A Cloud Microphysics Parameterization for Shallow Cumulus Clouds Based on Lagrangian Cloud Model

Y. Noh
Department of Atmospheric Sciences, Yonsei University, Korea

If Lagrangian motion of cloud droplets is calculated directly,

- Diffusion and settling of droplets can be calculated directly.
- No numerical diffusion of droplet spectrum
- Activation and condensational growth are calculated naturally following droplets.
- Direct interaction between droplets and turbulence.
- Time history of each droplet can be obtained.

→ Transition to raindrops of each raindrop (autoconversion, accretion) is calculated directly.

1. The Route to Raindrop Formation in a Shallow Cumulus Cloud Simulated by a LCM

(Hoffmann et al., *JAS* 2017)

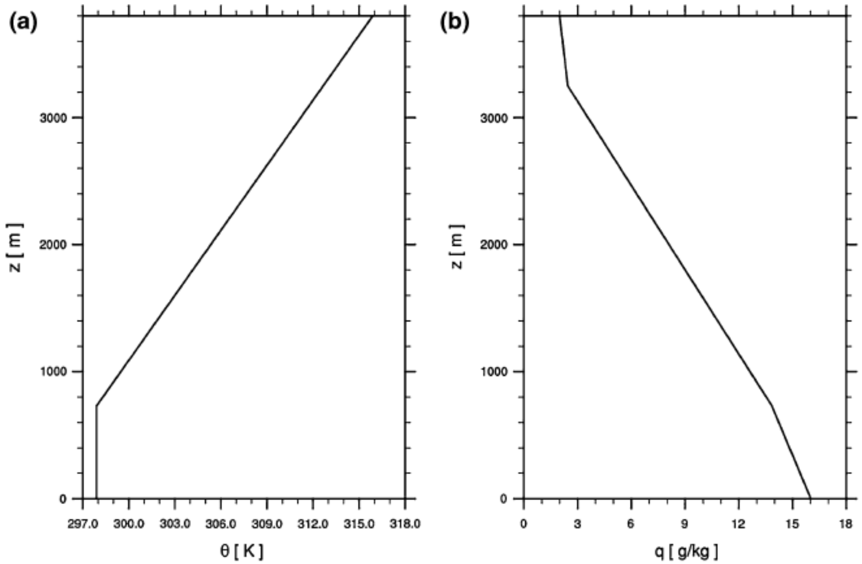
2. Analysis of Cloud Microphysics Parameterization from LCM

(Noh et al., *JAS* 2018)

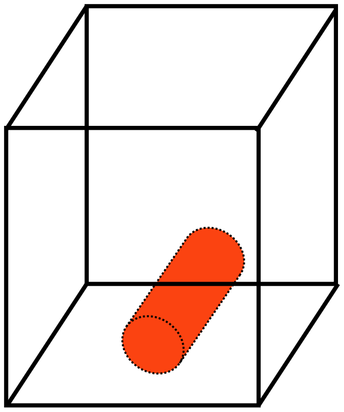
3. Development of Cloud Microphysics Parameterization Based on LCM

An Idealized Single Cloud Experiment (RICO)

- **MODEL : LES (PALM)**
  - ✓ Domain size : 1.28 km x 3.84 km x 3.84 km
  - ✓ Time step: 0.1 s
  - ✓ Resolution: 20 m
  - ✓ Collision kernel: Hall kernel (GRAV)
    AW kernel (TURB)

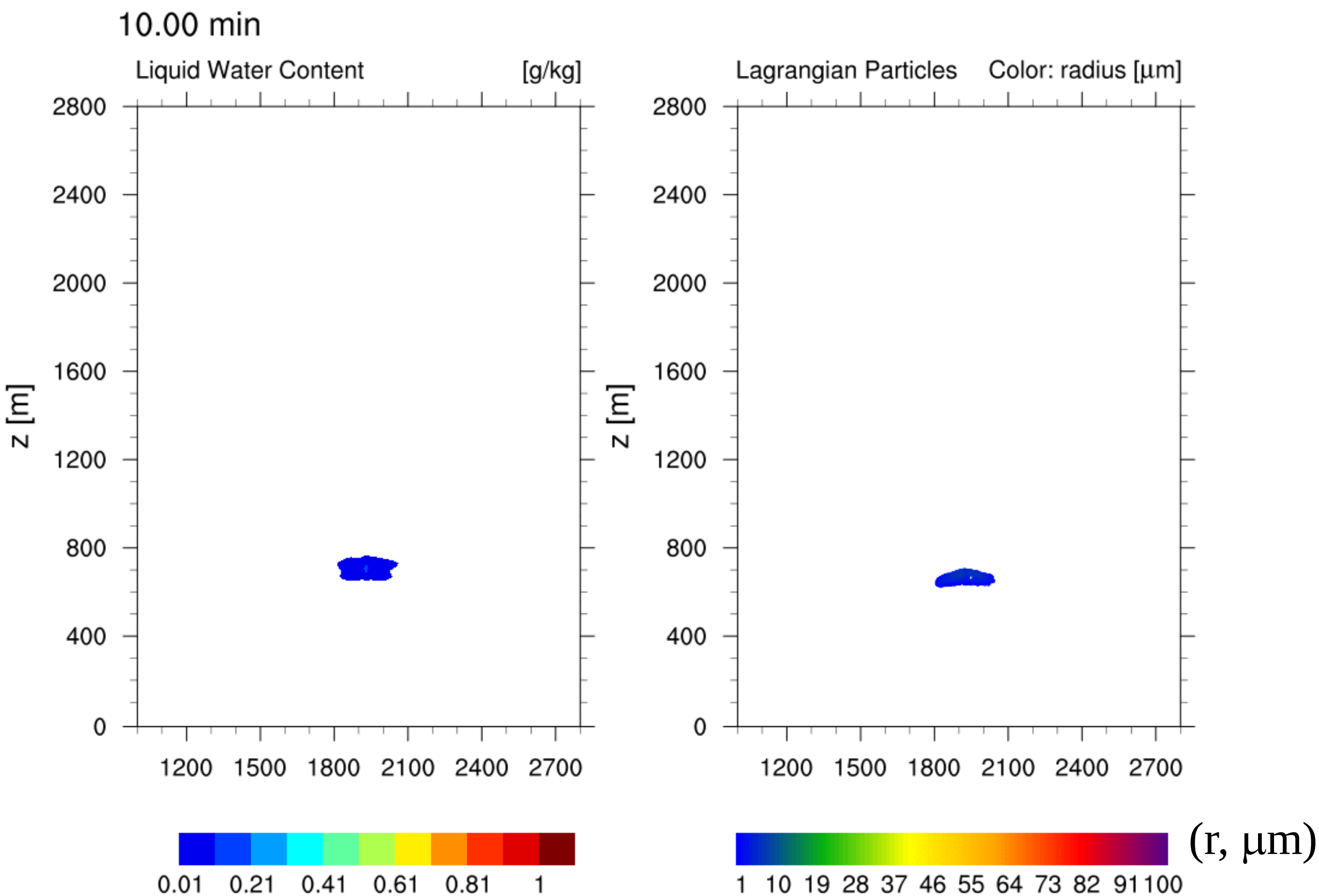


- **Particles information**
  - ✓ Initial particles size: 0.1 μm
  - ✓ Initial weighting factor: $9 \cdot 10^9$
  - ✓ Total number of particles: ~ 3.4×10^8 (~ 200 per grid box)
  - ✓ Particle concentration: 100 cm^{-3}
  - ✓ Bubble size : 1280 m x 150 m x 200 m, $\Delta T = 0.4$ K
  - ✓ Initial CCN concentration: 20, 70, 150 cm^{-3}



1. The Route to Raindrop Formation in a Shallow Cumulus Cloud Simulated by a LCM

Simulation of an Idealized Single Cloud



Evolutions of LWC and Lagrangian droplets

Questions to Raindrop Formation

It is difficult to explain the rapid growth of cloud droplets in the size range 15 - 40 μm, for which neither the diffusional growth and nor the collisional growth is effective.

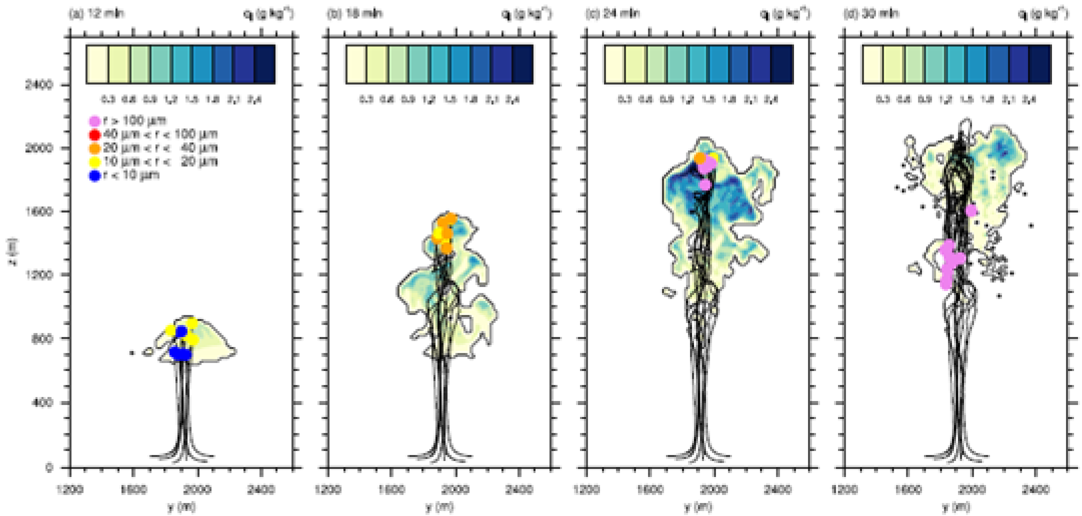
$$\rightarrow \quad K(R,r) = \pi (R+r)^2 \left| v(R) - v(r) \right| E(R,r)$$

• Entrainment and mixing broaden the droplet size distribution (DSD) (Baker et al. 1980, Cooper 1989, Lasher-Trapp et al. 2005)

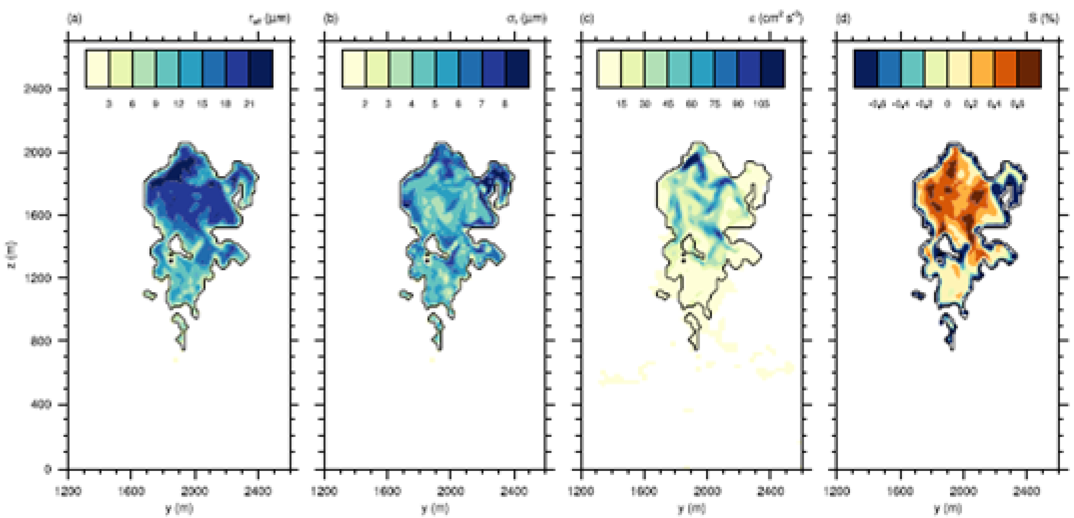
• Turbulence induces collection enhancement (TICE) (Pinsky and Khain 2002, Wang and Grabowski 2009)

