Enhancement of radar reflectivity factor due to turbulent droplet clustering in cumulus clouds

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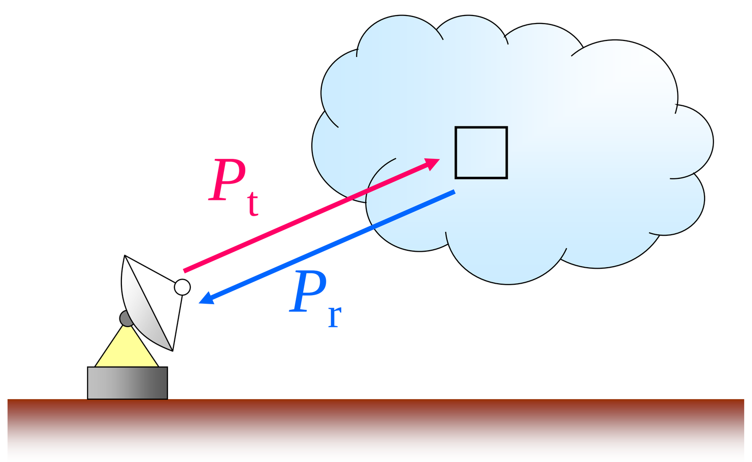
References. Matsuda, K. and Onishi, R.: Turbulent enhancement of radar reflectivity factor for polydisperse cloud droplets, *Atmos. Chem. Phys.*, 19, 1785-1799, 2019.

1. Introduction

To estimate cloud microphysical properties in radar observations, it is important to understand the relationship between the properties and the radar reflectivity factor. The observed radar reflectivity factor is increased by the Bragg scattering due to turbulent fluctuations of the refractive index and droplet number density (Knight & Miller, J. Atmos. Sci., 1998); i.e., clear-air and particulate Bragg scattering, respectively. In this study, we investigate the influence of particulate Bragg scattering due to **turbulent clustering** of cloud droplets with a broad size distribution.

2. Radar reflectivity factor $Z[\mathrm{mm^6\ m^{-3}}] \propto P_\mathrm{r}/P_\mathrm{t}$

r_p Droplet radius
n_p Droplet number density
k_m Wavenumber of microwave
$E_\mathrm{n}(k)$ Power spectrum of refractive index fluctuation



$Z =$ **Incoherent part** — Dependent on size and amount of cloud droplets: $Z_\mathrm{incoh} = 2^6 \langle r_\mathrm{p}^6 \rangle \langle n_\mathrm{p} \rangle$

$+$ **Particulate Bragg scattering** — Influence of spatial correlation of cloud droplets: $Z_\mathrm{PB} = \dfrac{2^5 \pi^2}{k_\mathrm{m}^2} E_\mathrm{r3np}(2k_\mathrm{m})$

$+$ **Clear-air Bragg scattering** — Influence of spatial variance of refractive index: $Z_\mathrm{CB} = \dfrac{2^3}{|K|^2 k_\mathrm{m}^2} E_\mathrm{n}(2k_\mathrm{m})$

3. Parameterization of $E_\mathrm{r3np}(k)$ based on direct numerical simulation (DNS) with Lagrangian particles

- Three-dimensional DNS of particle-laden homogeneous isotropic turbulence
  Taylor-microscale-based Reynolds number: $\mathrm{Re}_\lambda = 204$ (512^3 grid points)
  Lagrangian particles (droplets): $N_\mathrm{p} = 1.5 \times 10^7$ for each Stokes number

