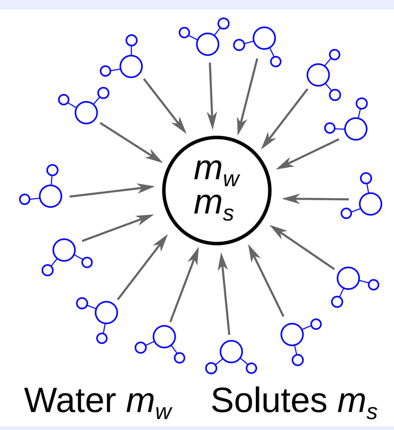


Condensation / evaporation

The droplet condensation mass rate depends on the Köhler equilibrium saturation S_{eq} and a kinetic term B [Fuk70]:

$$\dot{m}_w =: \gamma = R_p\, \big(S(r_v, T) - S_{\mathrm{eq}}\big)\, B(R_p, T, p)\,,$$

$$S_{\mathrm{eq}} = \frac{m_w}{m_w + \sum_s m_s\, i_s\, M_w / M_s}\, \exp\left(\frac{2\sigma_w}{R_v\, T\, \rho_p R_p}\right).$$



Time integration

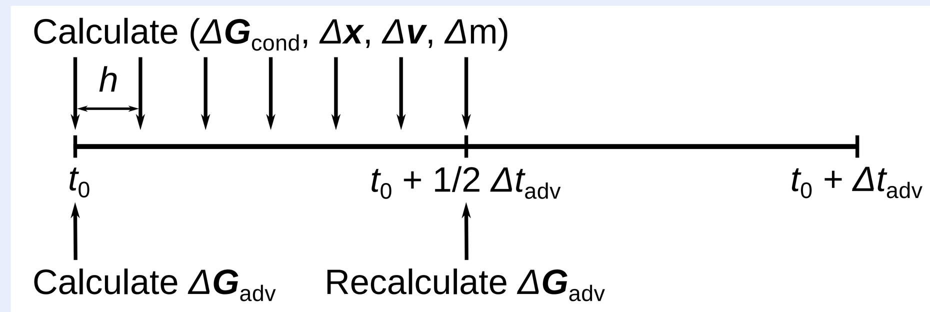
Equations of motion (EOM)

The fluid state variables $\boldsymbol{G} := (\Theta, r_v)^{\top}$ are discretized on a rectangular spatial Arakawa C-type grid. The time evolution of each cell consists of an advection and a condensation part. Together with the equations for particle position, velocity and water mass, the EOM are given by

$$\dot{\boldsymbol{G}} = \boldsymbol{f}_{\mathrm{adv}}(\boldsymbol{G}) + \boldsymbol{f}_{\mathrm{cond}}(\boldsymbol{G}, \phi_{\mathrm{cond}})\,, \qquad \dot{\boldsymbol{x}} = \boldsymbol{v}\,, \quad \dot{\boldsymbol{v}} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{v}, m, t)\,, \quad \dot{m} = \gamma(\boldsymbol{x}, m, t)\,.$$

Timescale separation

- "Slow" advection time step Δt_{adv}
- "Fast" particle interaction step h
- Discrete time $t_n^k = n\, \Delta t_{\mathrm{adv}} + k\, h$



Subloop integration scheme

The particle interactions are integrated in a subloop $t_n^0 \le t < t_n^N = t_{n+1}^0$.

We use the midpoint rule for particle positions and purely implicit methods for velocities and masses.

Assuming the fluid field $\boldsymbol{u}(\boldsymbol{x}, t)$ is known, the shown scheme describes the evolution of state variables and all particles in any given grid cell during one sub step $t \to t + h$.

$$\boldsymbol{x}_*^k = \boldsymbol{x}^k + \frac{h}{2}\boldsymbol{v}^k$$

$$m^{k+1} = m^k + h\,\gamma\left(\boldsymbol{x}_*^k, m^{k+1}, t^{k+1}\right)$$

$$\boldsymbol{v}^{k+1} = \boldsymbol{v}^k + h\,\boldsymbol{a}\left(\boldsymbol{x}_*^k, \boldsymbol{v}^{k+1}, m^{k+1}, t^{k+1}\right)$$