• Effects of giant aerosol particles (Ochs 1978, Johnson 1982)

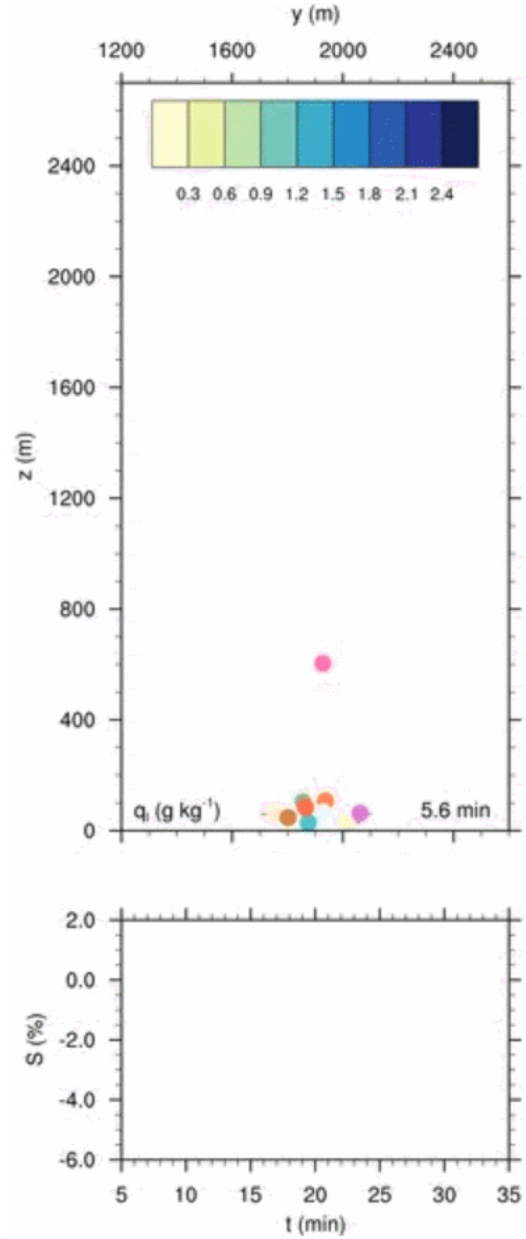
⇨ ***The best way to investigate raindrop formation is how and under which condition cloud droplets grow to raindrops by tracking Lagrangian droplets in LCM.***



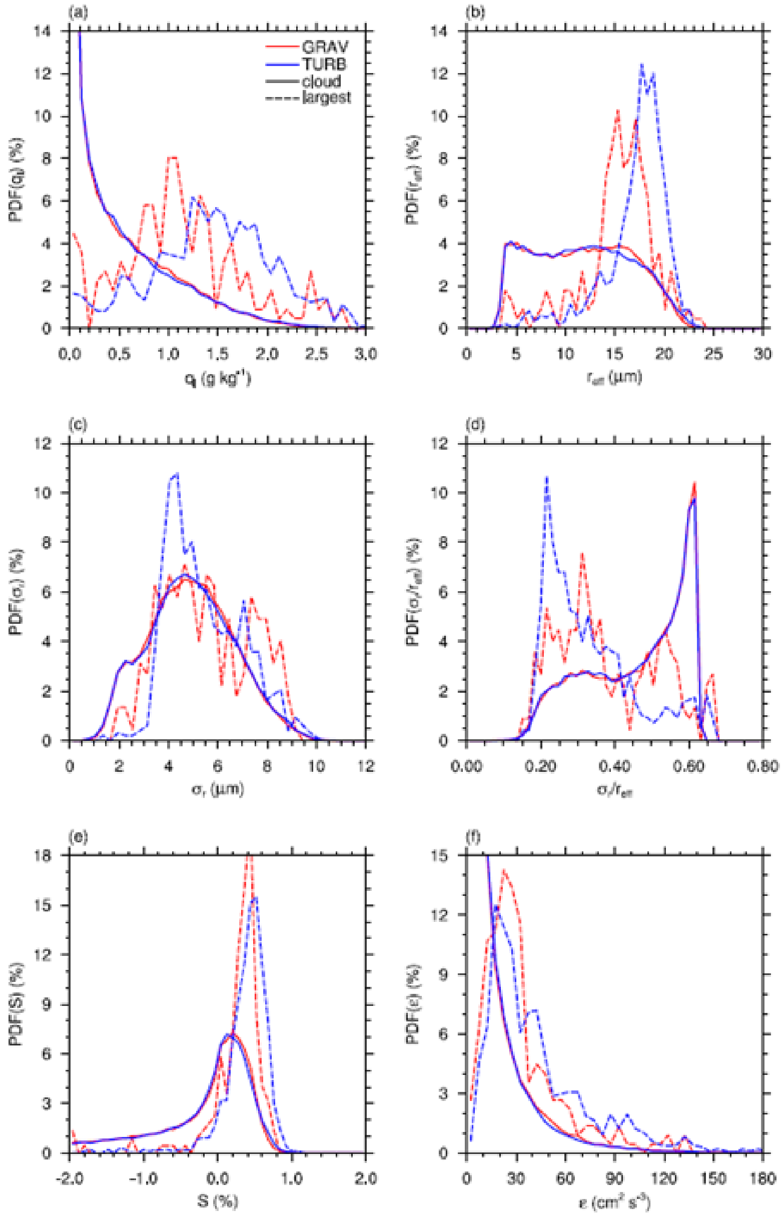
Evolution of q_l with largest droplets



Distributions of r_{eff}, σ_r, ε, and S



Raindrop formation is triggered when droplets with a radius of 20 µm appear in near the cloud top, characterized by a large q_l, r_{eff}, σ_r, ε, and S.



Pdf of variables

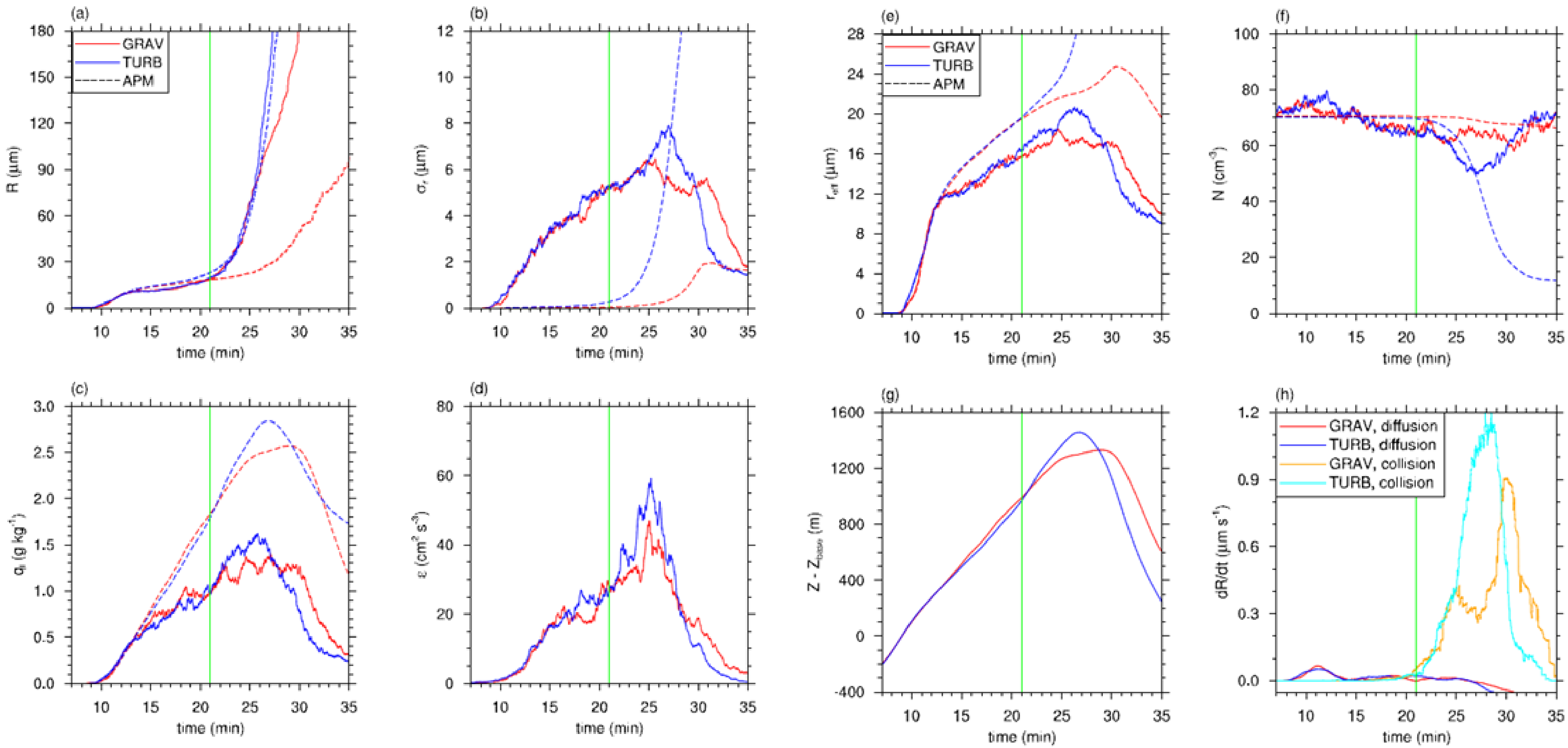
--- : potential raindrops

— : whole cloud

red: GRAV

blue: TURB

- Raindrop is formed in the region of high q_l , r_{eff} , ε, σ_r and S
- TURB – higher q_l , r_{eff} , ε, and S
- GRAV – higher σ_r



Time series following potential raindrops

(--- : adiabatic parcel model (no DSD broadening), — : LCM; red: GRAV, blue: TURB)

- Raindrop formation is triggered, when largest droplets grow to r = 20 μm.
- TURB - Raindrop formation is triggered in time, regardless of DSD broadening
  GRAV – Raindrop formation is severely delayed without DSD broadening
- TURB does not accelerate the timing of raindrop formation, but it enhances the collisional growth rate substantially, leading to stronger precipitation.

