Condensational and collisional growth of cloud droplets in a turbulent environment

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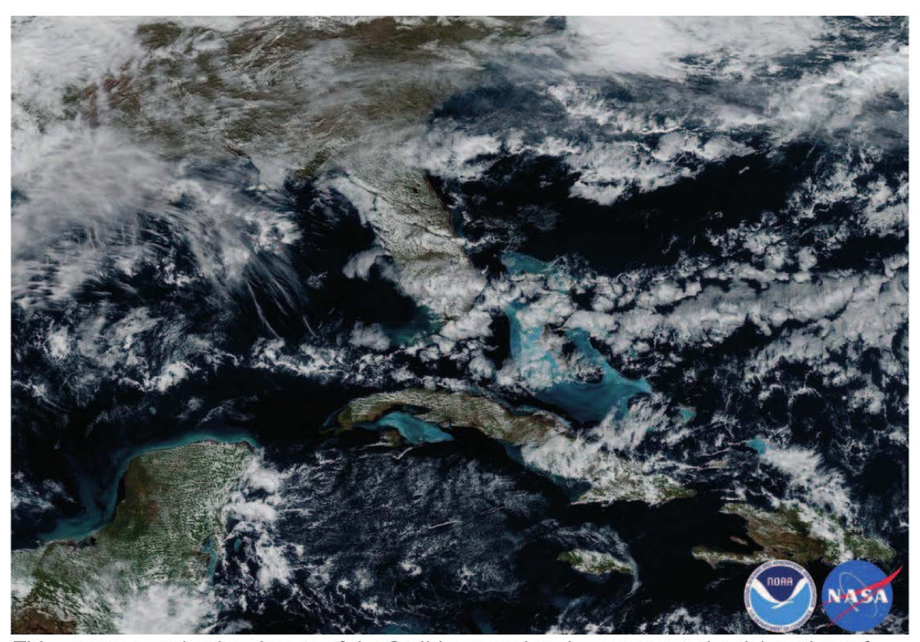
Abstract

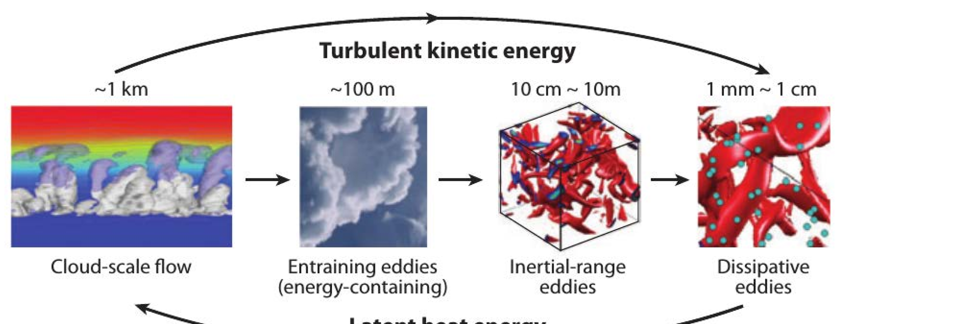
The effect of turbulence on combined condensational and collisional growth of cloud droplets is investigated using high-resolution direct numerical simulations. The motion of droplets is subjected to both turbulence and gravity. We solve the thermodynamic equations that govern the supersaturation field together with the hydrodynamic equations describing the turbulence. The collision-coalescence process is approximated by a superparticle approach assuming unit collision and coalescence efficiency, i.e., droplet coalesce upon collision. Condensational growth of cloud droplets due to supersaturation fluctuations depends on the Reynolds number, while the collisional growth was previously found to depend on the mean energy dissipation rate. Here we show that the combined processes depend on both Reynolds number and the mean energy dissipation rate. Droplet size distributions broaden either with increasing Reynolds number or mean energy dissipation rate in the range explored here. Even though collisional growth alone is insensitive to Reynolds number, it is indirectly affected by the large scales of turbulence through condensation. This is argued to be due to the fact that condensational growth results in wider droplet-size distributions, which triggers collisional growth. Since turbulence in warm clouds has a relatively small mean energy dissipation rate, but a large Reynolds number, turbulence mainly affects the condensational growth and thus influences the collisional growth indirectly through condensation. Thus, the combined condensational and collisional growth of cloud droplets is mostly dominated by Reynolds number. This work, for the first time, numerically demonstrates that supersaturation fluctuations enhance the collisional growth. It supports the findings from laboratory experiments and the observations that supersaturation fluctuations are important for precipitation.