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}_*^k + \frac{h}{2}\boldsymbol{v}^{k+1}$$

$$\boldsymbol{G}^{k+1} = \boldsymbol{G}^k + \Delta\boldsymbol{G}_{\mathrm{adv}}^k + h\,\boldsymbol{f}_{\mathrm{cond}}(\boldsymbol{G}^k, \phi_{\mathrm{cond}}^k)$$

$$\phi_{\mathrm{cond}}^k = \sum_{\alpha \in \mathrm{cell}} \frac{\Delta m_\alpha^k}{V_0\, h}$$

Advection scheme

The change by advection is spread over the subloop using a modified Runge-Kutta method:

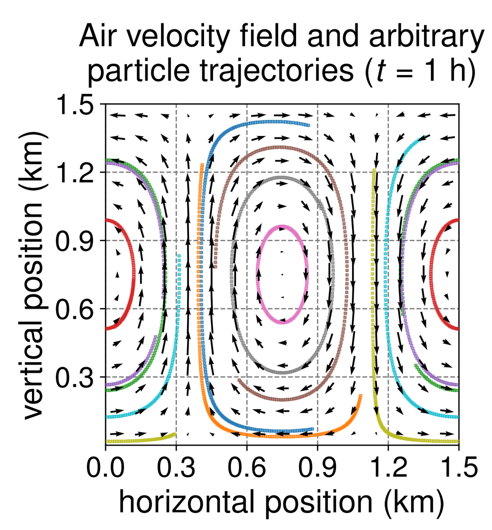
$$\Delta\boldsymbol{G}_{\mathrm{adv}}^k / h = \begin{cases} \boldsymbol{f}_{\mathrm{adv}}\left(\boldsymbol{G}^0\right), & 0 \le k < \frac{N}{2} \\ 2\,\boldsymbol{f}_{\mathrm{adv}}\left(\boldsymbol{G}^{N/2}\right) - \boldsymbol{f}_{\mathrm{adv}}\left(\boldsymbol{G}^0\right), & \frac{N}{2} \le k < N \end{cases}$$

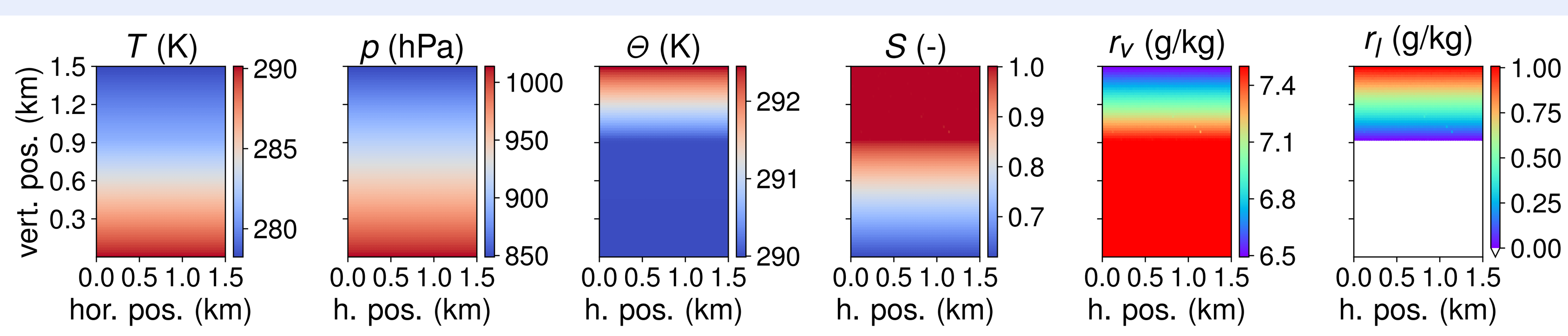
ICMW 2012 test case: Drizzling stratocumulus cloud

We use a 2D kinematic framework ([Muh13], case 1) with periodic boundary conditions, stationary dry air density profile and velocity field $\rho_{\mathrm{dry}}\boldsymbol{u} = \nabla \times (\psi(x,z)\hat{\boldsymbol{e}}_y)$ defined by the stream function [Ras11]

$$\psi(x,z) = -J_{\max} X / \pi \sin(\pi z / Z) \cos(2\pi x / X)\,.$$

Starting from a hydrostatic atmosphere, the total water content is decomposed into water vapor and liquid water by saturation adjustment with boundary conditions $r_{\mathrm{tot}} = r_v + r_l = 7.5\,\mathrm{g/kg}$ and $\Theta_{\mathrm{liq}} = 289\,\mathrm{K}$.