Time to reach raindrops (τ_R) from the collision box model

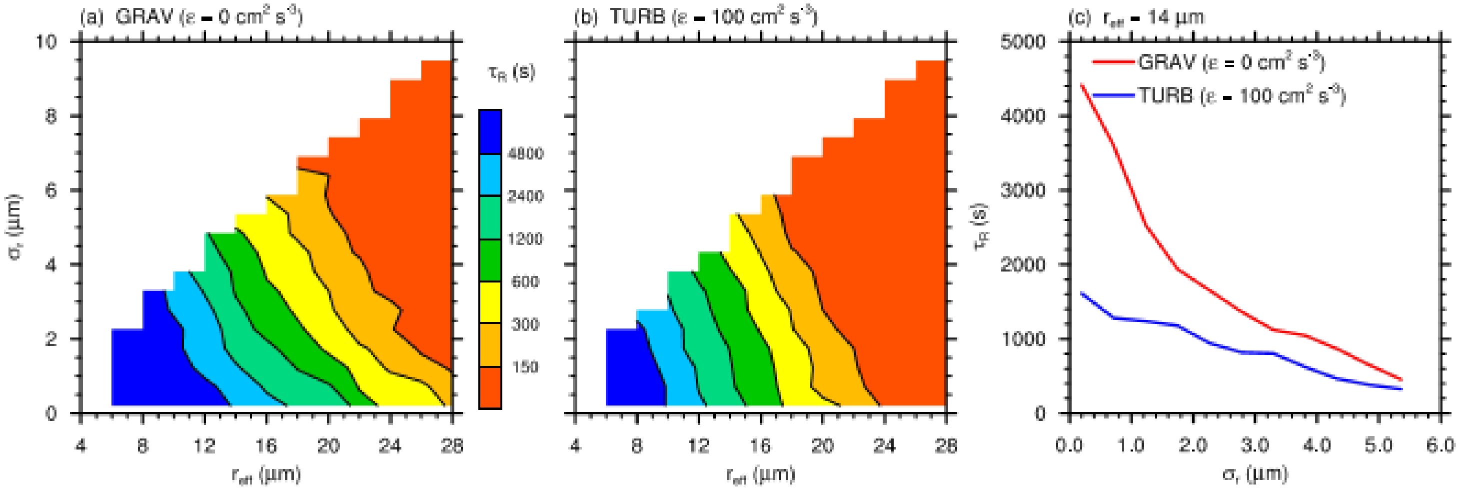
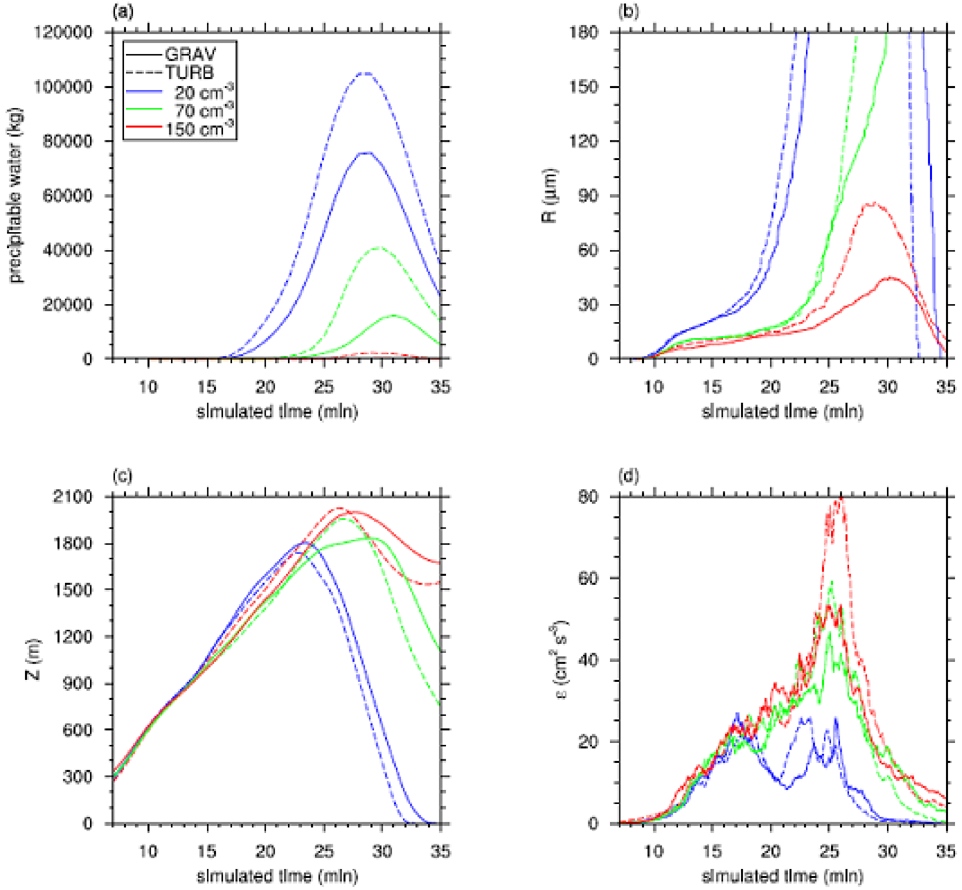


FIG. 9. The variation of the time to reach raindrops τ_R from box simulations of the collisional growth process starting from different log-normally shaped drop size distributions with different σ_r and r_{eff}: (a) GRAV ($\varepsilon = 0\,\mathrm{cm}^2\,\mathrm{s}^{-3}$), (b) TURB ($\varepsilon = 100\,\mathrm{cm}^2\,\mathrm{s}^{-3}$), and (c) the variation of τ_R with σ_r for $r_{\mathrm{eff}} = 14\,\mu\mathrm{m}$.

- small $\varepsilon \rightarrow \tau_R$ becomes very large for small σ.
- large $\varepsilon \rightarrow \tau_R$ does not vary much with σ.



Times series of variables from different initial droplet concentrations (Nc): (a) ql (r > 40 μm), (b) R, (c) Z, and (d) ε (solid: GRAV, dotted: TURB) (blue: 20 cm^{-3}, green: 70 cm^{-3}, red: 150 cm^{-3}).

- Delayed raindrop formation for larger Nc.
- stronger effect of TICE for larger Nc.

Conclusion

- Raindrop formation is triggered
  - when droplets with a radius of 20 μm appear in the region near the cloud top

- Raindrop is formed in the region of high q_l , r_{eff} , ε, σ_r and S.
  - q_l , r_{eff} , ε, and S are higher in TURB
  - σ_r is higher in GRAV.

- TURB - Raindrop formation is triggered in time, regardless of DSD broadening.
  GRAV – Raindrop formation is severely delayed without DSD broadening.

- TURB does not accelerate the timing of raindrop formation, but it enhances the collisional growth rate substantially, leading to stronger precipitation.

- As aerosol concentration (N) increases,
  - faster and stronger precipitation
  - stronger effect of turbulence

Parameterization of Cloud Microphysics

ex) two moment scheme (warm cloud)

$$\frac{dq_C}{dt} = Cd - A - C + Df$$

$$\frac{dN_C}{dt} = Ac - Ev - A - C + Df$$

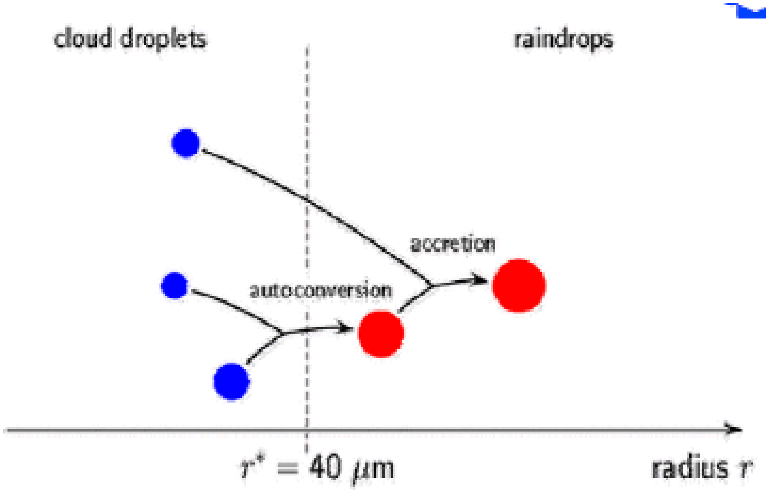
$$\frac{dq_R}{dt} = -Sd + A + C - Ev + Df$$

$$\frac{dN_r}{dt} = Sd - Ev + A - Sc + Df$$

q_c = cloud water mixing ratio ($r < r^*$)
q_r = rainwater mixing ratio ($r > r^*$)
N_c = cloud drop concentration
N_r = raindrop concentration
Cd = condensation
Df = diffusion
Ac = activation
Sd = sedimentation
Ev = evaporation
Sc = selfcollection

A (= autoconversion):
cloud droplets + cloud droplets → raindrops

C (= accretion):
cloud droplets + raindrop → raindrops



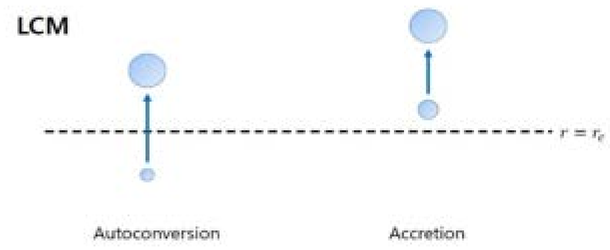
Parameterizations of Autoconversion (A) and Accretion (C)

Schemes	Autoconversion rate (kg m^{-3} s^{-1})	Accretion rate (kg m^{-3} s^{-1})
Kessler (1969)	$A = \alpha q_c H(q_c - q_{cT})$ ($\alpha = 10^{-3}$, $q_{cT} = 5 \times 10^{-4}$)	$C = \beta q_c q_r^{7/8} N_c^{1/8}$ ($\beta = 0.29$)
Tripoli and Cotton (1980)	$A = \alpha q_c^{7/3} N_c^{-1/3} H(R - R_T)$ ($\alpha = 3268$, $R_T = 7$ μm)	$C = \beta q_c q_r$ ($\beta = 4.7$)
Beheng (1994)	$A = \alpha d q_c^{4.7} N_c^{-3.3}$ ($\alpha = 3.0 \times 10^{34}$, $d = 9.9$ for $N_c < 200$ cm^{-3}, $d = 3.9$ for $N_c > 200$ cm^{-3})	$C = \beta q_c q_r$ ($\beta = 6.0$)
Khairoutdinov and Kogan (2000)	$A = \alpha q_c^{2.47} N_c^{-1.79} \rho^{-1.47}$ ($\alpha = 7.42 \times 10^{13}$)	$C = \beta (q_c q_r)^{1.15} \rho^{-1.3}$ ($\beta = 67$)

N_c = the droplet concentration
q_c = cloud water mixing ratio, q_r = rain water mixing ratio

Calculation of Autoconversion and Accretion

LCM can calculate autoconvesion and accretion of individual droplets directly by capturing the moment at which r grows larger than r^*.