Physical picture



This very recent daytime image of the Caribbean region demonstrates the rich variety of cloud features and organizational patterns over ocean and land.

"Understanding Clouds to Anticipate Future Climate" *Sandrine Bony et al, 2017*



Grabowski et al, 2013
Bodenschatz et al, 2010

Can we understand clouds without turbulence?

To address the bottleneck problem on warm rain formation, we study the coagulational growth of cloud droplets by direct numerical simulation (DNS). We track superparticles in a turbulent gas flow and we solve the corresponding equations by PENCIL CODE (https://github.com/pencil-code/).

$w_l \sim nr^3$ *Beals et al. (2015)*

$n(t) = \int_0^\infty f(r,t)\,\mathrm{d}r$ *Shaw (2003)*

Direct numerical simulation

Eulerian equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = S_\rho, \tag{1}$$

$$\frac{D\boldsymbol{u}}{Dt} = \boldsymbol{f} - \rho^{-1}\nabla p + \rho^{-1}\nabla \cdot (2\nu\rho\mathbf{S}) + B\boldsymbol{e}_z + \boldsymbol{S}_u, \tag{2}$$

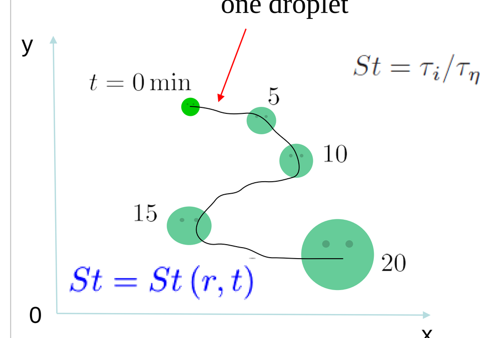
$$\frac{DT}{Dt} = \kappa\nabla^2 T + \frac{L}{c_p}C_d, \tag{3}$$

$$\frac{Dq_\mathrm{v}}{Dt} = D\nabla^2 q_\mathrm{v} - C_d, \tag{4}$$

$$C_d(\boldsymbol{x},t) = \frac{4\pi\rho_1 G}{\rho_\mathrm{a}(\Delta x)^3}\sum_{j=1}^{N_\Delta} s(\boldsymbol{x},t)\,r(t) = \frac{4\pi\rho_1 G}{\rho_\mathrm{a}}\sum_{j=1}^{N_\Delta} s(\boldsymbol{x},t)\,f(\boldsymbol{x},t)r(t)\,\delta r \qquad s(\boldsymbol{x},t) = \frac{q_\mathrm{v}(\boldsymbol{x},t)}{q_\mathrm{vs}(T)} - 1$$

Momentum equation for droplets: $\frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{V}_i \qquad \frac{d\boldsymbol{V}_i}{dt} = \frac{1}{\tau_i}(\boldsymbol{u} - \boldsymbol{V}_i) + \boldsymbol{g}$

Detection of the collision:



Track collision numerically ?