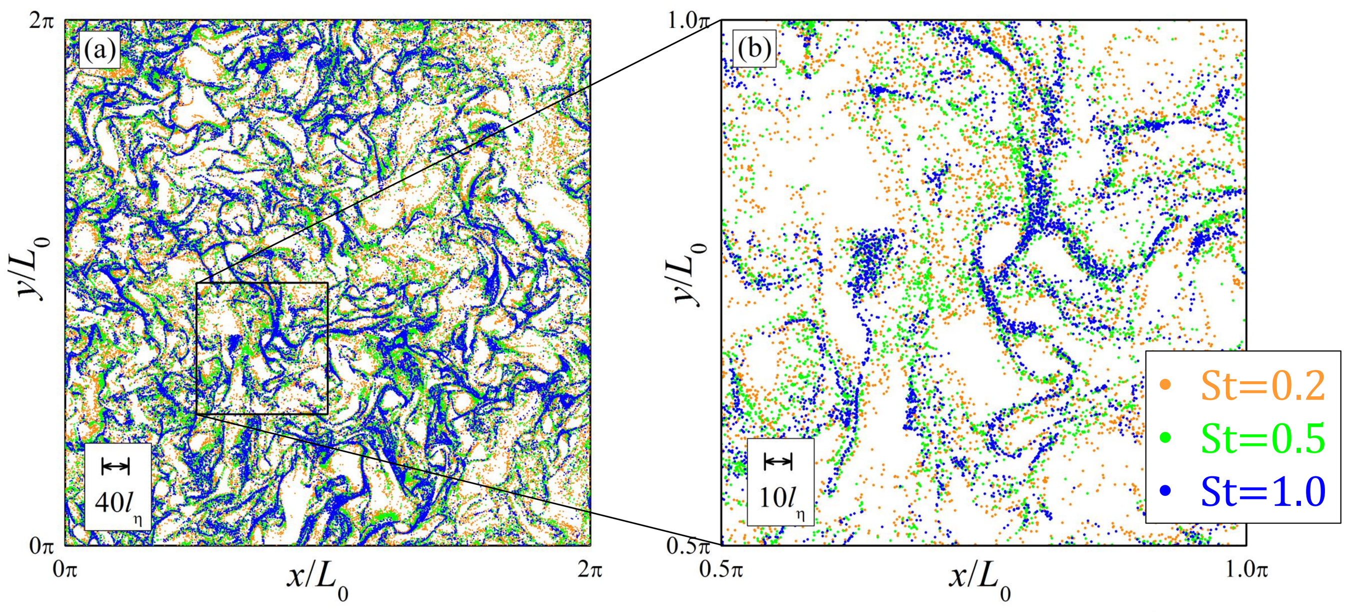


Fig. 1: Turbulent droplet clustering (2D slice at $0 < z < 4l_\eta$) obtained by DNS. Droplets concentrate in low-vorticity regions due to centrifugal force.

Stokes number St

$$\mathrm{St} \equiv \frac{\tau_\mathrm{p}}{\tau_\eta} \propto \left(\frac{r_p}{l_\eta}\right)^2$$