- collision equation

$$\frac{dM_n}{dt}\delta t = \sum_{m=1}^{n-1} W_m \frac{M_n}{W_n} P\left[\frac{K(r_n, r_m) W_n \delta t}{\Delta V}\right] - \sum_{m=n+1}^{N_P} W_n \frac{M_m}{W_m} P\left[\frac{K(r_m, r_n) W_m \delta t}{\Delta V}\right]$$

gain of mass from super-droplets with smaller W_m

loss of mass to super-droplets with larger W_m

At every collision event ($r_n \rightarrow r_n'$)

$$\Delta M_{mn} = W_n \frac{M_m}{W_m} > 0$$

$\rightarrow A, C, S$ depending on r_n, r_n', and r_m

*A = autoconversion, C = accretion, S = self-collection

TABLE 1. Grouping of collision event to autoconversion, accretion, and self-collection (○: raindrop; ×: cloud droplet).

r_m	r_n	r'_n	Group	
×	×	×	Self-collection	
×	×	○	Autoconversion	$A = M_n + \Delta M_{mn}$
×	○	○	Accretion	$C = \Delta M_{mn}$
○	×	○	Accretion	$C = M_n$
○	○	○	Self-collection	

A and C occur at a very small fraction of grids during Δt (= 0.2 s)

$$\rightarrow \qquad A(q_c) = \frac{1}{N_{qc}} \sum_{i=1}^{N_{qc}} A_i \qquad\qquad C(Q_{rc}) = \frac{1}{N_{Qcr}} \sum_{i=1}^{N_{qc}} C_i$$

* $A_i(C_i)$ = autoconversion (accretion) at each grid point

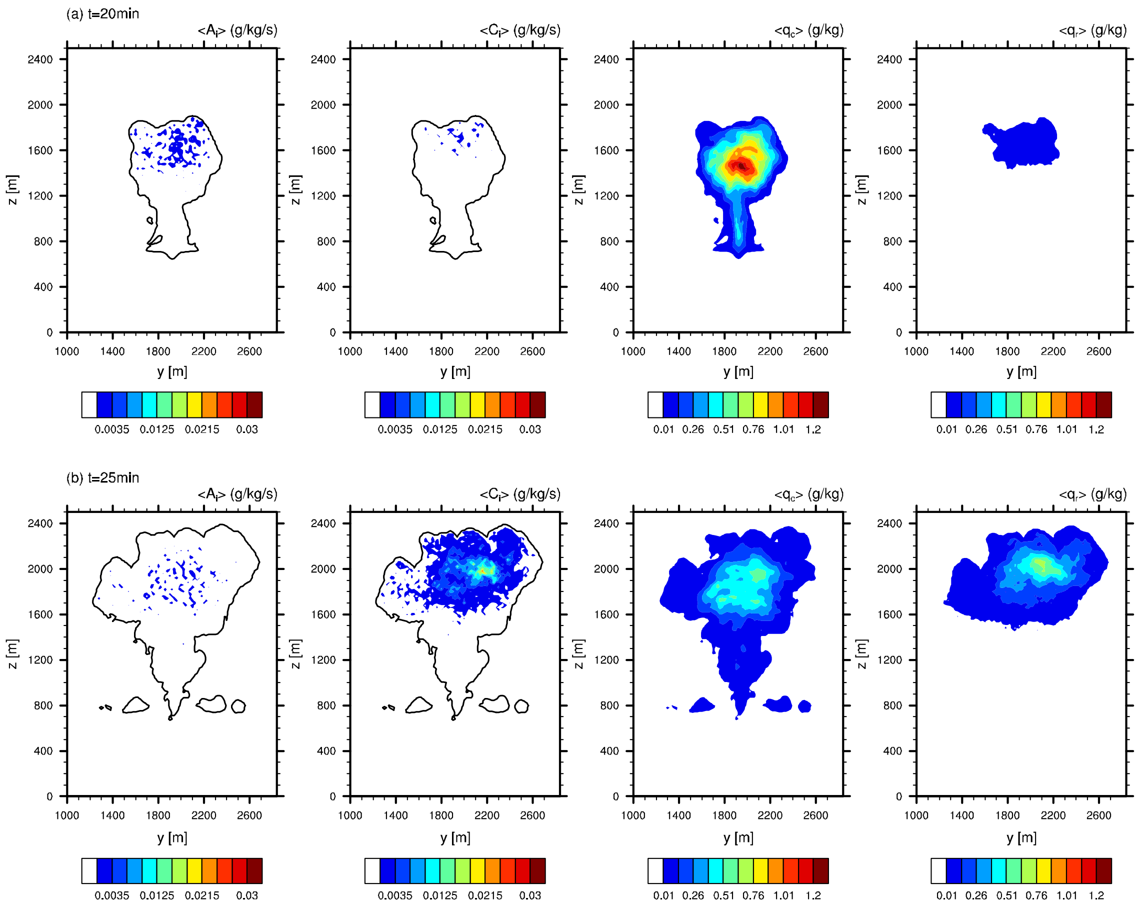
$N_{qc}(N_{Qrc})$ = number of grids with $q_c(Q_{rc})$ $\leftarrow Q_{cr} = q_c q_r$

SBM	LCM
- grid-averaged - continuous, deterministic	- following Lagrangian particles - intermittent, stochastic

* Calculation of A and C in the SBM

$$A = \int_0^{x^*} \left[\int_{x^*-x}^{x^*} K(x, x')x' n(x') dx' \right] n(x)\, dx \qquad C = \int_{x^*}^{\infty} \left[\int_0^{x^*} K(x, x')x' n(x') dx' \right] n(x)\, dx$$

$$x^* = (4/3)\rho \pi r^{*3}$$



t = 20 min

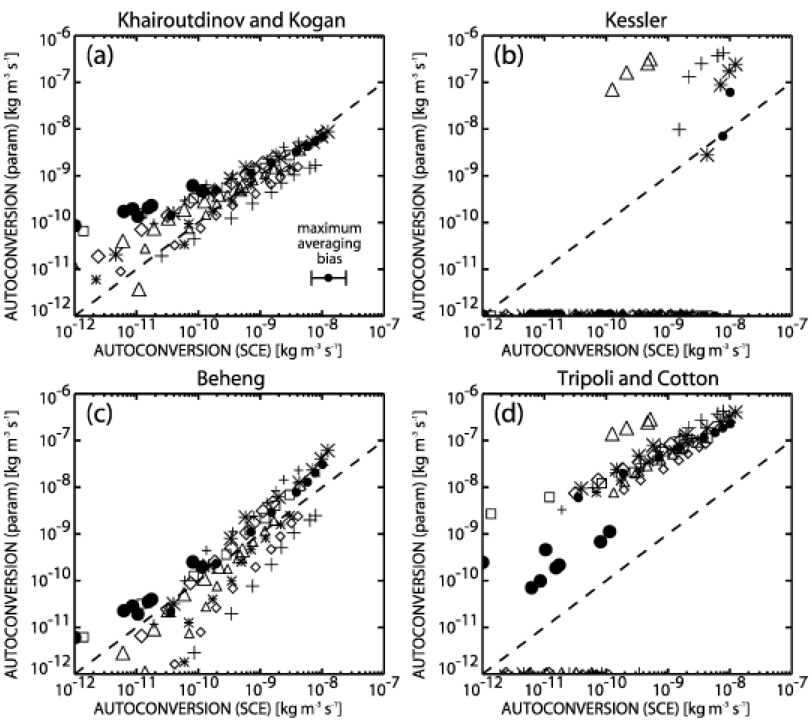
t = 25 min

A C q_c q_c

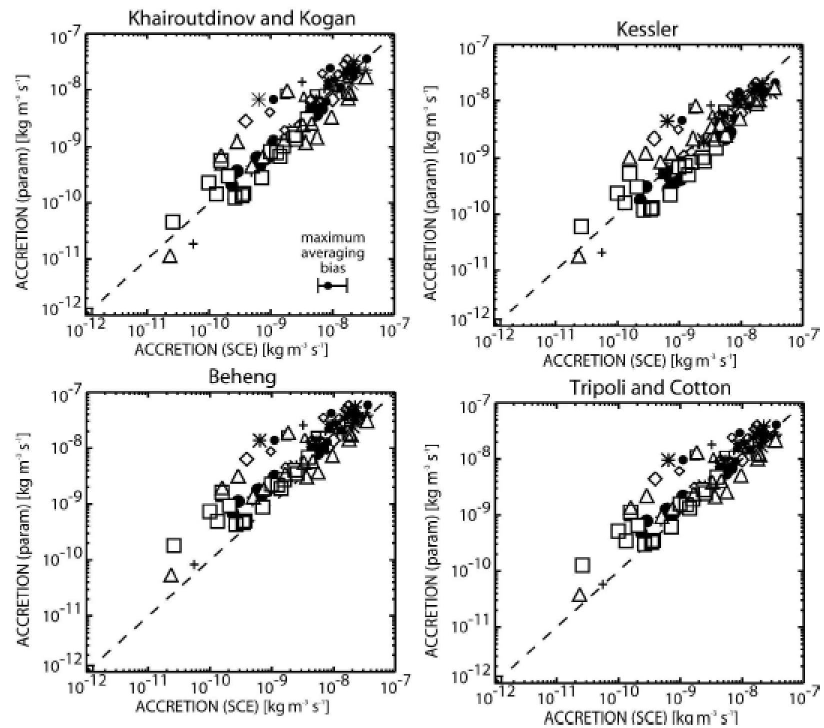
Distributions of A and C, q_c, and q_r averaged in the y-direction

Comparison of Parameterizations of A and C with SBM

- LES - Khairoutdinov and Kogan (2000), Kogan (2013)
- Observed DSD - Wood (2005), Hsieh et al. (2009)
- Idealized DSD - Seifert and Beheng (2001), Frankin (2008)



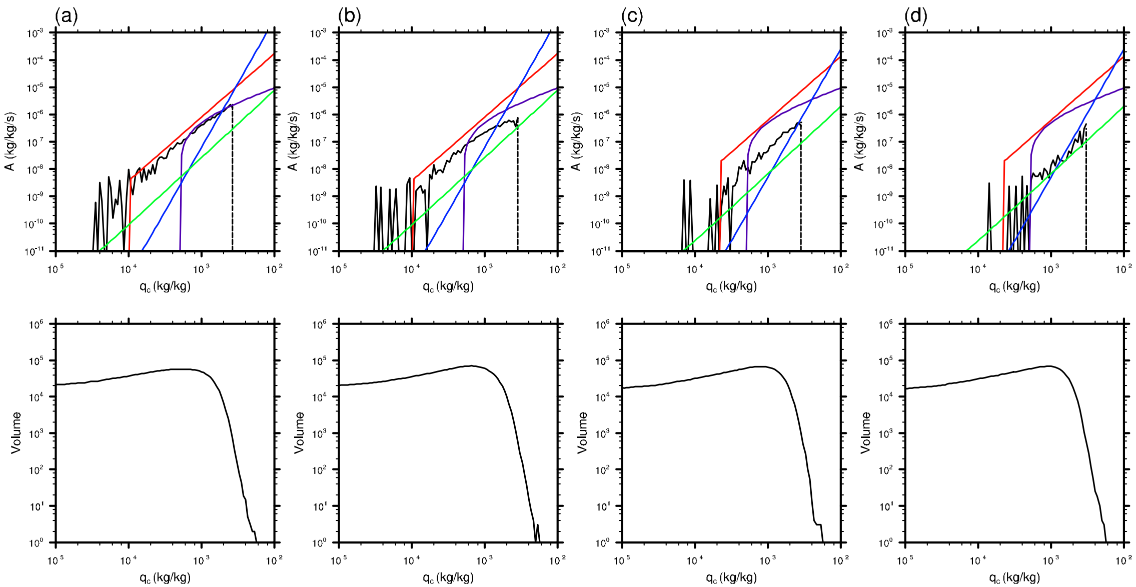
Autoconversion



Accretion

Wood (2005) – scatter plots between SBM results and parameterization

Comparison of Autoconversion from LCM with Parameterizations.