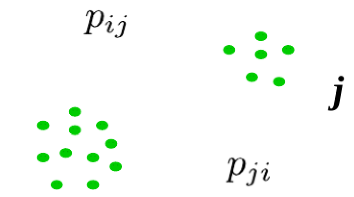
mean free path: $\lambda = \frac{1}{n\sigma}$, $\sigma = \pi(r_i + r_j)^2$

collision frequency: $\tau_c^{-1} = \frac{\delta v}{\lambda}$, $\delta v = |\mathbf{v}_i - \mathbf{v}_j|$

probability of collision: $p = \frac{\delta t}{\tau_c}$ — δt: numerical time step

random number $< p$ $\Longrightarrow$ **collision!**

Stochastic scheme:



Dullemond et al. (2008)
Shima et al. (2009)
Johansen et al. (2012)
Xiang-Yu Li et al.(2017)

Table 1. Summary of the simulations.

Run	$N_p/10^6$	mesh points	f_0	L (m)	u_rms ($\mathrm{m\,s^{-1}}$)	Re_λ	$\bar\epsilon$ ($\mathrm{m^2 s^{-3}}$)	$\eta \cdot 10^{-4}$ (m)	τ_η (s)
A	8.4	256^3	0.02	0.125	0.17	57	0.039	4	0.016
B	67	512^3	0.02	0.25	0.21	94	0.04	4	0.016
C	67	512^3	0.02	0.50	0.27	158	0.036	4	0.017
D	67	512^3	0.0072	0.44	0.13	98	0.005	7	0.044
E	67	512^3	0.01	0.37	0.15	97	0.01	6	0.032
F	67	512^3	0.014	0.30	0.18	94	0.02	5	0.022

Here, N_p is the number of superparticles used in the simulation, f_0 is the amplitude of the random forcing (see text), and L is the domain size.

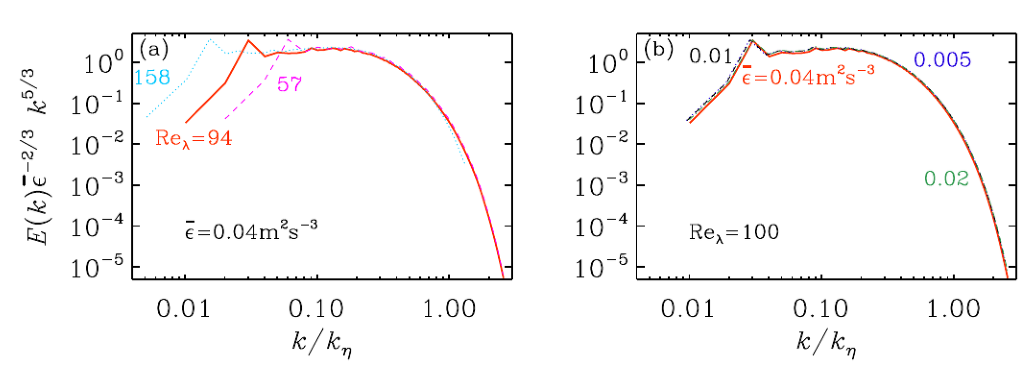
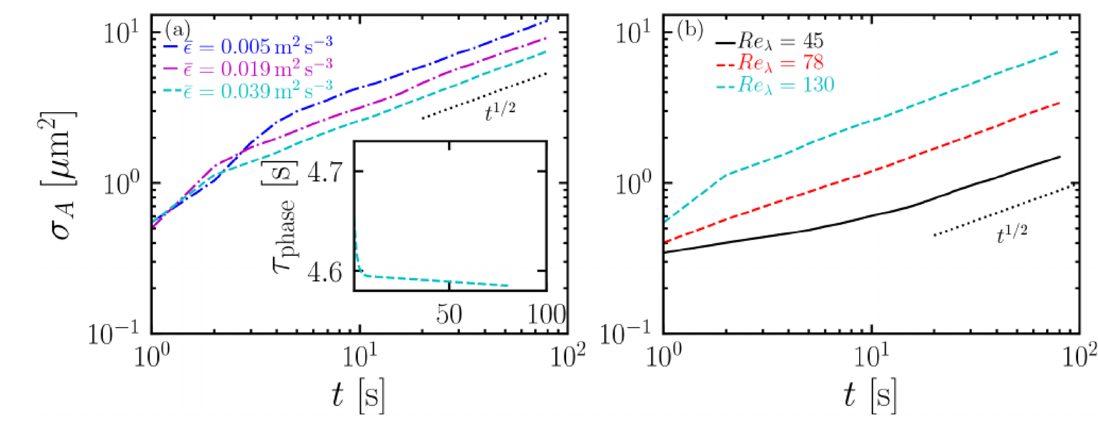