τ_p Droplet relaxation time
τ_η Kolmogorov time
l_η Kolmogorov scale

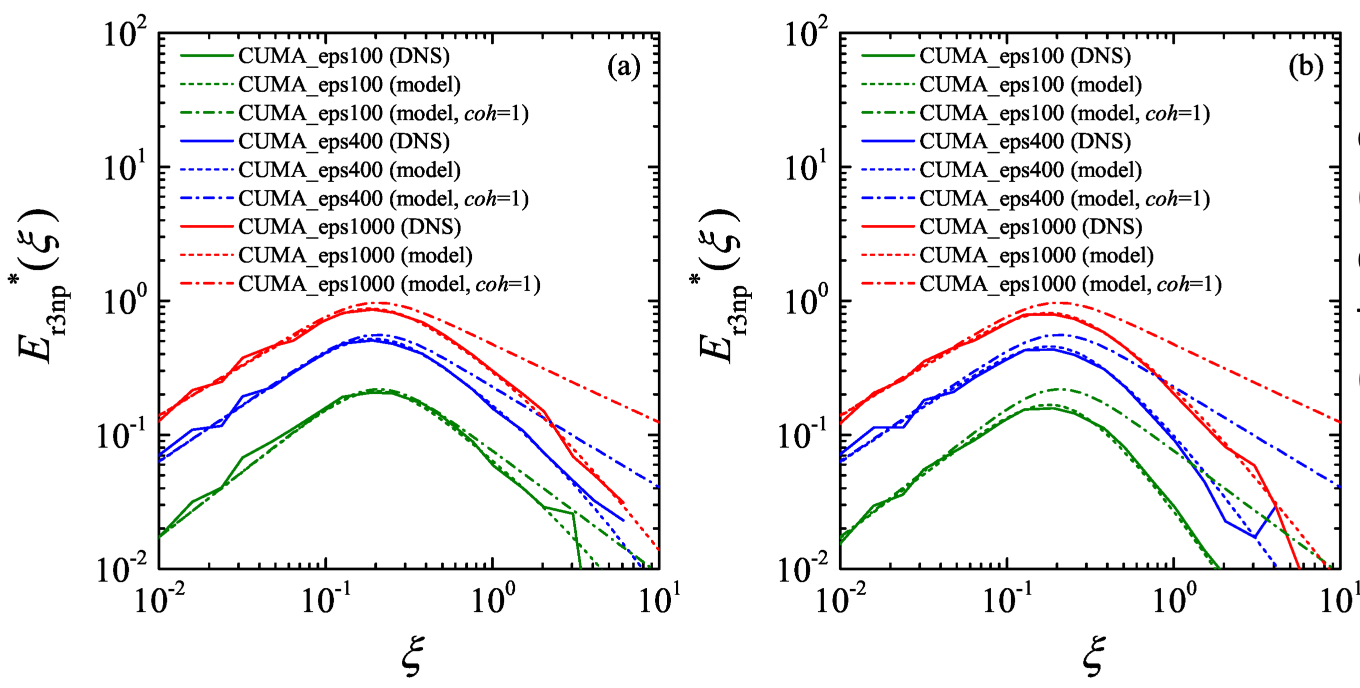


Fig. 2: Comparison of $E_\mathrm{r3np}(k)$ obtained from DNS data (droplets with broad size distribution) and estimated by the proposed parameterization: (a) $g = 0$ and (b) $g = 9.8$ m/s².

Parameterization developed in this study

r_p^3-weighted number density spectrum $E_\mathrm{r3np}(k)$:

$$E_\mathrm{r3np}(k) = \int_0^\infty \int_0^\infty r_\mathrm{p}^3 r_\mathrm{p}'^3 q_\mathrm{r}(r_\mathrm{p}) q_\mathrm{r}(r_\mathrm{p}') \boxed{C_\mathrm{np}(2k_\mathrm{m}|r_\mathrm{p}, r_\mathrm{p}')} dr_\mathrm{p} dr_\mathrm{p}'$$

Cross spectrum of number density fluctuations $C_\mathrm{np}(k|r_\mathrm{p1}, r_\mathrm{p2})$:

$$C^*_\mathrm{np}(k|\mathrm{St}_1, \mathrm{St}_2) = coh(k|\mathrm{St}_1, \mathrm{St}_2)\sqrt{E^*_\mathrm{np}(k|\mathrm{St}_1) E^*_\mathrm{np}(k|\mathrm{St}_2)}$$

Coherence (based on present DNS results):

$$coh(k|\mathrm{St}_1, \mathrm{St}_2) = \exp(-k/k_c)$$

Critical wavenumber k_c:

$$k_c l_\eta = \underbrace{\frac{0.191}{|\mathrm{St}_1 - \mathrm{St}_2|}}_{\text{St dependence based on present DNS results}} \underbrace{\left[1 + \frac{1}{3a_0}\mathrm{Fr}^{-2}\right]^{-\frac{1}{2}}}_{\text{Correction for gravitational settling (Lu et al., New J. Phys., 2010)}}$$

Power spectrum $E^*_\mathrm{np}(k|\mathrm{St})$: Matsuda et al. *J. Atmos. Sci.* 2014

Normalization of cross spectrum

$$C^*_\mathrm{np}(k|\mathrm{St}_1, \mathrm{St}_2) \equiv \frac{C_\mathrm{np}(k|r_\mathrm{p1}, r_\mathrm{p2})}{\langle n_\mathrm{p1} \rangle \langle n_\mathrm{p2} \rangle l_\eta}$$

$q_\mathrm{r}(r_\mathrm{p})$ PDF of droplet radius
k Wavenumber
Fr Froude number
a_0 Normalized acceleration variance

4. Application to cloud large-eddy simulation (LES) data

- Multiscale simulation model, *MSSG* (*Multi-Scale Simulator for the Geoenvironment*)
- Spectral-bin cloud microphysics scheme (Onishi & Takahashi, J. Atmos. Sci., 2012)
- The protocol of RICO model intercomparison project (van Zanten et al., 2006)
- Extra 1h simulation with higher resolution: $\Delta_x = \Delta_y = 25$ m and $\Delta_z = 20$ m