$$A(q_c) = \frac{1}{N_{qc}} \sum_{i=1}^{N_{qc}} A_i$$

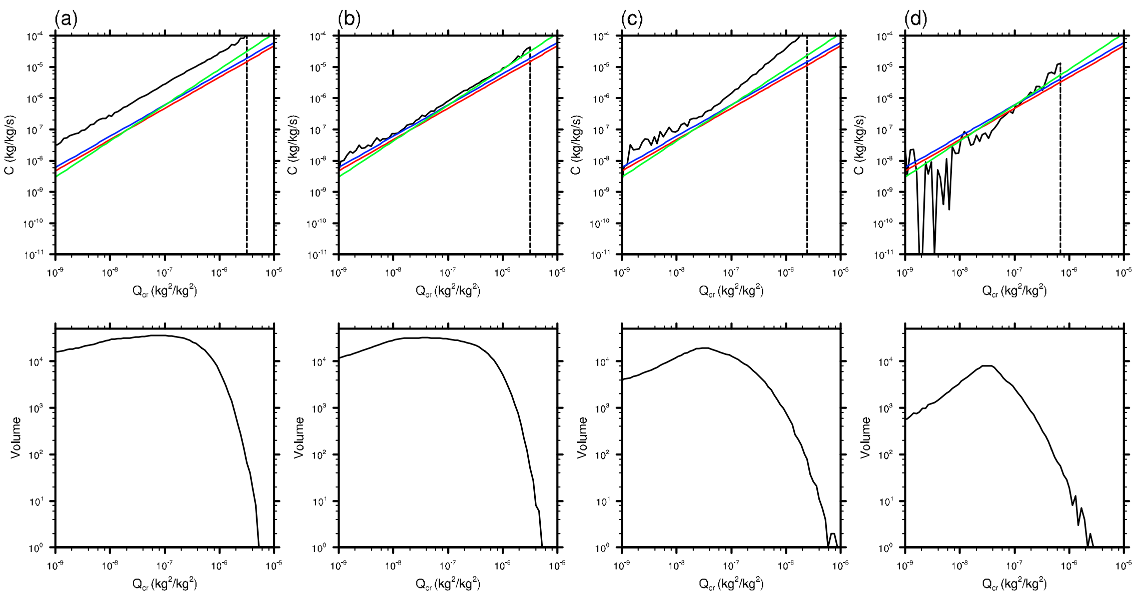
Variation of A (upper) and N_{qc} (lower) with q_c

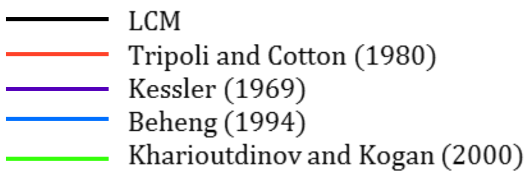
(a, c: TURB; b, d: GRAV; a, b: N = 70; c, d: N = 150)

- The Kessler type autoconversion is verified ($A \sim q_c^n H(R - R_T)$)
- The variation pattern generally follows the Tripoli and Cotton scheme ($A \sim q^{7/3}$), although α and R_T are different.

Tripoli and Cotton (1980); $A = \alpha N^{-1/3} q_c^{7/3} H(R - R_T)$ $\quad$ $*R = (q_c / N)^{1/3}$

Comparison of Accretion from LCM with Parameterizations.





$$C(Q_{rc}) = \frac{1}{N_{Qcr}} \sum_{i=1}^{N_{qc}} C_i$$

Variation of C (upper) and N_{Qcr} (lower) with Q_{cr}

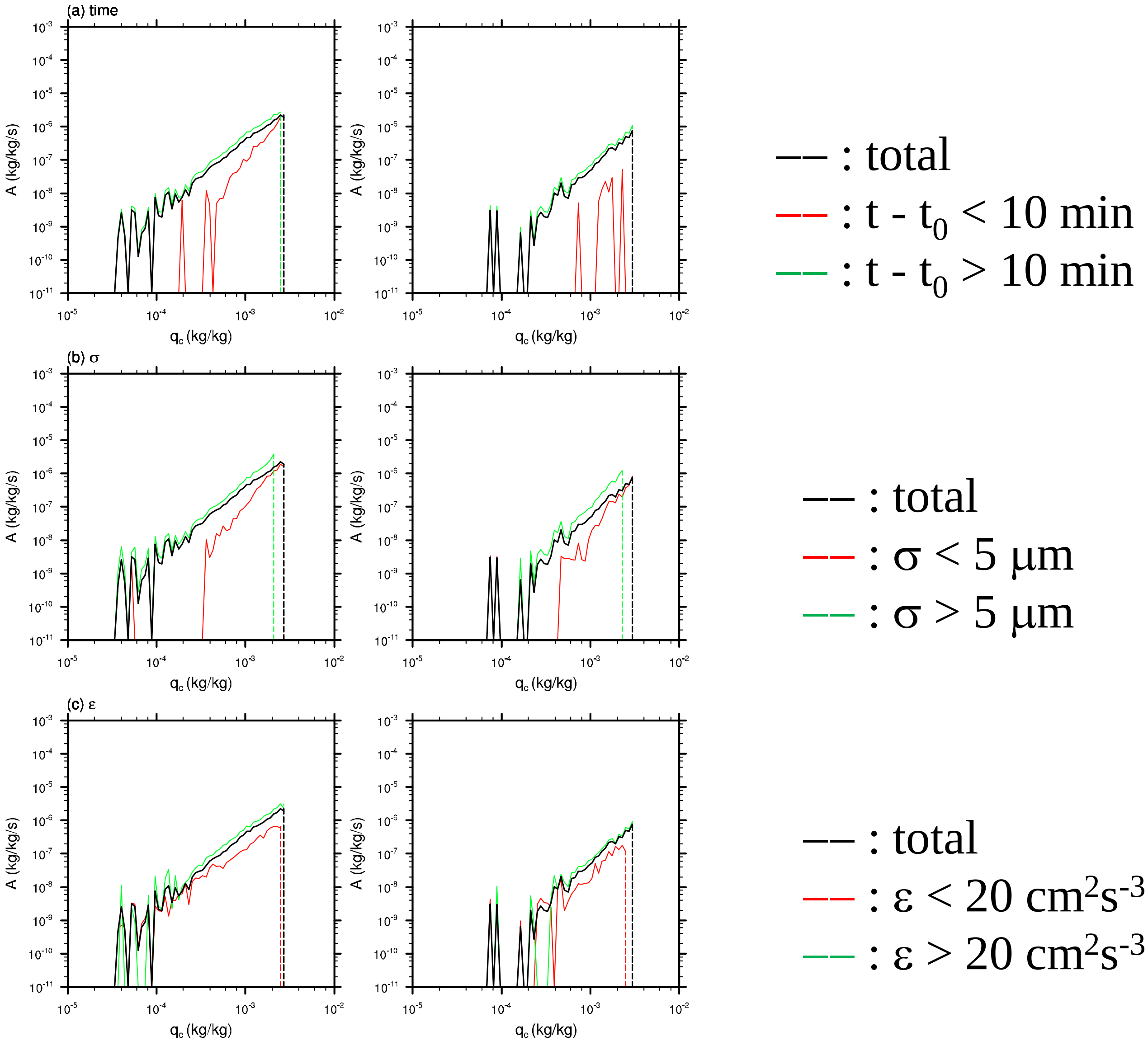
(a, c: TURB; b, d: GRAV; a, b: N = 70; c, d: N = 150)

Other Factors to Affect A

- Turbulence-Induced Collection Kernel Enhancement (TICE): ε
  - Seifert et al. (2010), Franklin (2008), Seifert and Onishi (2016)

- Dispersion of DSD: σ
  - Berry and Rheinhardt (1974), Beheng (1994), Liu and Daum (2004)

- Aging Period: $t - t_0$
  - Straka and Rasmussen (1974), Cotton and Anthes (1989)