Figure 1. Turbulent kinetic-energy spectra for (a) different Re_λ = 57 (magenta dashed line), 94 (red solid line), and 158 (cyan dotted line) at fixed $\bar\epsilon = 0.04\mathrm{m^2 s^{-3}}$ (see Runs A, B, and C in Table 1 for details) and for (b) different $\bar\epsilon = 0.005\ \mathrm{m^2 s^{-3}}$ (blue dotted line), 0.01 (black dashed line), 0.02 (green dash-dotted line) and 0.04 (read solid line) at fixed Re_λ = 100 (see Runs B, D, E, and F in Table 1 for details).

- **X.-Y. Li**, A. Brandenburg, N. Haugen, and G. Svensson. Eulerian and Lagrangian approaches to multidimensional condensation and collection. *Journal of Advances in Modeling Earth Systems*, 9(2):1116–1137, 2017
- **X.-Y. Li**, A. Brandenburg, G. Svensson, N. Haugen, B. Mehlig, and I. Rogachevskii. Effect of turbulence on collisional growth of cloud droplets. *Journal of the Atmospheric Sciences*, 75(10):3469–3487, 2018
- **X.-Y. Li**, G. Svensson, A. Brandenburg, and N. E. L. Haugen. Cloud-droplet growth due to supersaturation fluctuations in stratiform clouds. *Atmospheric Chemistry and Physics*, 19.1 (2019): 639-648
- **X.-Y. Li**, Axel Brandenburg, Gunilla Svensson, Nils Haugen, Bernhard Mehlig, and Igor Rogachevskii. Condensational and collisional growth of cloud droplets in a turbulent environment. *arXiv*:1807.11859, 2018
- **X.-Y. Li**, B. Mehlig, G. Svensson, A. Brandenburg, and N. E. L. Haugen. Fluctuations andgrowth histories of cloud droplets: superparticle simulations of the collision-coalescence process. *arXiv*:1810.07475, 2018