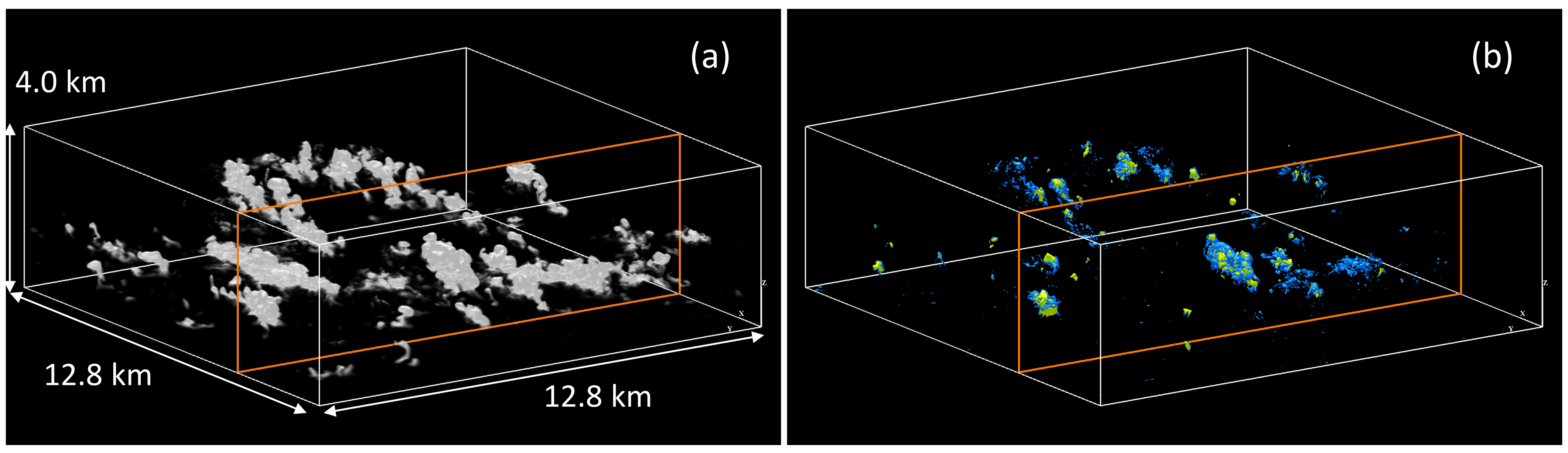
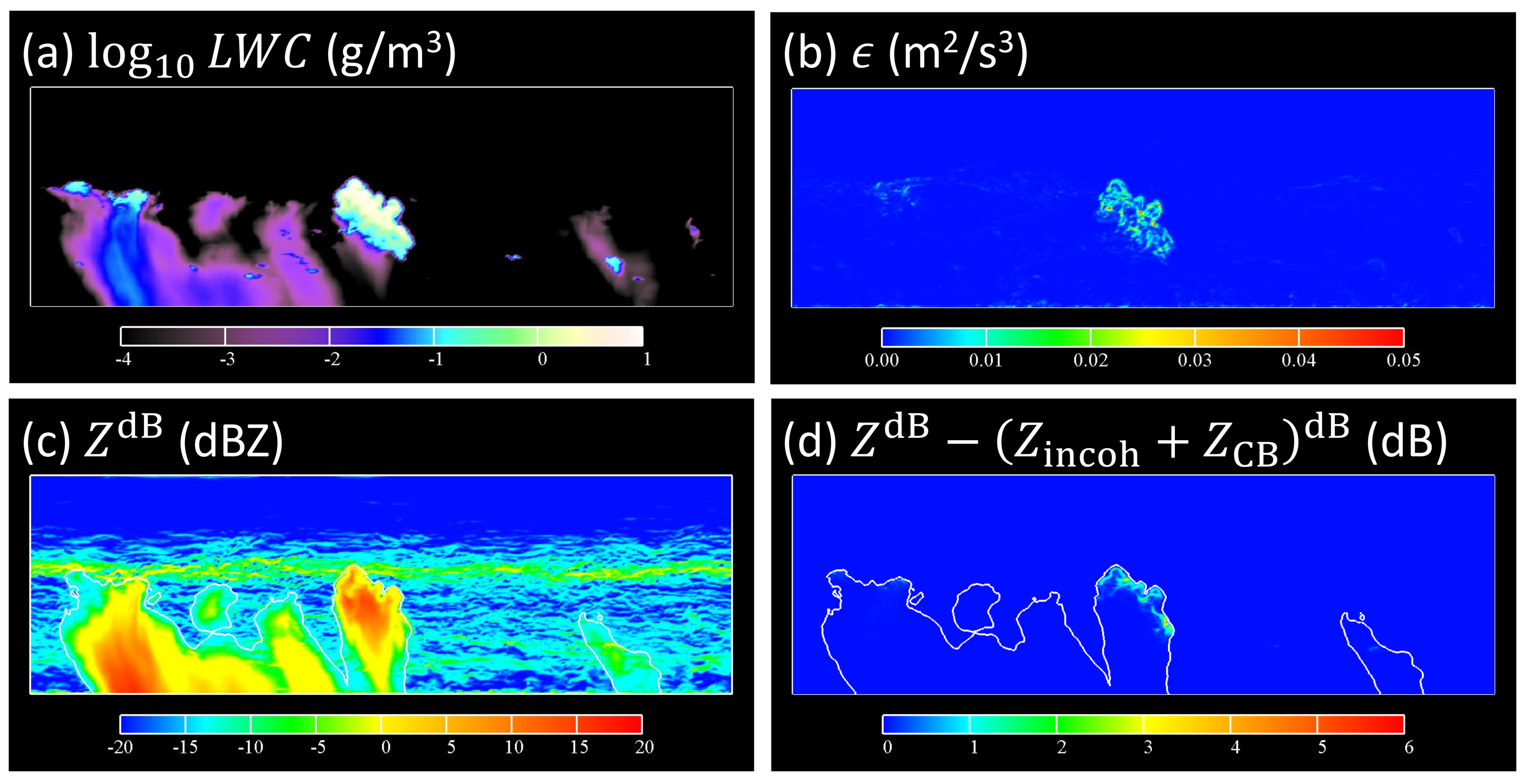