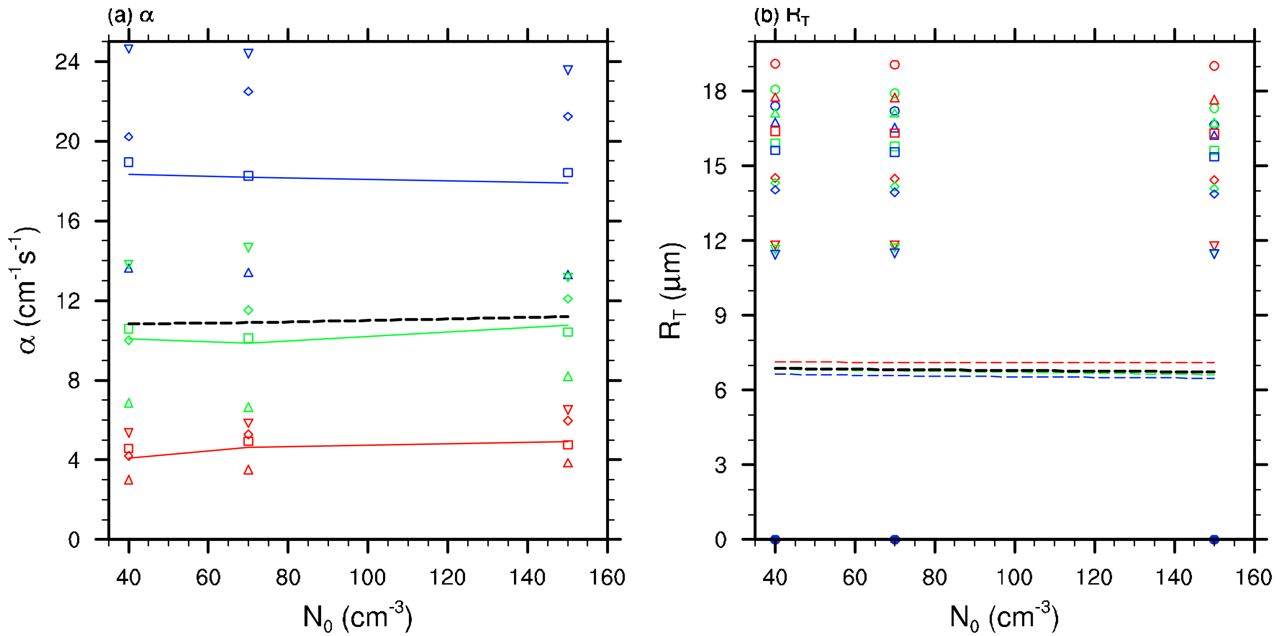


N_0 = 70 cm^{-3} N_0 = 150 cm^{-3}

Variations of *A* with q_c for different subgroups

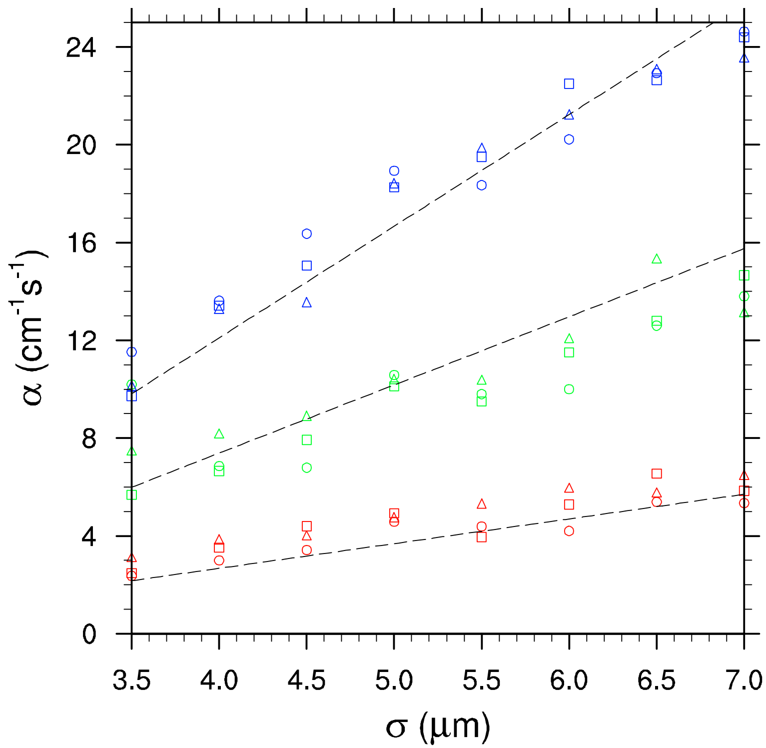
Box Collision Model Simulations

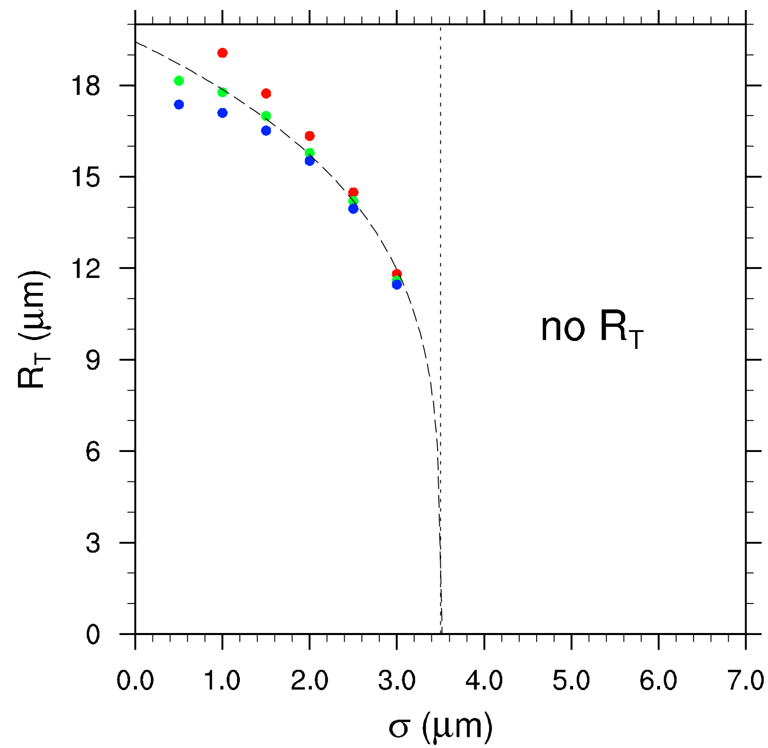
- only the collision algorithm
- starting with nog-normal DSD with N_0 (= 40, 70, 150 cm^{-3}), σ (= 0.5, .. 7 μm), and r_0 (= 1, .. 18 μm)
- ε = 0, 200, 400 cm^{2}s^{-3},



→ The TC parameterization $A = \alpha N_c^{-1/3} q_c^{7/3} H(R - R_T)$ is followed, although α and R_T vary widely.

*R_T is determined by the radius at which $A / N^{-1/3} q_c^{7/3}$ becomes smaller than 1/10 of that at r = 18 μm





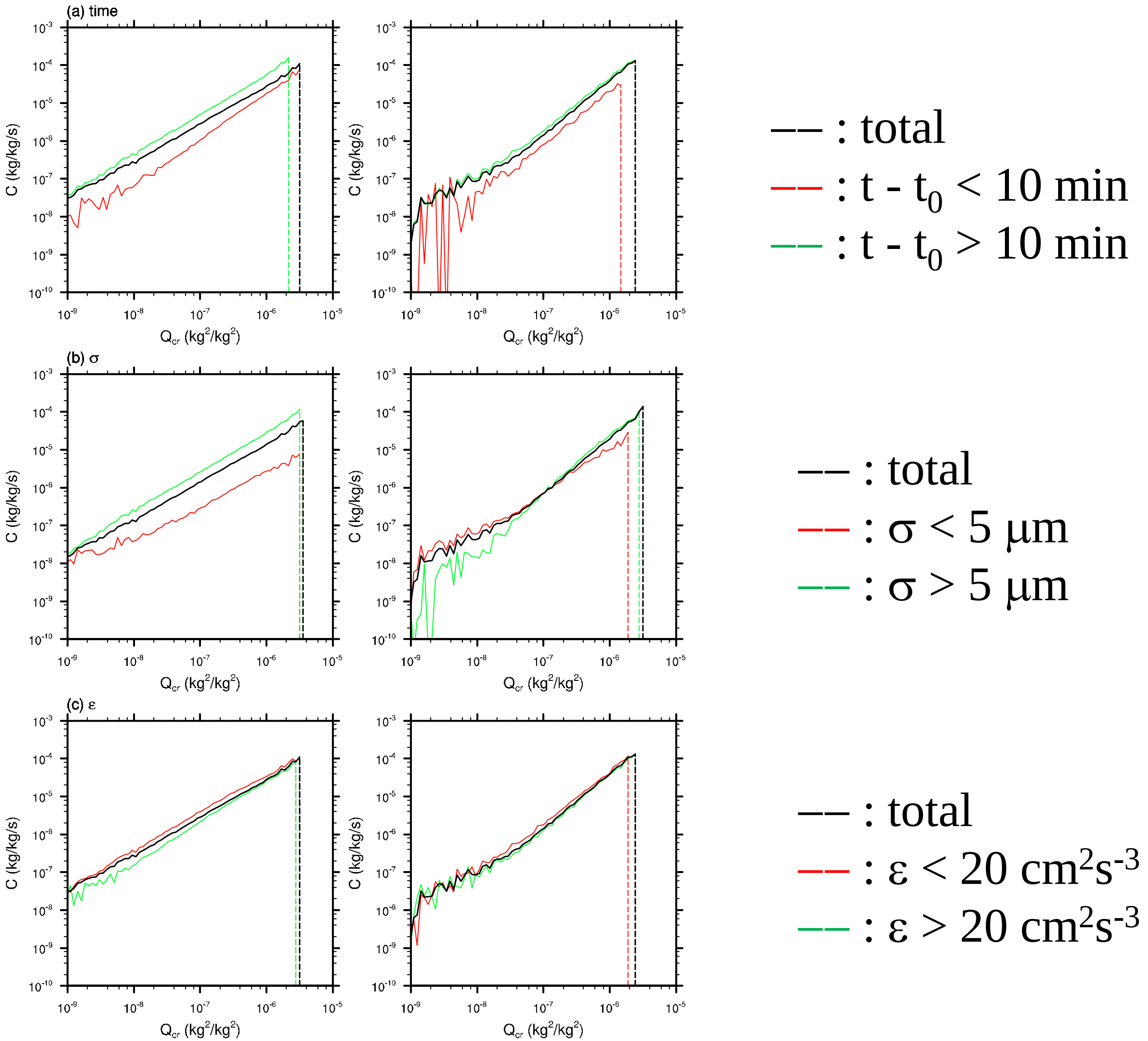
Variations of α with σ and ε

Variations of R_T with σ

(●: $\varepsilon = 0\ \mathrm{cm^2s^{-3}}$, ●: $\varepsilon = 200\ \mathrm{cm^2s^{-3}}$, ●: $\varepsilon = 400\ \mathrm{cm^2s^{-3}}$)

$$\alpha = a(\sigma - \sigma_\alpha)(1 + b\varepsilon)$$

$$\begin{aligned} R_T &= d_R^{1-m}(\sigma_R - \sigma)^m, \quad && \text{if } \sigma < \sigma_{R-} \\ &= 0\,, && \text{if } \sigma \geq \sigma_R \end{aligned}$$



N_0 = 70 cm^{-3} N_0 = 150 cm^{-3}

Variations of C with q_c for different subgroups

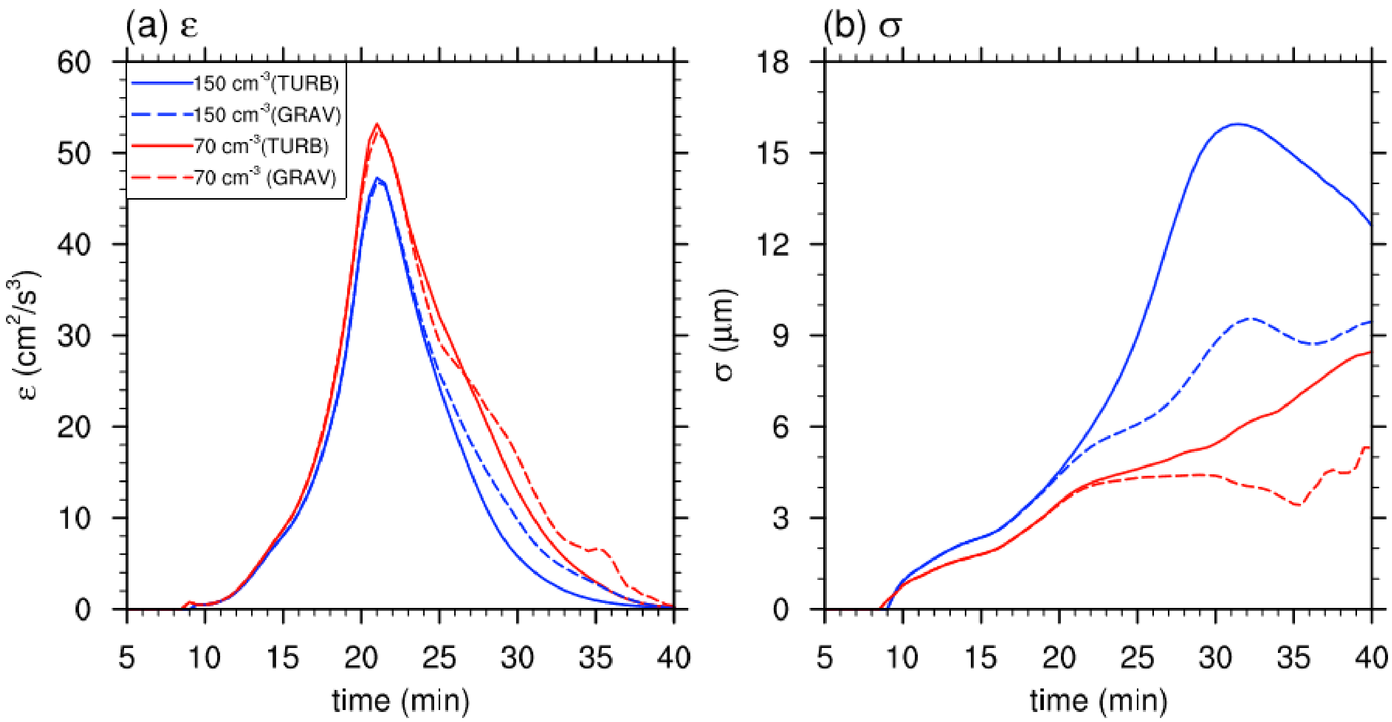
A New Microphysics Parameterization Based on LCM Results

- autoconversion: $A = \alpha N_c^{-1/3} q_c^{7/3} H(R - R_T)$

TC80	New Parameterization
$\alpha = 3268$ $R_T = 10\ \mu\text{m}$	$\alpha = a(\sigma - \sigma_\alpha)(1 + b\varepsilon)$ $R_T = d_R^{1-m}(\sigma_R - \sigma)^m, \quad \text{if } \sigma < \sigma_{R-}$ $\quad = 0, \quad \text{if } \sigma \geq \sigma_R$ $\sigma = \sigma(t - t_0, N_0)$ $\varepsilon = \varepsilon(t - t_0)$ (a = 1.0 $cm^{-1}\mu m^{-1}$, b = 8.0 x 10^{-3} $cm^{-2}s^{-3}$, σ_a = 1.35 μm)

- accretion: $C = \beta q_c q_r$

TC80	New Parameterization
$\beta = 4.7$	$\beta = 6.3 - 29.0$



Both ε and σ increase with time in the early stage.