Results

Condensational growth of cloud droplets

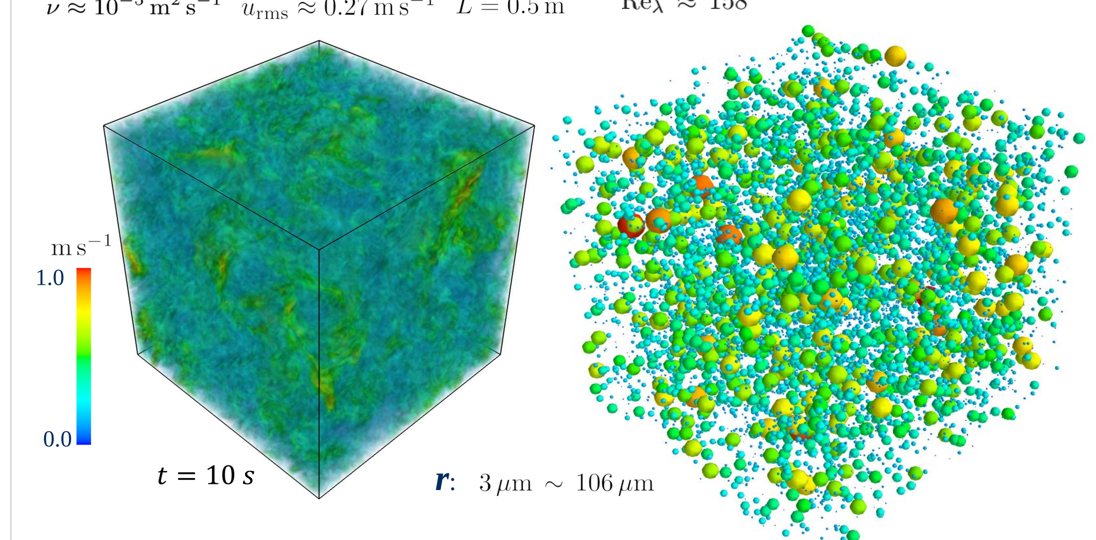


Time evolution of σ_A for different **(a)** $\bar\epsilon$ at $Re_\lambda = 130$ and **(b)** Re_λ at $\bar\epsilon = 0.039\,\mathrm{m^2\,s^{-3}}$.

Broadening of droplet size distribution due to supersaturation fluctuations.

Gollision-coalescence in 3-D turbulence

$\nu \approx 10^{-5}\,\mathrm{m^2\,s^{-1}}$ $u_\mathrm{rms} \approx 0.27\,\mathrm{m\,s^{-1}}$ $L = 0.5\,\mathrm{m}$ $\mathrm{Re}_\lambda \approx 158$



Time evolution of the droplet-size distribution

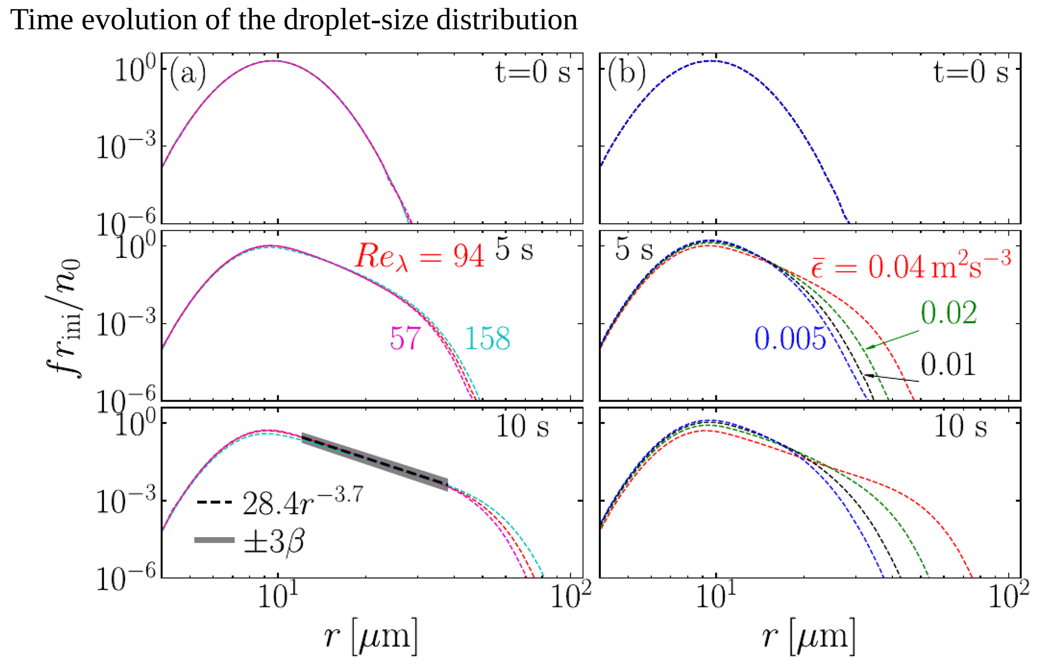
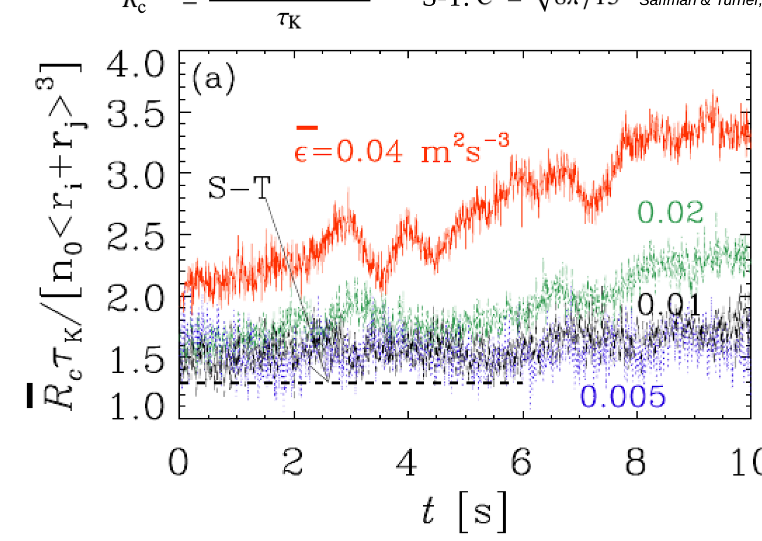