Fig. 3: Cloud LES data: (a) volume rendering of optical depth and (b) isosurfaces of (blue) the energy dissipation rate $\epsilon = 100$ cm²/s³ and (yellow) the vertical velocity $u_3 = 3$ m/s.



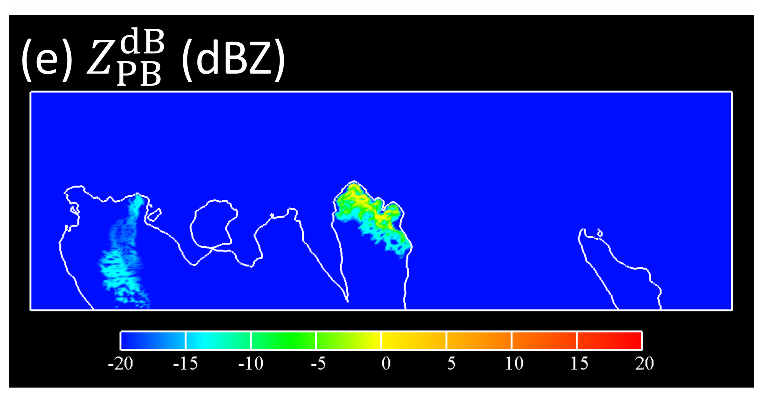


Fig. 4: Liquid water content (LWC), energy dissipation rate ϵ, and radar reflectivity factors for S-band microwaves in the cross section. The solid lines in (c)–(e) indicate $Z^\mathrm{dB}_\mathrm{incoh} = -18$ dBZ.

5. Conclusion

- The DNS of particle-laden turbulence has been performed to obtain turbulent droplet clustering data (Fig. 1). The clustering data were used to calculate and parameterize the number density fluctuation spectrum, which represents the clustering influence on the particulate Bragg scattering. The proposed parameterization can reproduce the spectrum in a sufficient accuracy even for the case with gravitational droplet settling (Fig. 2).
- The proposed parameterization has been applied to cloud LES data (Fig. 3). The radar reflectivity factor was calculated considering the particulate and clear-air Bragg scattering. The increase of radar reflectivity factor due to turbulent clustering can be larger than the observation error level. The large influence is observed inside turbulent clouds, where the liquid water content and the energy dissipation rate are sufficiently large (Fig. 4).

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