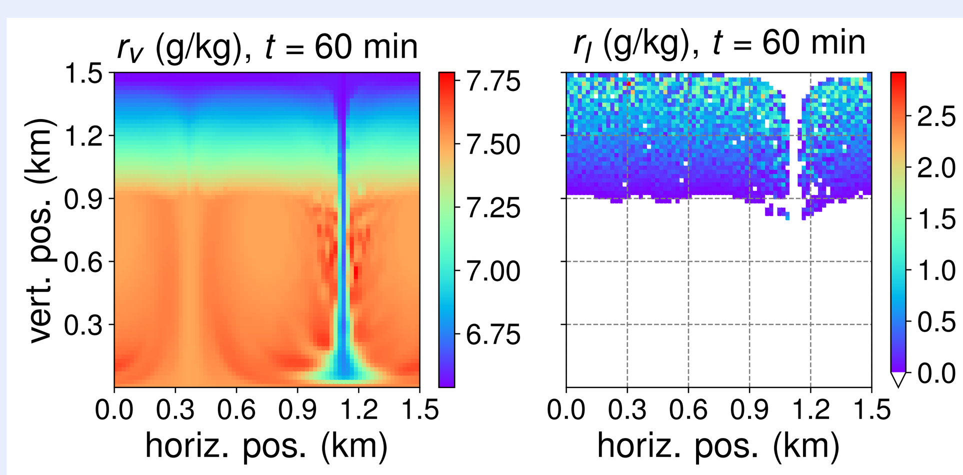


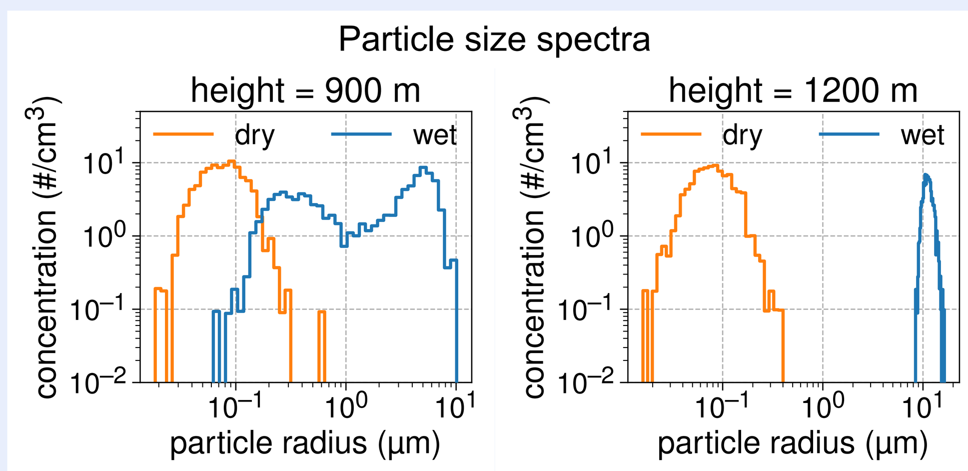
Simulation results

We present results of a one hour simulation run with monomodal dry size spectrum of NaCl CCN (initial total conc. of 100 cm^{-3}). The grid consists of 75 x 75 cells and includes 22500 super-droplets in total ($\xi \sim 1 \times 10^{10}$).

For heavy, wet particles gravity leads to a deviation of the trajectories from circular paths and thereby to the formation of downdraft tunnels for r_l and r_v.

For this monomodal case, all droplets get activated and the wet size distribution inside the cloud shows a narrow peak.





Outlook

- Implement model for collision and break-up of droplets
- Extend for multi-component solutions and ice crystal systems
- Investigate turbulence effects and the importance of disregarded interactions
- Extend to 3D and couple to ASAM CFD solver

References

[Fuk70] Fukuta, N.; Walter, L. A. *J. Atmos. Sci.* 1970, **27**, 1160.
[Muh13] Muhlbauer, A.; and Coauthors. *Bull. Am. Meteorol. Soc.* 2013, **94**(5), 45.
[Ras11] Rasinski, P.; Pawlowska, H.; Grabowski, W. W. *Atmos. Res.* 2011, **102**, 120.
[Shi09] Shima, S.; Kusano, K.; Kawano, A.; Sugiyama, T.; Kawahara, S. Q. *J. Royal Meteorol. Soc.* 2009, **135**, 1307.
[Zhu07] Zhu, H. P.; Zhou, Z. Y.; Yang, R. Y.; Yu, A. B. *Chem. Eng. Sci.* 2007, **62**(13), 3378.