FIG. 2. Droplet size distribution for the same simulations as in Fig. 1. (a) Different Re_λ at fixed $\bar\varepsilon$. (b) Different $\bar\varepsilon$ at fixed Re_λ. Here β is the standard deviation and 3β is the significance level.

Mechanism

$R_\mathrm{c}^\mathrm{S.T.} = \frac{C(\mathrm{Re}_\lambda)\,n_0\,(2r)^3}{\tau_\mathrm{K}}$ S-T: $C = \sqrt{8\pi/15}$ *Saffman & Turner, 1955*



For small $\bar\epsilon$, at early times, C ≈1.6

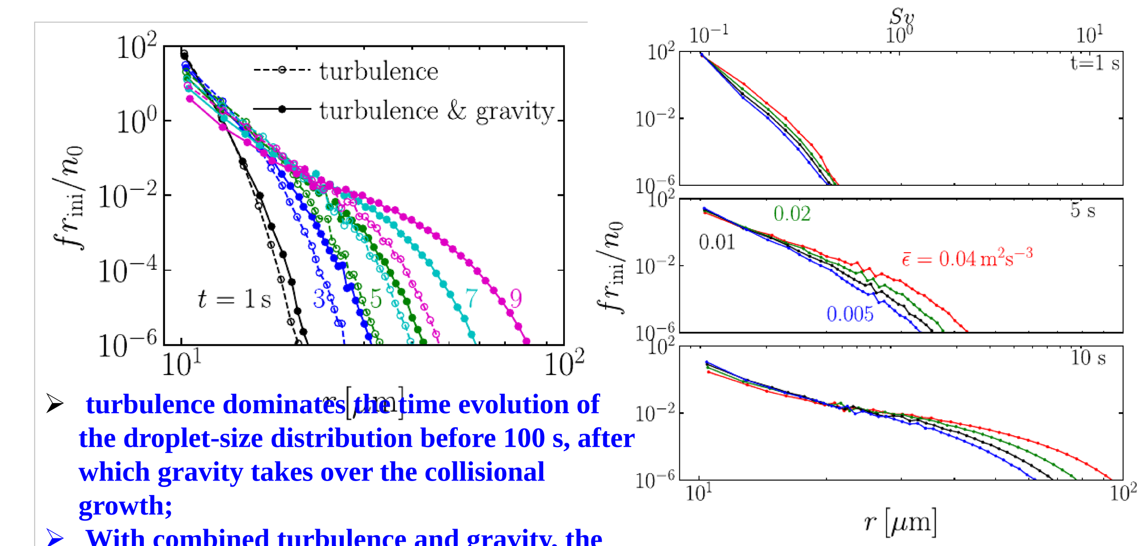
$\bar R_c \sim \sqrt{\bar\epsilon}.$

Scaling law breaks at later times and for large $\bar\epsilon$

Growth of cloud droplets strongly depends on the mean turbulent energy-dissipation rate per unit mass, and only weakly on the Reynolds number of the turbulent flow.