* Cotton and Anthes (2001)

- The 'aging period' is necessary to commence autoconversion in order to avoid the early production of rain water too low in the cloud.

Conclusion

• Autoconversion and accretion are calculated directly from LCM results.

• Various parameterizations of autoconversion (A) and accretion (C) are evaluated. (Kessler (1969), Tripoli anc Cotton (1980), Khairoutdinov and Kogan (2000), Beheng (1994))

• The Kessler-type autoconversion parameterization is verified for the first time.

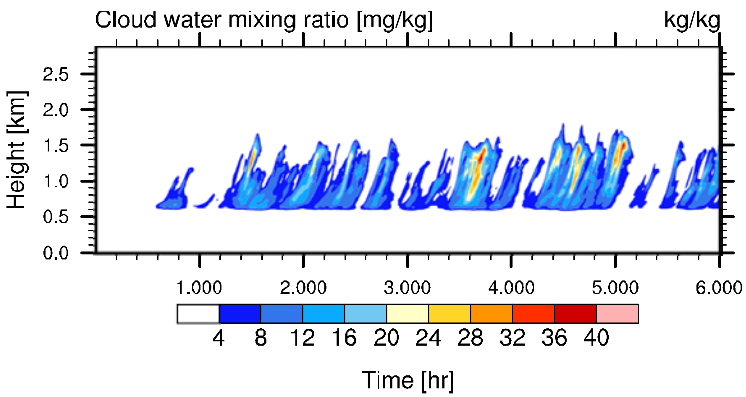
• The effects of DSD (σ), turbulence (ε), and aging on A are clarified.
$\rightarrow$ Applied to improve the autoconversion parameterization by Tripoli and Cotton (1980).

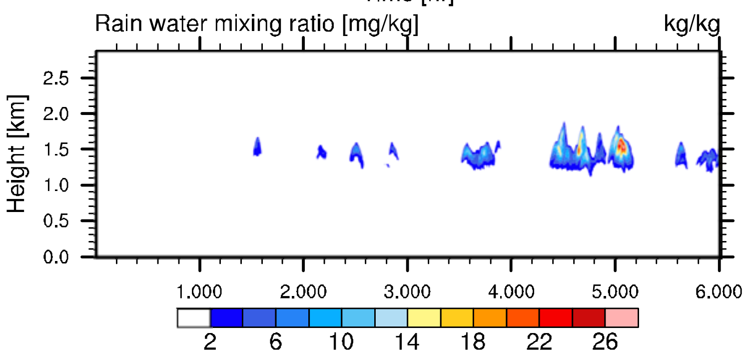
- The aging period is realized to prevent too early precipitation.

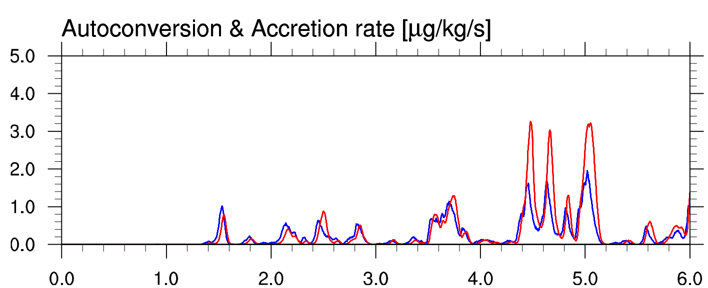
- The variation of σ and ε may depend on cloud dynamics, and should be parameterized.

3. Development of Cloud Microphysics Parameterization Based on LCM

Cloud Field Simulation (BOMEX)

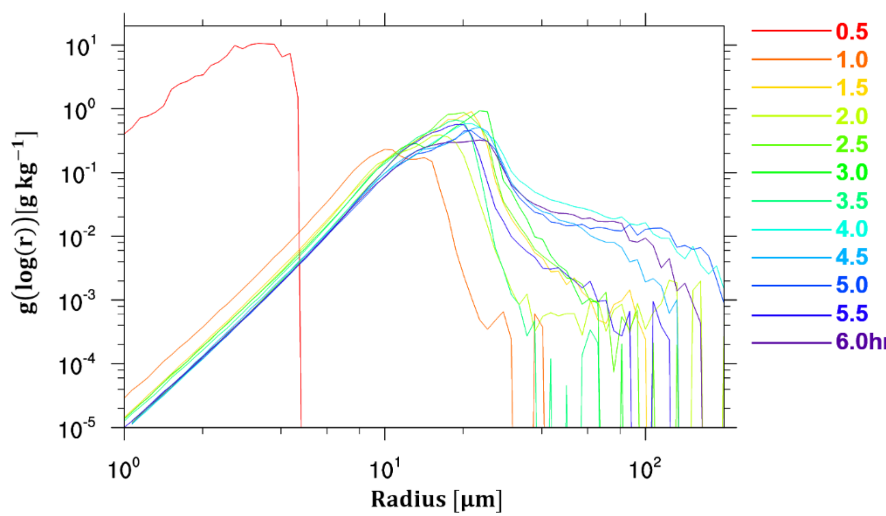




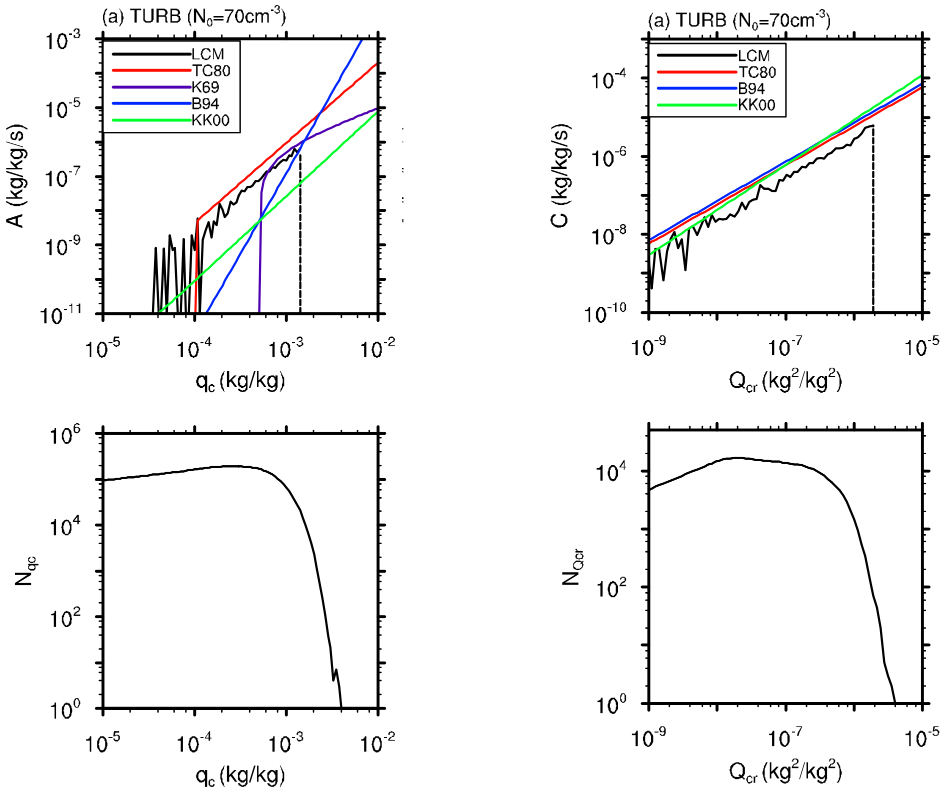


Simulation setup

- Domain size: $2.4km \times 2.4km \times 2.88km$ $(\Delta x = \Delta y = \Delta z = 20\mathrm{m})$
- Timestep: $0.2s$
- Initial particle size: $0.01\mu m$
- Initial droplet concentration: $70cm^{-3}$
- 100 superdroplet / grid



Comparison of A and C from LCM with Parameterizations.

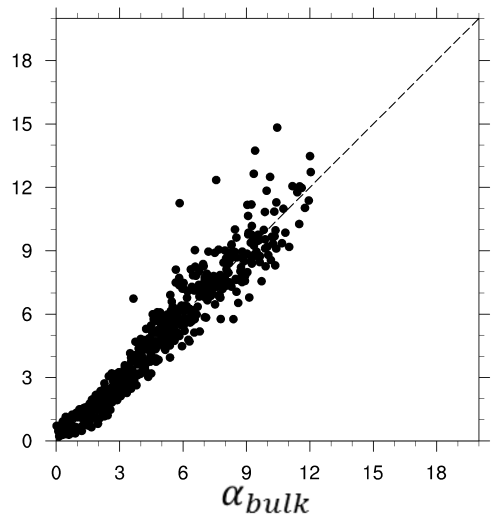


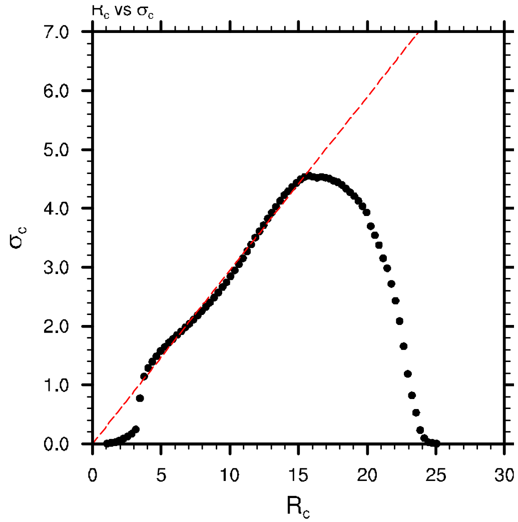
Tripoi and Cotton (1980);