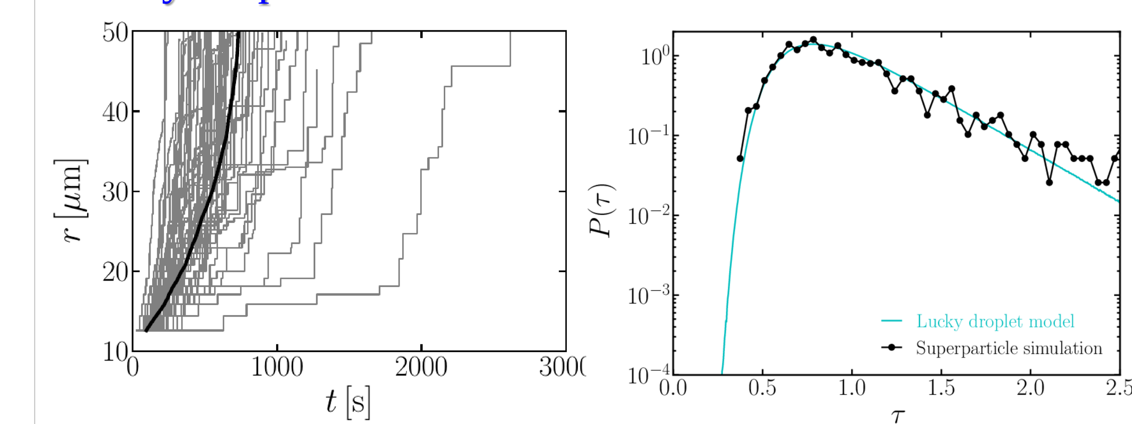
Gollision-coalescence in 3-D turbulence & gravity

Initial distribution: monodisperse cloud droplets (10 μm)



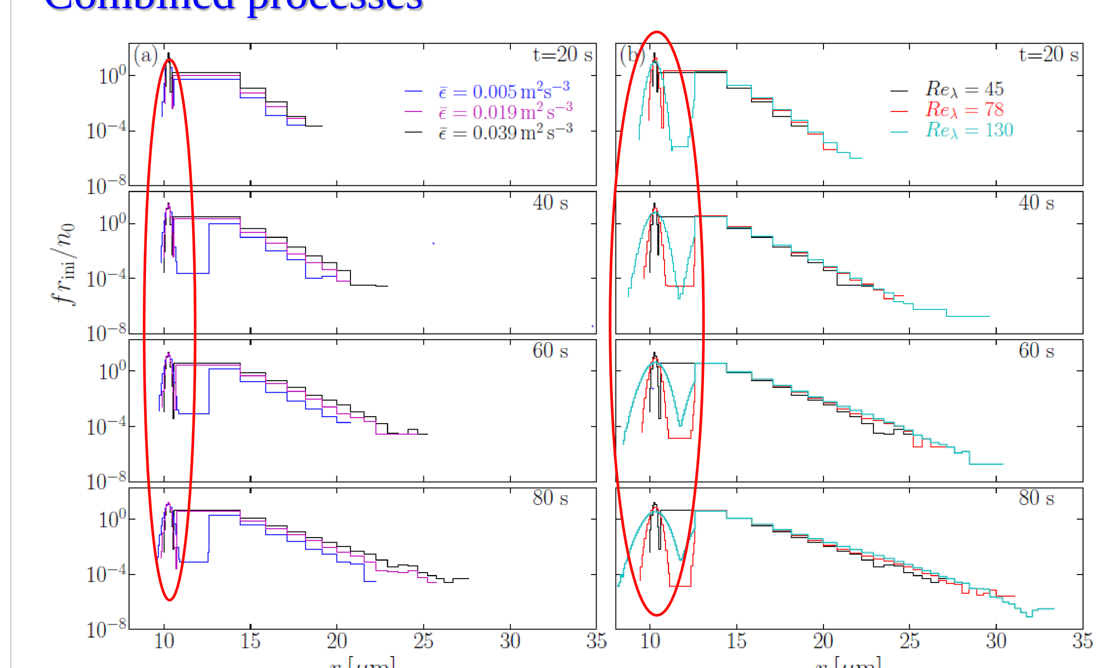
- **turbulence dominates the time evolution of the droplet-size distribution before 100 s, after which gravity takes over the collisional growth;**
- **With combined turbulence and gravity, the time scale to reach drizzle sized droplets is about 800 s, which is relevant to the time scale of the rapid warm rain formation.**

Lucky droplet model

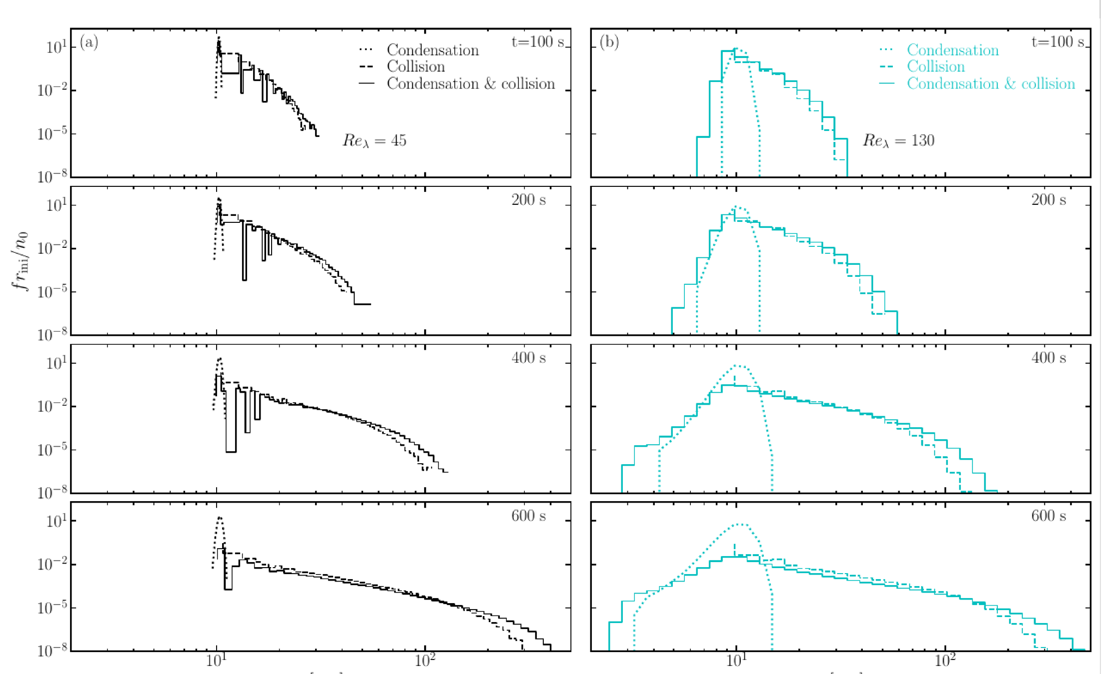


$\mathscr{T}_\mathcal{N} = \sum_{k=1}^{\mathcal{N}} t_k \qquad p_k(t_k) = \lambda_k \exp(-\lambda_k t_k) \qquad \lambda_k = \lambda_1 k^{4/3}$

Combined processes



The combined condensational and collisional growth of cloud droplets is mostly dominated by Reynolds number.



Conclusion

The combined condensational and collisional growth of cloud droplets is mostly dominated by Reynolds number. This work, for the first time, numerically demonstrates that supersaturation fluctuations enhance the collisional growth. It supports the findings from laboratory experiments and the observations that supersaturation fluctuations are important for precipitation.

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