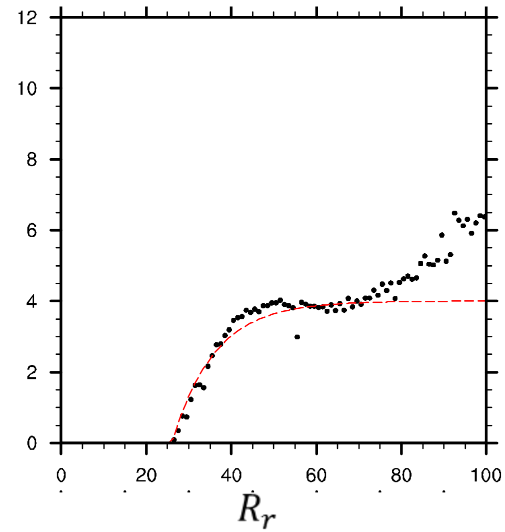
$$A = \alpha N_c^{-1/3} q_c^{7/3} H(R - R_T)$$

$$C = \beta q_c q_r$$

Analysis of LCM Cloud Field Simulation Data







$$\alpha = a(\sigma_c - \sigma_{c0})(1 + b\varepsilon) + \alpha_0$$

$$\sigma_c = cR_c$$

$$\beta = \beta_0\{1 - \exp[-\gamma(R_r - R_*)]\}$$

$$A = \alpha N^{-1/3} q_c^{7/3} H(R - R_T)$$

$$C = \beta q_c q_r$$

*Cohard and Pinty (2000), Lim and Hong (2010): $n_c(D) = (N_c / \Gamma(2))\lambda^2 D \exp(-\lambda D)$

$$\lambda \propto R_c^{-1}$$

Enhancement of A and C with σ (τ) and ε in SB

(Seifert and Beheng 2001, 2006; Seifert et al. 2010)

$$f_r(D) = \alpha D^{-\beta D} \qquad f_c(x) = A x^{\nu} e^{-Bx}$$

$$A = \frac{k_c}{20x^*}\frac{(\nu+2)(\nu+4)}{(\nu+1)^2} q_c^2 \overline{x}_c^2 \left[1 + \frac{\Phi_A(\tau)}{(1-\tau)^2}\right] \qquad \Phi_A = 600\tau^{0.68}(1-\tau^{0.68})^3$$

$$C = k_r q_c q_r \Phi_C(\tau) \qquad \Phi_C = \left(\frac{\tau}{\tau + 5\times10^{-4}}\right)^4$$

$$k_c = k_{c0}\left\{1 + \varepsilon \,\mathrm{Re}_\lambda^{1/4}\left[\alpha_{cc}\exp\left[-\left(\frac{\overline{r}_c - r_{cc}}{\sigma_{cc}}\right)^2\right] + \beta_{cc}\right]\right\}$$

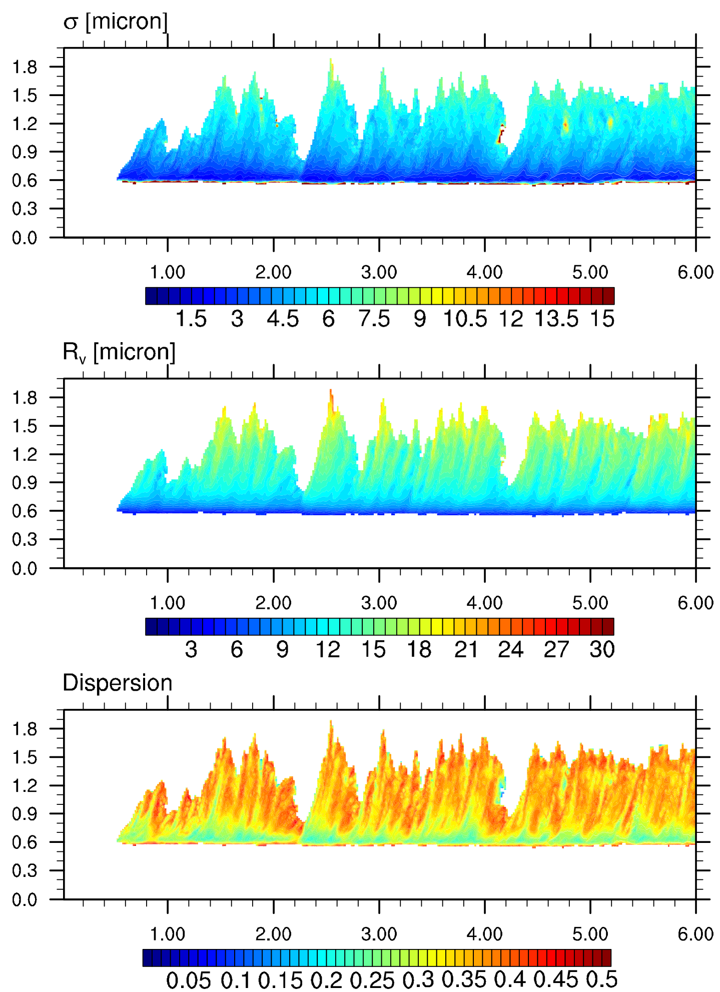
$$\tau = q_r/(q_r + q_c) \qquad \overline{x}_c \propto q_c / N_c$$

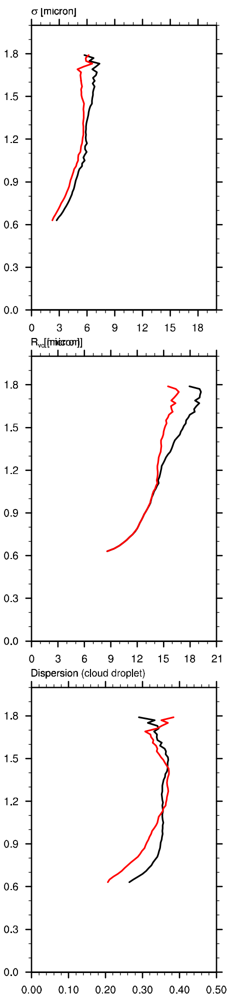
Relation between σ and R

σ

R

σ/R





(σ/R, σ_c/R_c)

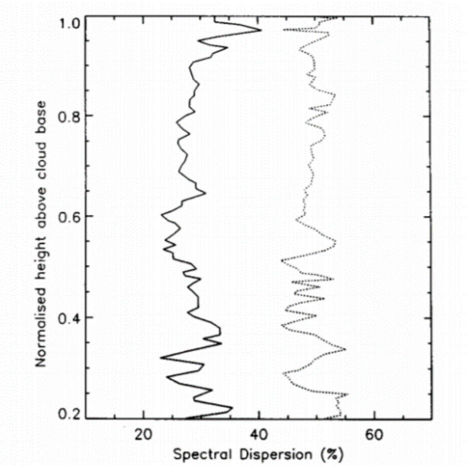
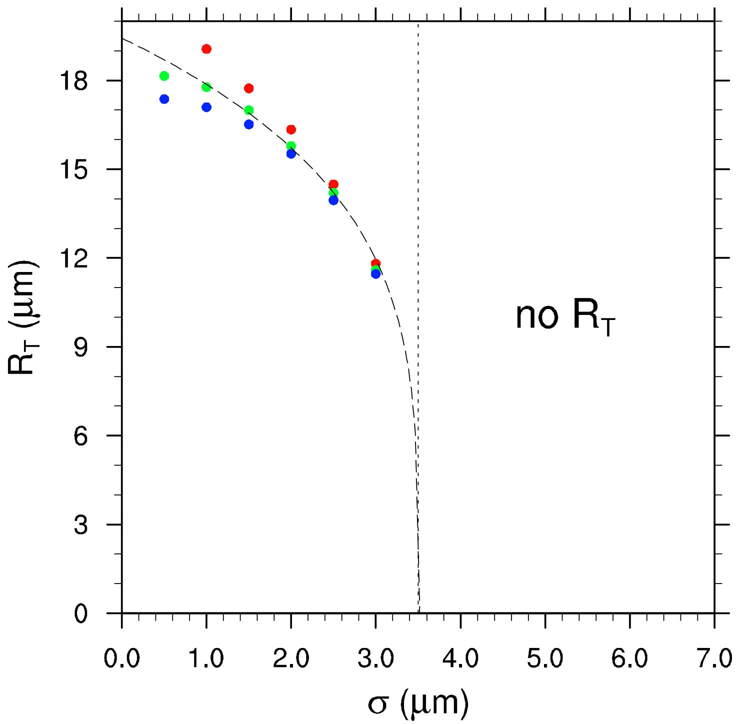


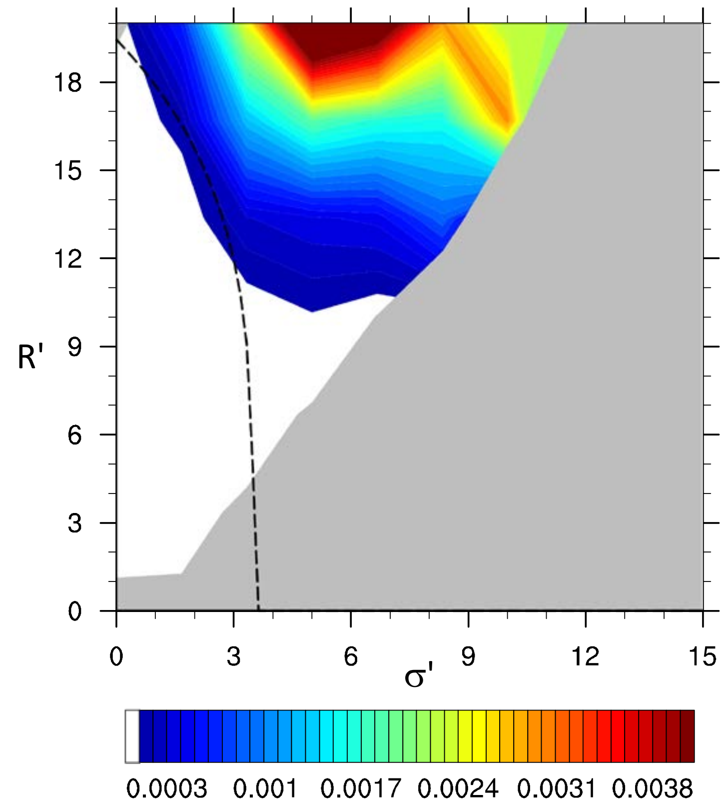
FIG. 4. A plot of spectral dispersion (d) against normalized height in stratocumulus cloud in a maritime air mass (solid line) during FATE on 6 November 1991 and a continental air mass (dashed line) over the North Sea on 26 February 1991.

(Martin et al. 1994)

Box Model



Cloud Field Simulation



$$P(R',\sigma') = \frac{N_{auto}(R',\sigma')}{N(R',\sigma')}$$

$$R_T = d_R^{1-m}(\sigma_{cR} - \sigma_c)^m, \quad \text{if } \sigma < \sigma_R$$
$$= 0, \quad \text{if } \sigma \geq \sigma_R$$

, if R_T and σ_c are independent.

But $\sigma_c = cR_c \quad \rightarrow \quad R_T = 2.5\ \mu\text{m}$

A New Microphysics Parameterization Based on LCM Results

- autoconversion: $A = \alpha N^{-1/3} q_c^{7/3} H(R - R_T)$

Parameterization	Constants
$\alpha = a(\sigma_c - \sigma_{c0})(1 + b\varepsilon) + \alpha_0$ $R_T = d$ $\sigma_c = cR_c$	a = 0.91 $cm^{-1}\mu m^{-1}$ b = 0.081 $s^3\ cm^{-2}$ σ_{c0} = 0.37 μm α_0 = 3.83 $cm^{-1}\mu m^{-1}$ c = 0.029 d = 2.5 μm

- accretion: $C = \beta q_c q_r$

Parameterization	Constants
$\beta = \beta_0\{1 - \exp[-\gamma(R_v - R_*)]\}$	β_0 = 4.0 γ = 0.01 μm^{-1}